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On the Applicability of Time-Series Models for Structural Reliability Analysis

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Abstract. The assessment of time-dependent reliability problems is still a challenging task. Besides the difficulty to characterize a problem from real-world data, most of known solutions rely on approximations suitable only for specific cases or on burdensome simulation approaches. This is due to the difficulty in working with general stochastic processes, particularly for situations of non-ergodicity. A time-series model is a particular case of stochastic process that operates in continuous state space and discrete time set. Such models can be used to represent a wide range of random phenomena that spans through time, usually with simpler formulation. They are also relatively simple to build from data tables, which are usually all the information available about time-dependent behavior of random engineering systems. This work presents a preliminary study where data generated from continuous stochastic processes commonly used in structural reliability are used to build different time-series models, which are then used to replace the original stochastic process in reliability analysis. Auto Regressive, Moving Average and Auto Regressive Moving Average models are considered. The same time-dependent reliability problem is solved considering each case, and details about the solutions are addressed.

Keywords: Time-dependent Reliability, Time-series models, Structural Reliability

1 Introduction

Structural design is usually guided by normative codes, which in most cases assess structural safety via semi-probabilistic procedure. Although this suffices for most practical cases, such approach does not allow for failure probability quantification, an utmost important information in order to design optimal structures, whose safety is guaranteed at the same time that monetary costs and environmental impact are minimum. Time-independent structural reliability analysis provides a more general framework. Quantities describing structural parameters and load conditions are modeled as random variables, whose uncertainty is propagated trough mechanical models in order to quantify its output uncertainties (e.g. stresses, strains, displacements), so that failure probabilities can be calculated. Although the task can be demanding, several approaches have been proposed to efficiently address this problem (see for example Santos and Beck [1] and Kroetz et al. [2]). Despite being a relevant step forward, time-independent reliability disregards time-fluctuation in its parameters. Natural hazards usually span trough time with varying intensity, thus being better characterized by stochastic processes. Unfortunately, complete information about these phenomena is seldom available. In order to build a time-dependent reliability analysis, one usually has limited information, arising from point-wise measurements, for example, environmental conditions parameters obtained in a meteorological station.

In time-dependent reliability analysis, time-varying parameters are usually modeled as classical continuous stochastic processes (e.g. Gaussian and Wiener processes), whose sampling in general involves the spectral decomposition of auto-correlation matrices. On the other hand, fewer works about reliability point to a tentative of describing time-dependent random parameters using auto-regressive time-series models. These are particularly suited for modeling stochastic behavior from discrete data obtained in regular intervals of time, in this context called *lags*. As an advantage, auto regressive models are usually of simpler formulation, which allows for lower computational burden and could lead to easier derivation of future time-dependent reliability techniques. In this paper, we test the applicability of three different time-series models by solving a reliability problem with each model built from the same data set, and compare the results.

2 Time Series Analysis

Time series analysis is the employment of statistical techniques in order to understand the behavior of a variable from a set of observations on the values that it assumes at different times. In time series analysis, time-dependent behavior of a system is characterized and separated from its random fluctuation, so that one can exploit the knowledge of each part in order to forecast its future behavior. In this sense, it can be understood as a bivariate analysis where time is the independent variable [3].

Although technically any stochastic process can model a time-series, the term ”time-series model” is usually employed in reference to econometric models that deal with a few past realizations of the variable of interest to predict its future values. Such models have been used in different applications regarding reliability in the works of Walls and Bendell [4], Ho and Xie [5], Billinton and Wangdee [6], Li et al. [7], among others. It can be concluded that time-series models are adequate for reliability analysis. Despite that, none of these works deal with the computation of small values of failure probability. This is a key aspect in structural reliability, since many problems still require the usage of Monte Carlo simulation. It is well known that the main drawback of this technique is the excessive computational burden associated with low order of magnitude failure probabilities. An early work by Mignolet and Spanos [8] have addressed the potential applications of such models in structural reliability analysis, concluding that autoregressive moving average models can be suitable for describing time-dependent loads caused by phenomena like ocean waves and earthquakes. To the best knowledge of the authors, literature lacks a study about the impacts of replacing classical continuous stochastic processes by time series models in structural analysis.

3 Time Series Models and Prediction

3.1 Autoregressive model (AR)

One of the most simple yet powerful tools in time series analysis is the representation of a variable of interest through autoregressive models. As the name suggests, it consists in forecasting a variable considering its own past values, or performing a regression of the variable of interest against itself. [9]. The model degree p represents the number of lags considered in the regression (i.e. the number of past values considered, spaced by the constant amount of time called *lag*). A general autoregressive model of degree p is represented by AR(p), and can be written as shown in Equation 1:

$$l_t = \beta_0 + \beta_1 l_{t-1} + \beta_2 l_{t-2} + \cdots + \beta_p l_{t-p} + \epsilon_t, \tag{1}$$

where i counts the lags, l_t is the current value of the variable of interest l, β_i is a constant for $i = 0$ and represents the partial autocorrelation between l_t and l_{t-i} for $i > 0$ and ϵ_t is the unpredictable error in current prediction, usually modeled as white noise. Since each lag term will have different impact on the prediction of current term, a reasonably good prediction can be built disregarding the lags whose partial autocorrelation are low (arbitrarily close to zero). Several techniques can be used to determine the AR model's coefficients, including Burg's lattice-based method and variations [10] and Yule-Walker estimates [11]. In this work, least-squares regression is adopted.

3.2 Moving average model (MA)

Another useful model of simple general expression is the moving average model, represented by MA(q), where q is the model degree [12]. It consists in representing the value of a time-dependent variable by weighting the errors of previous time steps, so that predicted value deviates from the process's mean by a linear combination of the q previous errors. In this sense, the error is the difference between the true value of a random process in an instant and its correspondent prediction. In MA forecasting, the error term is modeled as white noise. A general expression for MA(q) is given by Equation 2:

$$l_t = \mu + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_q \epsilon_{t-q} + \epsilon_t, \tag{2}$$

MA(q) models in general present non zero autocorrelation for the first q lags, whilst zero for other lags. Thus, observing the correlogram of the data series from which the model will be built can provide useful information about how many terms it should have. As a rule of thumb, one can choose de model's degree as the number of terms where the correlation function is sufficiently large, neglecting terms where the correlogram is close to zero. The estimation of MA(q)'s coefficients classically involves a transformation from the original model to a correspondent AR(∞) model, which is then truncated and has its coefficients determined by one of the aforementioned techniques. The MA(q) are then estimated considering the relationship between both models. In this work, parameter estimation is performed automatically with the help of matlab ARIMA model from its Econometric toolbox.

3.3 Autoregressive moving average model (ARMA)

An ARMA model is a combination of an autoregressive model and a moving average model, so that both past values of the variable of interest and past prediction errors are combined in prediction. The adopted notation is inherited from individual models: an ARMA(p, q) is composed by an autoregressive model of degree p and a moving average model of degree q. A general expression is given by Equation 3:

$$l_t = \beta_0 + \beta_1 l_{t-1} + \beta_2 l_{t-2} + \cdots + \beta_p l_{t-p} + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_q \epsilon_{t-q} + \epsilon_t \tag{3}$$

The order of each part of an ARMA model is usually chosen using the Bayesian Information Criterion, which is basically an adaption from maximum likelihood estimation, where a modified likelihood function is maximized for each of several candidate models with different number of parameters, so that the best one is chosen [13]. This leads to the construction of L^2 candidate models, where $L = p_{max} + q_{max}$ is the sum of the maximum admitted degree for each part of the model. This could lead to excessive evaluations, and more simplistic approaches are often adopted, for example, choosing the optimum model only from a subset where $p = q$. This assumption usually has little influence on the average quality of selected ARMA models, since the loss caused by not having the best ARMA(p, q) model between the candidates is compensated by the fact that selection from a small number of candidates is more accurate [14]. Hence, the latter approach is adopted in this work.

4 Time-dependent Reliability Analisys

In most structural reliability problems uncertainties are modeled as random variables, so that a time-independent analysis is performed in order to determine structural probabilities of failure. However, when loads and structural integrity vary significantly in time (e.g. due to strength degradation or structural repair), it is convenient to study the evolution of failure probabilities in time. In this context. a *cumulative probability of failure* $P_{fc}(t_1, t_2)$ is considered to compute the probability of violating a certain limit at any point of a given time interval $[t_1, t_1]$:

$$P_{f_c}(t_1, t_2) = \mathbb{P}\left(\exists \tau \in [t_1, t_2] : g(\tau, \boldsymbol{X}(\tau, \omega)) \leq 0\right). \tag{4}$$

In this notation g is a limit state function of the random vector $\boldsymbol{X}$. Although several analytical techniques have been proposed in order to compute P_{fc}, they are mostly very limited or problem-specific (see Kroetz et al. [15] for details). Thus, a general time-dependent Monte Carlo approach is adopted herein. Consider a time interval of interest discretized in N points. Realizations of time-dependent variables, obtained from measurements or random process discretization, are then used to compute a given limit state function for each point in time. Let these values be gathered in an array G of length N. In this sense, deterministic quantities are constant throughout G, and so are random variables after being sampled once. Thus each position $i = 1, ..., N$ in G stores the value of a limit state function in time $t_i = (i-1) \cdot \Delta t$, where $\Delta t = \frac{\mathcal{T}}{N-1}$ is the sampling step. For each time interval $[t_i,\ t_{i+1}]$, a counter k_{i+1} is defined. Let k be an array whose position k_{i+1} stores a failure counter that refers to the interval $[t_i,\ t_{i+1}]$, so that all counters k_n, with $n = i+1, \ldots, N$ are increased whenever the limit state is violated for the first time (i.e. all the remaining counters after the outcrossing are increased). A brute Monte Carlo estimation for the cumulative probability of failure until an arbitrary instant t_i, i.e. $P_{fc_{MC}}(0, t_i)$, is given by:

$$P_{fc_{MC}}(0, t_i) = \frac{k_i + k_0}{N_{MC}}, \tag{5}$$

where k_0 counts the number of failures at $t = 0$. Since this general approach relies only in point-wise evaluation of limit states, any time-series model can be used as input, allowing for the comparison between performance of different time-series models.

5 Comparison of Time Series Models

Consider the time-dependent reliability problem of a degrading steel bending beam, whose length is $L = 5m$ and rectangular cross-section (b_0, h_0). This beam is submitted to dead loads $\rho_{st} = b_0 h_0 (Nm^{-1})$, as well as a pinpoint load F applied at midspan (See Figure 1). This example is based on the problem proposed by Sudret [16].

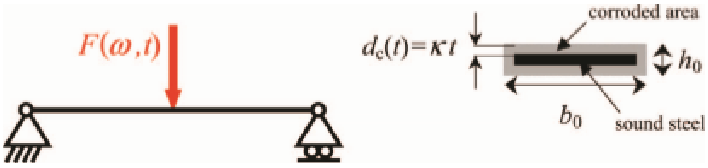


Figure 1. Corroded beam under a midspan load

The structure is subject to corrosion, in such a way that corrosion depth d_c all around the section increases linearly in time ($d_c = \kappa t$). It is assumed that the corroded areas have lost all mechanical stiffness. The limit state function associated with the formation of a plastic hinge is described by Equation 6:

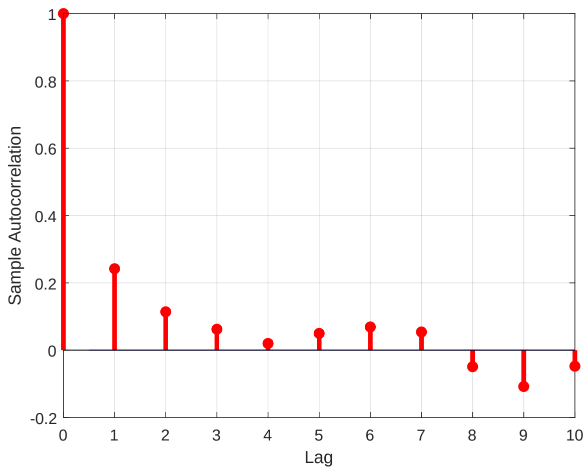
$$g(,t,\mathbf{X}) = \frac{(b_0 - 2\kappa t)(h_0 - 2\kappa t)^2 f_y}{4} - (\frac{FL}{4} + \frac{\rho_{st} b_0 h_0 L^2}{8}), \quad (6)$$

where the random variables are detailed in Table 1, and F is a time-variant load described by a Gaussian stochastic process of mean $\mu = 6kN$ and coefficient of variation $cov = 0.3$.

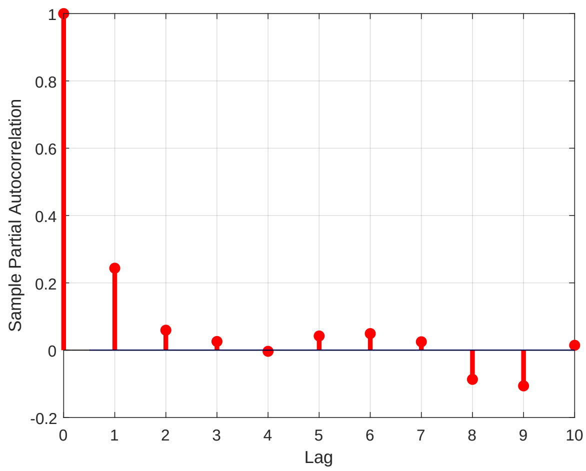
Table 1. Corroded bending beam – random variables and parameters

Parameter	Distribution	Mean	COV
Steel yield stress (f_y)	Lognormal	240MPa	10%
Beam breadth (b_0)	Lognormal	0.2m	5%
Beam Height (h_0)	Lognormal	0.04m	10%

Consider now that F is an environmental load, and instead of actually knowing its properties, all there is available is a historical series of intensity measurements. In this example, a table is built by monthly discretization of the original process via the EOLE method [17] for a 10 years time span. The information obtained from a single sample is used as input for constructing the time-series models. After sampling, the autocorrelation (ACF) and partial autocorrelation (PACF) functions are plotted in correlograms for the first ten lags, as shown in Figure 2.



(a) ACF for first 10 lags



(b) PACF for first 10 lags

Figure 2. Autocorrelogram and Partial Autocorrelogram for old sample.

A qualitative analysis of the correlograms suggests that a 2^{nd} order MA model could be adequate, since lags have smaller autocorrelations after that. The same conclusion can be drawn for the AR model from the study of the partial autocorrelation function. Thus AR(2), MA(2) and ARMA$(2, 2)$ model are used in this example. The problem is solved as described in Section 4 for each constructed model, and also considering the original process to serve as reference. In all cases, 10^6 Monte Carlo samples are used. Results are gathered in Table 2 and illustrated in Figure 3.

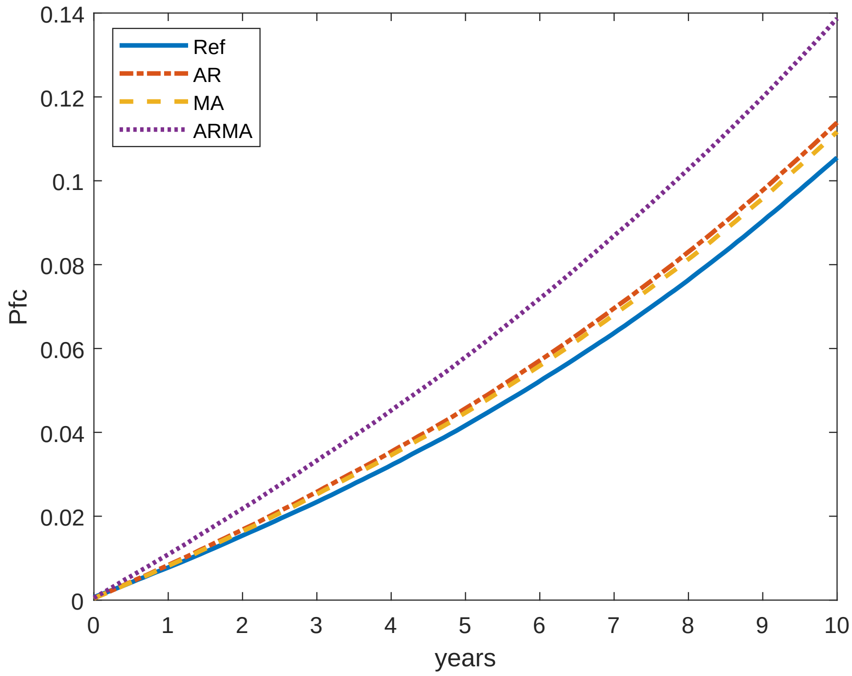


Figure 3. Cumulative failure probabilities for models built from a single sample

Table 2. Results for single sample

Model	P_{fc}	error
Ref	0.1055	–
AR(2)	0.1139	7.9%
MA(2)	0.1117	5.9%
ARMA(2,2)	0.1387	31%

Although the general behavior of failure probability evolution seems to have been captured by the models, a considerable error is observed in simulations. This does not necessarily mean that the models are bad, since they could be adequately representing information from a sample that itself is not a good representation of the original stochastic process (the sample presented e mean of 6105N and a standard deviation of 1788N). In order to test that, more samples were taken, until a sample whose mean and standard deviation significantly close to the original was obtained (errors not larger than 0.1% were admitted). Rebuilding the models considering a better quality sample, results were significantly improved, as can be seen in Table 3 and Figure 4.

6 Conclusion

In this work AR, MA and ARMA time-series models have been tested in the context of structural reliability. The models were employed in a benchmark problem whose original solution involves classical expansion of random process discretization. Accurate results were obtained, particularly when simulated empirical data reflected the underlying stochastic process characteristics. Results suggest that time-dependent structural reliability problems can be treated with simpler models than classically employed continuous stochastic processes. Although

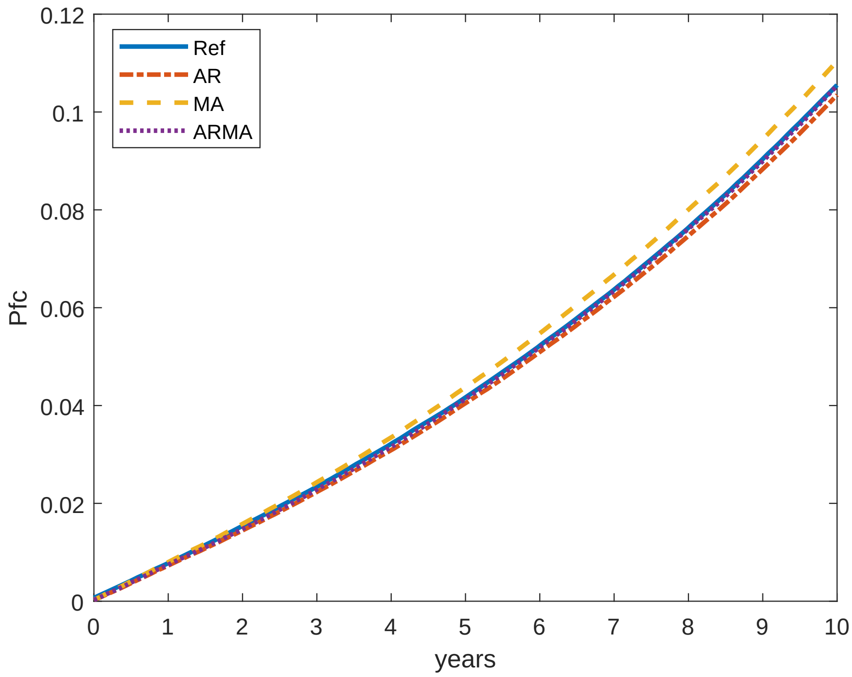


Figure 4. Cumulative failure probabilities for models built from good quality sample

Table 3. Results for better quality sample

Model	P_{fc}	error
Ref	0.1055	–
AR(2)	0.1035	1.9%
MA(2)	0.1104	4.6%
ARMA(2,2)	0.1054	0.1%

errors tend to accumulate in time, a larger data set tends to hold more representative mean and standard deviations, what leads to smaller errors. Since general time-dependent reliability problems must rely on burdensome Monte Carlo simulation, the usage of simpler models may help in the future not only in reducing computation time, but also in the development of new solution approaches.

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