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Matching numerically a rheological model to the Eurocode creep through dynamic analysis

Alexandre de Macêdo Wahrhaftig[1], Kaique Moreira Matos Magalhães[2]

[1,2]*Department of Construction and Structures, Federal University of Bahia (UFBa)*
Rua Prof. Aristídes Novis, nº 2, 5º andar, Federação, CEP:40210-91, Salvador, BA, Brazil
[1]*alixa@ufba.br,* [2]*kaique.magalhaes@ufba.br*

Abstract. A dynamic analysis of vibration for considering a three-parameter rheological model to fit the same results as predicted for creep by the Eurocode (EN 1992) criteria is performed based on the adjustment of its parameters. The use of a rheological model of three parameters as a valid alternative for real problems brings a huge facility for mathematical implementation and manipulation due the simplicity of the solution. For adjustment of the elastics and the viscous parameters, a numerical simulation to calculate the fundamental frequency of an actual reinforcement concrete pole is carried out in comparison with the standard Eurocode criteria. In this determination, the geometry variation, a concentrated force present at the free end of the structural element, and the self-weight of the structure are considered.

Keywords: numerical modelling, concrete structures, dynamic analysis, viscoelasticity, Eurocode.

1 Introduction

Columns constitute continuous systems, and their analysis can be reduced to an analogous system containing a single degree of freedom. The vibration mode is restricted to a configuration previously established by a mathematical function that describes the vibratory movement, and the properties of the system can be expressed as generalized coordinate functions. Rayleigh [1] in his study on the vibration of elastic systems applied this technique considering the function valid throughout the problem domain. For cases, where the properties of the structural elements vary along their length, the formulation developed for calculating the stiffness and mass must be solved by observing the intervals defined in the geometry, being the integrals obtained using the Rayleigh method can be solved within the limits established for each interval. To analytically define the fundamental frequency for the case modeled in this study, all the elastic stiffness components are considered in the calculation, including the conventional stiffness, which depends on the material behavior, the geometric stiffness, which depends on the normal force acting on the structure (Wahrhaftig [2]-[6]), and the soil parcel, which accounts for the soil–structure interaction.

The structure selected for this study is a slender reinforced concrete (RC) having both full and hollow circular section with variable geometry, for which the natural frequency, was calculated considering all nonlinearities presents in the system. In this work, the geometric nonlinearity was taken in consideration by using the geometric stiffness parcel into the total stiffness of the system. The nonlinearity of the material was computed by reducing its flexural stiffness, reflecting the development of cracking in the concrete when bended, which is dependent on the magnitude of the stress. Another kind of material nonlinearity is creep, which occurs due the viscoelastic behavior of the concrete, it being considered in two ways. One of them is the mathematical model for creep predicted by European Standard EN 1992-1 [7] and the other one is a three-parameter viscoelastic model whose parameters are adjusted in order to meet the results obtained when using the Eurocode. In this sense, the use of the three-parameter viscoelastic model to represent the creep of concrete brings an enormous facility of employment for actual cases due the reduced number of variables which are manipulated. Indeed, just one of them is necessary because two of the three parameters can be expressed in terms of that, the modulus of elasticity of the concrete, a data easily calculated for any standard procedure or obtained in laboratory.

2 Basic principles of a vibratory movement

Figure 1 presents the bar model of a structure in undamped free vibration. Consider the following trigonometric function, taken as valid throughout its domain:

$$\phi(x) = 1 - \cos\left(\frac{\pi x}{2L}\right), \tag{1}$$

where x is the location of the calculation, originating at the base of the cantilever, L is the length of the column. The generalized coordinate is the horizontal displacement at the free end.

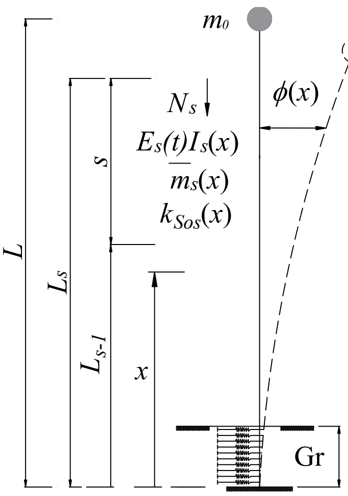


Figure 1. Frame element model in free vibration

That model represents a column under an axial compressive load, Ns(x), with either constant or variable properties along its length. These properties include the geometry, elasticity/viscoelasticity, and density, given by $Is(x)$, Es(t), and $\overline{m}_s(x)$, respectively, where s denotes the considered segment. Applied springs of variable stiffness k_{so}(x) act as the lateral soil resistance until the foundation elevation Gr. The system is under the action of gravitational normal forces, originating from the distributed mass along the length of the column, and of a lumped mass at the tip m0. In the case of vibration of a cantilevered column that is clamped at its base and free at its tip, the shape function given in Eq. (1) satisfies the boundary conditions of the problem. The use of Eq. (1) as a shape function for an actual structure with varying geometry has been validated by Wahrhaftig [8]. This validation involved a comparison with a computational solution derived using computational modeling by finite element method (FEM) and other mathematical expressions.

By applying the principle of virtual work and its derivations, the dynamic properties of the subject system are obtained. The elastic/viscoelastic conventional stiffness is given by:

$$k_{0s}(t) = \int_{L_{s-1}}^{L_s} E_s(t) I_s(x) \left(\frac{d^2\phi(x)}{dx^2}\right)^2 dx \text{, with } K_0(t) = \sum_{s=1}^{n} k_{0s}(t), \tag{2}$$

where for a segment s of the structure, Es(t) is the viscoelastic modulus of the material with respect to time; Is(x) is the variable moment of inertia of the section along the segment in relation to the considered movement, obtained by interpolation of the previous and following sections, and if it is constant, it is simply Is; k0s(t) is the temporal term for the stiffness; K0(t) is the final conventional stiffness varying over time; and n is the total number of segment intervals given by the structural geometry. In Eq. (2), obviously, t vanishes when the analysis considers a material with purely elastic, time-independent behavior. The geometric stiffness appears as a function of the axial load, including the self-weight contribution, and is expressed as

$$k_{gs}(m_o) = \int_{L_{s-1}}^{L_s} \left[N_0(m_o) + \sum_{j=s+1}^{n} N_j + \overline{m}_s(x)\left(L_s - x\right)g \right] \left(\frac{d\phi(x)}{dx} \right)^2 dx \text{, and } K_g(m_0) = \sum_{s=1}^{n} k_{gs}(m_0) \tag{3}$$

where $k_{gs}(m_0)$ is the geometric stiffness in segment s, Kg(m0) is the total geometric stiffness of the structure with n as defined previously, and $N_0(m_0) = m_0 g$ is the concentrated force at the top, all of which are dependent on the mass m0 at the tip. Further, Ns is the normal force from the upper segments, given by

$$N_s = \int_{L_{s-1}}^{L_s} \overline{m}_s(x) g dx \, . \tag{4}$$

Then, the total generalized mass is given by $M(m_0) = m_0 + m$, considering that

$$m = \sum_{s=1}^{n} m_s \text{, with } m_s = \int_{L_{s-1}}^{L_s} \overline{m}_s(x)\left(\phi(x)\right)^2 dx \text{, and } \overline{m}_s(x) = A_s(x)\rho_s \, , \tag{5}$$

where $\overline{m}_s(x)$ is the mass distributed to each segment *s*, which is obtained by multiplying the cross-sectional area, $A_s(x)$, by the density, ρ_s, of the material in the respective interval. Therefore, $\overline{m}_s(x)$ is the mass per unit length, and *m* is the generalized mass of the system owing to the density of the material, with *n* as previously defined. If the cross section has a constant area over the interval, $A_s(x)$ will be just A_s; consequently, the distributed mass will also be constant. Similarly, if the mass m_0 does not vary, all the other parameters that depend on it will also be constant.

One approach for considering the participation of the soil in the vibration of the system is to consider it as a series of vertically distributed springs that act as a restorative force on the system. With $k_{Sos}(x)$ denoting the spring parameter, the effective soil stiffness (as a function of the location *x* along the length) is generally defined as

$$K_{So} = \sum_{s=1}^{n} k_s \text{, with } k_s = \int_{L_{s-1}}^{L_s} k_{Sos}(x)\phi(x)^2 dx \text{, where } k_{Sos}(x) = S_{ops} D_s(x) \, , \tag{6}$$

where the parameter K_{So} is an elastic characteristic consisting of the sum of $k_{Sos}(x)$ along the foundation depth, which depends on the geometry of the foundation $D_s(x)$ and the soil parameter S_{ops}. Considering the normal force as positive, the total structural stiffness is obtained as

$$K(t, m_0) = K_0(t) - K_g(m_0) + K_{So} \, . \tag{7}$$

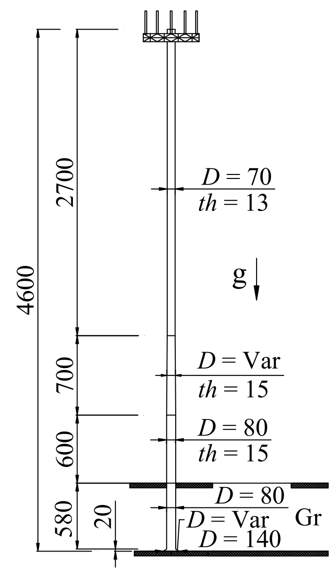
Finally, the natural frequency, in Hertz, as a function of the time and the mass at the tip, is calculated according to Eq. (8). Details of this analytical procedure can be seen in Wahrhaftig [9].

$$f(t, m_0) = \frac{1}{2\pi} \sqrt{\frac{K(t, m_0)}{M(m_0)}} \, . \tag{8}$$

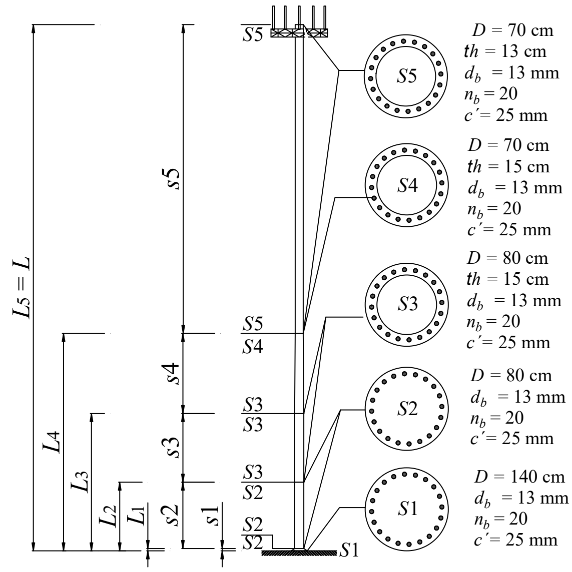
3 Matching numerically a rheological model to the Eurocode creep criteria

The case selected for the present study involves calculating the fundamental frequency of an actual slender reinforced concrete pole with variable geometry that presents both geometrical and material nonlinearities, shown in Figure 2(a), where g denotes gravitational acceleration; Gr means ground, and Var a tapered section. Figure 2(b) brings the dimensions of sections and arrangement of the reinforcement, where *s* represents each structural segment; *S*, *D* and *th* are the type, the external diameter, and the wall thickness of the section; d_b represents the reinforcing bar diameter; n_b is the number of reinforcing bars, and *c´* is the reinforcing cover.

The structure is 46 m high, which includes a 40 m superstructure with a hollow circular section and a 6 m deep, full circular-type foundation. The slenderness ratio of the tower structure is upper than 400.The modulus of elasticity adopted for the superstructure and foundation are 30 GPa and 25 GPa, considering characteristic resistances at 28 days after production (f_{ck}) of 45 and 20 MPa, respectively. A set of antennas and a platform are installed at the tip of the structure, constituting a concentrated mass of 1098 kg. Cables and a ladder are installed along the entire length, adding a distributed mass to the system of 40 kg/m. The densities of the reinforced concrete were defined as 2600 kg/m^3 and 2500 kg/m^3 for the super- and infrastructure, respectively. The physical nonlinearity of the material was computed for the superstructure and the foundation reducing the gross moment of inertia by a multiplier factor equal to 0.3, allowing the performing of a simplified nonlinear analysis according to Eurocode [7]. The foundation is a relatively deep shaft having a bell diameter of 140 cm, bell length of 20 cm, shaft diameter of 80 cm, and shaft length of 580 cm. The lateral soil resistance is represented by an elastic parameter equal to 2669 kN/m^3.



(a) Geometry (cm)



(b) Sections and structural segments

Figure 2. Subject reinforced concrete pole

As the studied structure is very slender creep must be considered on calculation. The best-known mechanical models for representing the creep are the Maxwell model, formed by spring in series with a dashpot, and the Kelvin–Voigt model, containing a spring and dashpot in parallel. One model used to represent the viscoelasticity of solids is the three-parameter model, in which the elastic parameter E_e is connected to the viscoelastic Kelvin–Voigt model with parameters E_v and η, which is a simplification of the Group I Burgers model.

The three-parameter model is an appropriate model for describing the viscoelastic nature of many solids and is often used to study the phenomenon in various scientific fields. The total deformations of the Kelvin–Voigt model are obtained summing the deformation of the elastic model, and the deformation of the Kelvin–Voigt model, from which is possible to extract the temporal function for the modulus of elasticity of the three-parameter model:

$$E(t) = \frac{1}{\left[\frac{1}{E_e} + \frac{1}{E_v}\left(1 - e^{-\frac{E_v}{\eta}t}\right)\right]}. \tag{9}$$

Eq.(9) can easily be expressed as

$$E(t) = \frac{1}{\left[\frac{1}{E_e} + \frac{\psi(t)}{E_v}\right]}, \text{ with } \psi(t) = \left(1 - e^{-\frac{E_v}{\eta}t}\right). \tag{10}$$

It is important to note that the viscoelastic behavior of the considered material is completely represented by the temporal modulus of elasticity, and it can be used for static or dynamic applications. Wahrhaftig [10]-[11], for instance, used the previous solution in numerical simulations. It is possible to transform the parameters of the viscous part to being just a function of the modulus of elasticity of the elastic part, which can easily be calculated by any standard procedure or obtained in the simplest laboratory. Therefore, these parameters can be written as

$$E_v = \alpha E_e;\ \eta = \gamma E_e\,, \tag{11}$$

where α is a real positive number, and γ brings together a temporal unit.

On the other hand, the creep model in the European Standard EN 1992-1-1 (Eurocode 2) [7] considers the effects of the creep behavior and its variation with time. Eurocode 2 provides hypothetical and model limitations for creep calculation, wherein the creep coefficient φ is predicted as a function of the tangent modulus of elasticity E_c. The creep deformation of concrete is computed by multiplying the immediate deformation by the creep coefficient, which, after incorporating all appropriate parameters, conducts to a temporal modulus of elasticity in terms of the following equation:

$$E(t,t_0) = \frac{1}{\left[\dfrac{1}{E_c(t_0)} + \dfrac{\varphi(t,t_0)}{E_c(t_{28})}\right]}, \tag{12}$$

where $E_c(t_0)$ is the modulus of elasticity at the beginning of loading, and $E_c(t_{28})$ is the modulus of elasticity 28 days after the concrete production. If the structure is loaded when the concrete finished to be produced, the initial time t_0 is considered to be equal to t_{28}, $E_c(t_0)$ is equal to $E_c(t_{28})$, and Eq.(12) can be rewrite as Eq.(13), to take a similar form of the Eq. (10).

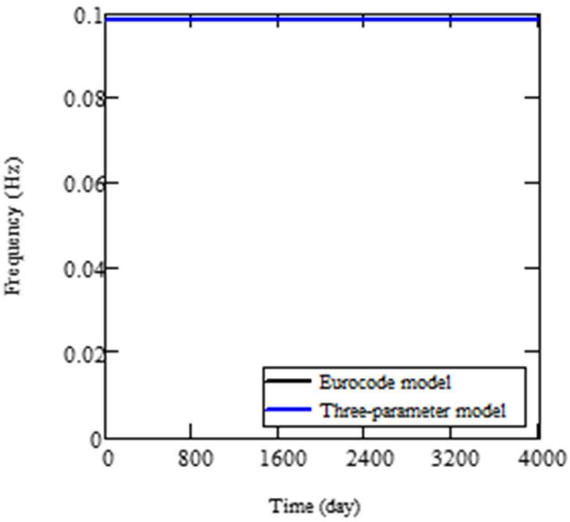
$$E(t) = \frac{1}{\left[\dfrac{1}{E_c} + \dfrac{\varphi(t)}{E_c}\right]}. \tag{13}$$

Considering for the three-parameter model that E_e is equal to E_c and setting α and γ as 3.913 and $10^{8.3066}$ seconds, respectively, and adopting an environmental humidity of 70%, the variation of the fundamental frequency for different instants in the lifetime of the structure can be obtained. It is important to mention that these chosen values for α and γ were defined so that simulation leads a good agreement for instants approaching to and after 2000 days. Therefore, they were intentionally defined so that the frequency met the same values as given by Eurocode.

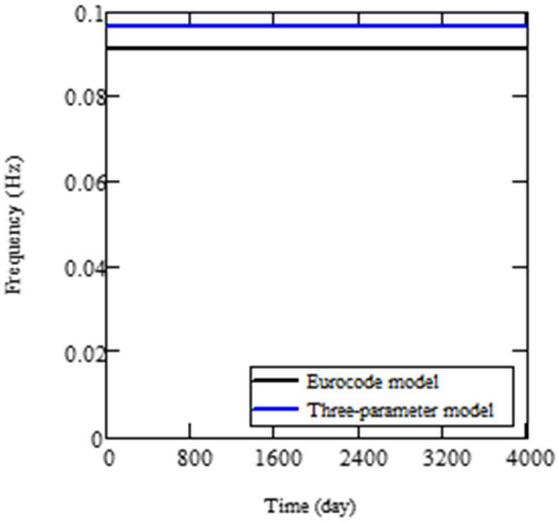
The choice of these coefficients has been done because of the convergence of the deformations occurs at 4000 days, at which time the interest of the structural engineering normally lies, being, however, possible to define other pairs of values for α and γ in case of a particular objective or even to choose that those can match both curves in the whole time interval. Therefore, the mentioned coefficients have been adjusted so that the frequency were equalized by both models considering a precision of 6 significant digits, as can be highlighted in Table 1. Figure 3 shows a comparison between results produced through both models, considering each selected instants of time. Details of this procedure can be found in Wahrhaftig [12].

Table 1. Frequencies for both models at selected instants.

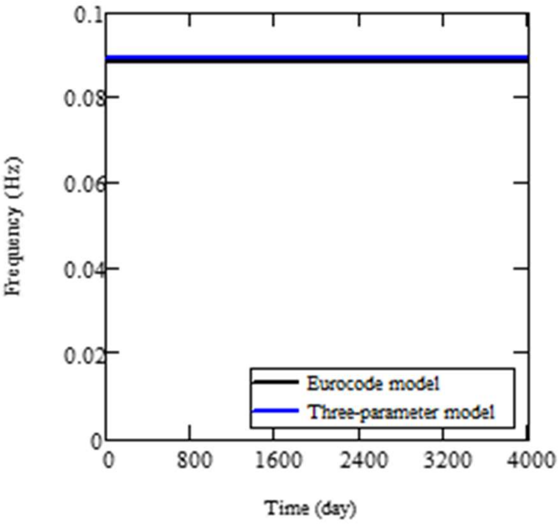
Time	Eurocode/Three-parameter model (matching)
0	0.098440 Hz
2000 days	0.087980 Hz
4000 days	0.087665 Hz



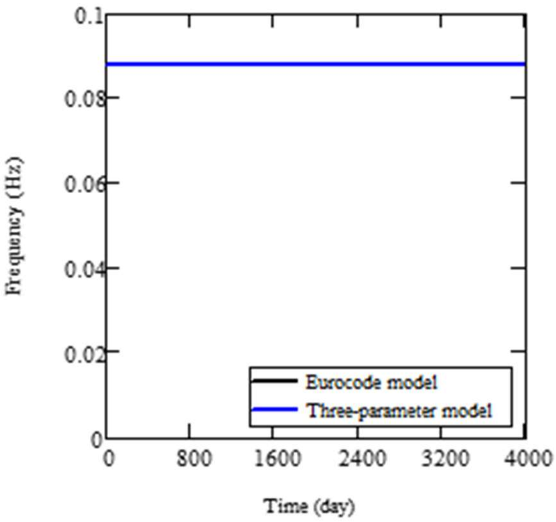
(a) t = 0



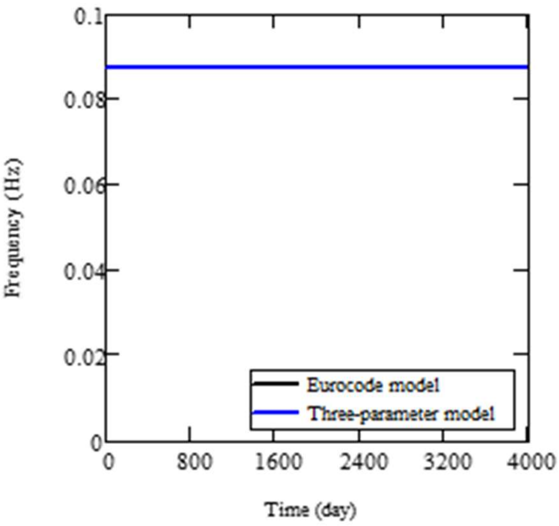
(b) t = 100 days



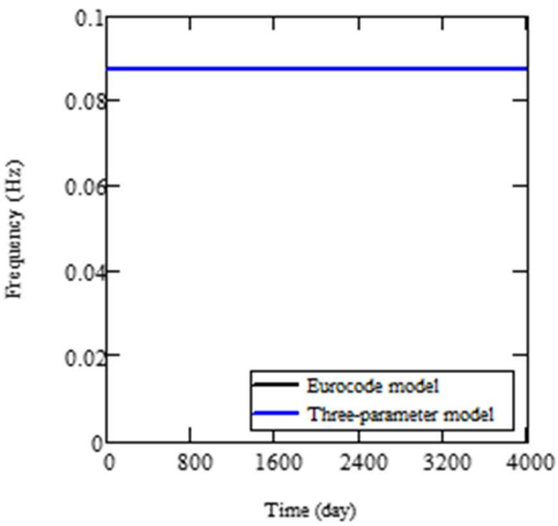
(d) t = 1000 days



(e) t = 2000 days



(f) t = 3000 days



(g) t = 4000 days

Figure 3. Comparative of frequencies to different times

4 Conclusions

- Because of the viscoelastic behavior of the material, the modulus of elasticity presents a variation along the time, reflecting on the structural frequency.
- A three-parameter viscoelastic model has been adjusted to fit the same results as predicted by Eurocode creep criteria for a specific interval of time.
- This article demonstrated the possibility of adjustment of a simple model to the standard one and how easy is its use for practical applications to calculate the first natural frequency of a structural system.
- For future works, a programing routine for obtaining a finer adjustment of the curve between the viscoelastic rheological model of three parameters and that of the model for creep as predicted by Eurocode must be developed.
- Further, comparative analyses considering other values of environmental humidity should be also performed.

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