XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Thermal Diffusion over a Portland Cement Concrete Gravity Dam

E.L. Madureira[1], G.B. Spinola[2], E. M. Medeiros[3], A. L. A. Silva[4]

[1]Departamento de Engenharia Civil – Universidade Federal do Rio Grande do Norte
Av. Sen. Salgado Filho S/N – Lagoa Nova, Natal, Rio Grande do Norte, Brasil
edmadurei@yahoo.com.br
[2]Departamento de Engenharia Civil – Universidade Federal do Rio Grande do Norte
Av. Sen. Salgado Filho S/N – Lagoa Nova, Natal, Rio Grande do Norte, Brasil
gabrielbessasp@gmail.com
[3]Departamento de Engenharia Civil – Universidade Federal do Rio Grande do Norte
Av. Sen. Salgado Filho S/N – Lagoa Nova, Natal, Rio Grande do Norte, Brasil
mm.edu@hotmail.com
[4]Departamento de Engenharia Civil – Universidade Federal do Rio Grande do Norte
Av. Sen. Salgado Filho S/N – Lagoa Nova, Natal, Rio Grande do Norte, Brasil
Arthur_leandro33@hotmail.com

Abstract. Temperature and its oscillations can influence some physical phenomena progress in concrete structures as the alkali-aggregate reaction swelling effect, the Creep volume variations, and the Shrinkage, and, consequently, affect its mechanical performance. The temperature distribution analysis drops over the Thermal Diffusion Theory that culminates in the Heat Diffusion Differential Equation. It is already known, at present, that the Finite Difference Technique represents as modest formulation, however it may be used to support the computational tools applied to the numerical analysis expeditiously, suitable to the endorsement of studies and designs. Its widely known in the Civil Engineering ambit the hot release due to the cement Portland hydration chemical reaction in concrete mass structures. The subject of this work is the numerical simulation of the thermal diffusion across concrete gravity dam focusing, specially, over the temperature fields evolution by time in its continuous solid mass. Such a subject will be hit from the implementation and application of one-dimensional and two-dimensional thermal diffusion models using an automatic language translated algorithm through the Finite Difference Approach on the Heat Diffusion Differential Equation. According the obtained results, the numerical modelling adopted in this paper simulates, in a suitable way, the dam behavior in face of the thermal diffusion, so that it represents strategically promising tool to perform similar tasks.

Keywords: Temperature, Diffusion, Finite Difference, Differential Equations, Numerical Simulation

1 Introduction

The thermal diffusion in continuous solid media is worth of attention that's why the temperature and their oscillations represent factors that exerts significant influence in a wide diversity of natural phenomena, such as the alkali-silica reaction swelling effect, the concrete creep and its shrinkage, that affect cement Portland concrete members, even interfering with the performance of civil construction structures. Its widely known in the Civil Engineering ambit, include, the heat release due to the cement Portland hydration chemical reaction in a concrete dam.

In this way, the prediction of the thermal field evolution by time across the solid mass of structural members are of interest as regards analysis involving its performance. In this sense, the inclusion of modules intended to the temperature field simulation is useful.

The temperature fields numerical analysis can be carried out from versatile approximated methods, such as the Finite Element Technique. However, in some cases, notably if the solid object of study presents low values thermal diffusion parameters, as in the case of concrete, the application of the Finite Element Method is hampered, that is why, in particular, disturbs arising in the numerical stability from its use, originated from the numerical behavior referring to the convergence and equilibrium criteria.

In the face of such a considerations, it may be suitable the support of prototypes developed according to an alternative numerical modeling such as the Finite Difference Technique that, despite its modest conception, is able to represent an effective strategic resource to answer to the demand now highlighted.

The aim of this work is the numerical simulation of the thermal diffusion across a massive concrete structure, paying attention, specially, to the analysis of its temperature field progress by time.

With a view to the fulfillment of the subject of this work was applied a computational algorithm drafted by using the FORTRAN automatic language, based on the finite difference Approach upon both.

2 Modelling

The rational study of the heat diffusion, through the analysis of its propagation forms, must involve the three processes of thermal energy transfer, namely, conduction, convection and radiation, Holman [1]. In practice, however, according to the reality of a special work, it may be suitable to prioritize that process which, effectively, predominates over the others remaining.

The physical modeling of temperature distribution problem across a solid mass, based upon the transient regimen by heat conduction, may be performed from the application of the correlated Diffusion Differential Equation, since be considered the initial and the boundary conditions regarding the situations that are been analyzed, Holman [1].

The thermal diffusion analysis that is proposed to perform in this work will take into account the artifice validated by Madureira at al [2], and, at this way, although the specimen studied is a three-dimensional body, due to its length to be much greater than its remain dimensions, the heat flow through the solid mass will be modeled as a two-dimensional version.

Once a temperature gradient in the analyzed sound body has been occurred it will settle a heat flow q(Watts/m^2). If an appropriated source generates thermal energy according to a specific rate represented by a function g = g(x, y, t)(Watts/m^3), the heat transfer in the system can be expressed from the heat balancing scheme of Fig. 1.

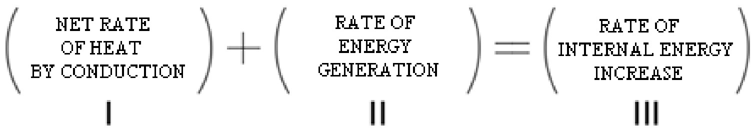

Figure 1. Energy Balance

Such a scheme can be represented in mathematical terms by the Heat Diffusion Equation which, in its two-dimensional version, can be expressed by Eq. 1.

$$\frac{\partial}{\partial x}\left(K\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial u}{\partial y}\right)+\ g\ =\ \rho c\frac{\partial u}{\partial t} \tag{1}$$

To those cases in which the material is homogeneous and, therefore, the thermal conductivity throughout the body volume features uniform distribution, Eq. 1 reduces itself to the form:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{1}{K}g\ =\ a^2\frac{\partial u}{\partial t} \tag{2}$$

Since it may let:

$$a^2\ =\ \frac{c\rho}{K}\ =\frac{1}{\alpha} \tag{3}$$

the "α" parameter is the material thermal diffusivity and indicates the heat propagation rate through a solid mass constituted by a similar kind of material.

If the heat diffusion process occurs with no an energy external source Eq. 2 takes the form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = a^2 \frac{\partial u}{\partial t} \tag{4}$$

The problem numerical solution may be obtained from the finite difference approach. For such a purpose the analyzed continuous solid should be subdivided into several elements, resulting, in this way, on a discrete mesh of points. The total observation time of the phenomenon is also subdivided from the consideration of some suitable instants of time accompanying the development of the phenomenon.

For each instant of time and for every point of the solid body the analytical derivatives of the function u = u(x,y,t), that appear on the Heat Diffusion Equation, Eq. 4, is replaced by its corresponding numerical versions that are written in the forms:

$$\left.\frac{\partial u}{\partial t}\right|_k = \frac{u_{i,j,k+1} - u_{i,j,k}}{\Delta t} \tag{5}$$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_i = \frac{u_{i+1,j,k} - 2u_{i,j,k} - u_{i-1,j,k}}{\Delta x^2} \tag{6}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_j = \frac{u_{i,j+1,k} - 2u_{i,j,k} - u_{i,j-1,k}}{\Delta y^2} \tag{7}$$

If it may be considered:

$$\Delta x = \Delta y \wedge \beta = \Delta t \left(\frac{a}{\Delta x}\right)^2 \tag{8}$$

and combining Eq. 4, Eq. 5, Eq. 6, and Eq.7 it may result into the recurrence form:

$$u_{i,j,k+1} = (1 - 4\beta)u_{i,j,k} + \beta\big(u_{i+1,j,k} - u_{i-1,j,k} + u_{i,j+1,k} - u_{i,j-1,k}\big) \tag{9}$$

If the prior aim to be accomplished is the complete problem solution, the initial condition and the conditions recognized, clearly, at the problem domain boundary, that reflect its reality, must be applied to the Eq. 9. The numeric values of the temperature distribution at further instant of time are so obtained, and, in this way, the thermal fields throughout the solid mass may be draft.

The problem featured in this paper may be solved from the Heat Diffusion Differential Equation analytical solution, too. For such a calculus journey it may be suitable to resource to the Bernoulli proposal apud Kreyszig[3] that, consider its bidimensional version solution as the multiplication involving three functions, each of them depending, solely, on an independent variable, x, y and t. By using such an artifice, the Heat Diffusion Differential Equation exchange itself on three Ordinary Differential Equations. According to Kreyszig[3] and Farlow[4], once the initial and the boundary conditions having been applied, the problem solution would be:

$$u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} sin(\vartheta x)\sin(\mu y)\, e^{-\lambda_{mn}^2 t} \tag{10}$$

since that:

$$\vartheta = \frac{m\pi}{L_x};\ \mu = \frac{n\pi}{L_y}; e, \lambda_{mn} = \left(\frac{\pi}{a}\right)^2 \left[\left(\frac{m}{L_x}\right)^2 + \left(\frac{n}{L_y}\right)^2\right] \tag{11}$$

and, the L_x and L_y parameters, represent the solid dimension over two coordinate directions. The B_{mn} coefficients are obtained from the EULER’s equation, in its form applicable to the Fourier’s series coefficients definition, apud Kreyszig[3], represented by:

$$B_{mn} = \frac{4}{L_x L_y} \int_0^{L_y} \int_0^{L_x} f(x,y) sin(\vartheta x) \sin(\mu y)\, dxdy \tag{12}$$

Where, f(x, y) is a real function, previously known, that describes the temperature field distribution at the instant that the diffusion phenomenon is triggered.

3 Computational Support

With a view to the acquisition of results aimed at the support of the tasks affecting the numerical simulation performed in this paper, a computational algorithm written by using the FORTRAN automatic language was drafted. The computational code is based on the approximation by finite differences of the Heat Diffusion Differential Equation.

Such algorithm was structured according to a logic strategy including the material diffusivity coefficient input, the data reading referring to the mesh topology resulted from discretization of the problem spatial domain, the information reading of the body geometric characteristics, the boundary conditions, the initial thermal distribution, as well as the phenomenon longevity.

The computational code starts from the division of the period along which the heat diffusion is analyzed in some suitable instants of time. After that, it generates a matrix for the initial time situation, in which each element corresponds to the temperature at a point on the solid body, according to the discretization mesh. The code then applies the recurrence expression, Eq. 9, to the values of the initial matrix and obtains a new matrix, representing the temperature distribution at the next instant of time. By applying the expression once more, this second matrix results in a third one, related to a further instant of time. This process is repeated over and over until the final instant of time is reached, stage when the thermal field of interest is generated.

The algorithm has in your logical schedule a manager module to perform the output of result in a neutral file focused to supply demands of the graphic postprocessor used for the visual display of thermal fields. Such a computational tool is drawn up in language recognized by the "Embarcadero Delphi" compiler that is compatible to the Windows platform in the programming language "C", Madureira and Silva [5]

4 Program Validation

The program validation has been verified from the comparison of the results obtained by using diffusion equation analytical resolution and those ones performed from its numerical version. The computational code was applied to solve the thermal diffusion through a two-dimensional square thin plate 3.00 m size, Fig. 2.
By examining Fig. 3, one may constate a good agreement between de curves presented in it.

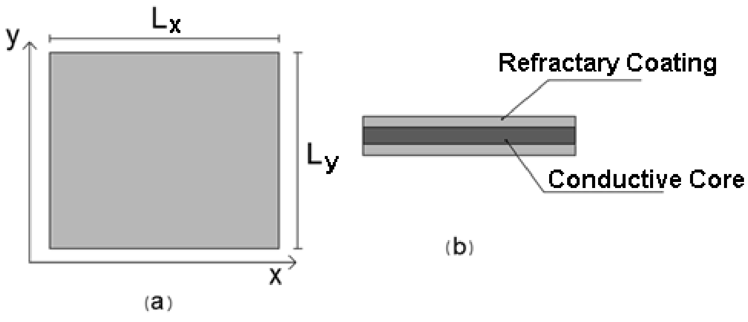

Figure 2. a - Plate; b – Cross section

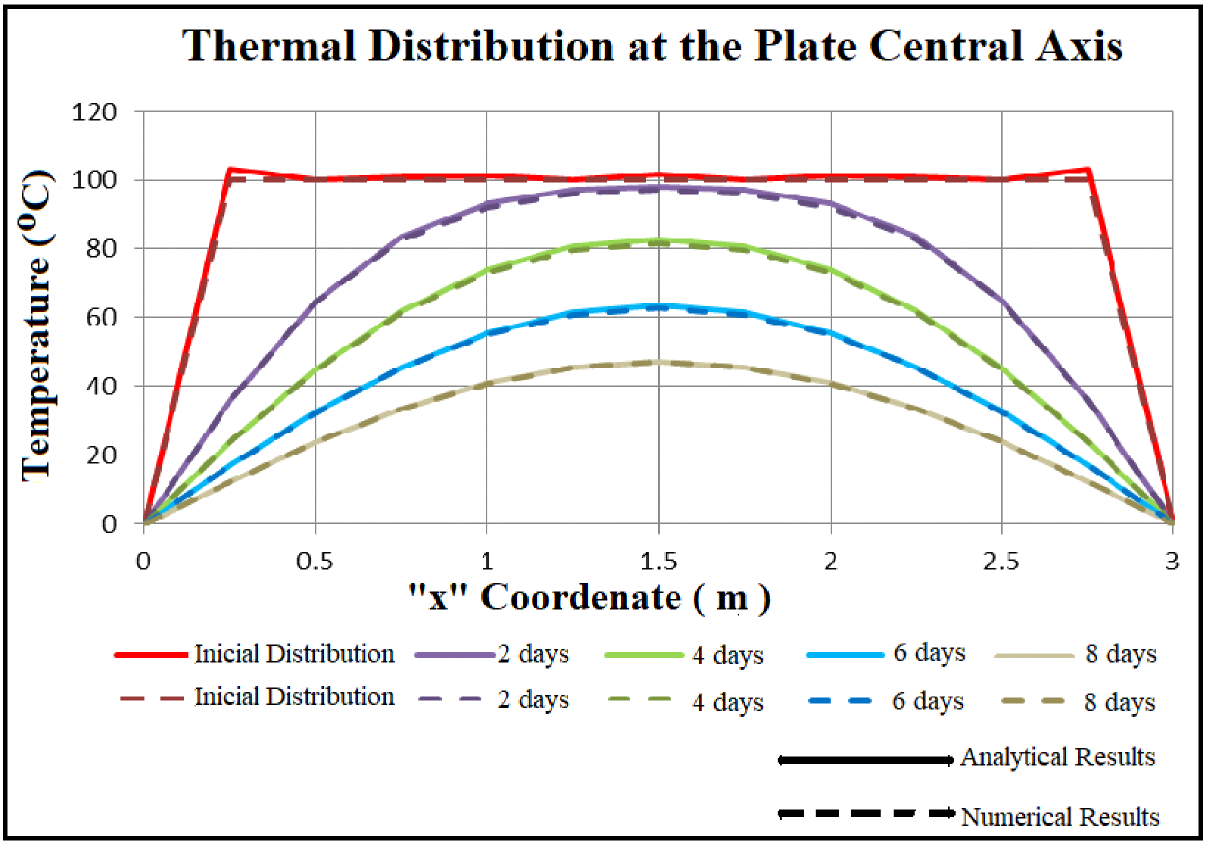

Figure 3. Analytical and numerical results comparison

5 Studied Specimens

The constructive member that is analyzed in this paper is a dam presenting 400.00 m length, 90.00 m height, 70.00 m and 5.00 m width at its bottom and its crest, respectively, vertical upstream surface and staggered downstream surface, Fig. 4. The perimeter surface over the threads BA and AG is exposed to atmospheric air and to solar radiation and so it is maintained at a temperature about 100 ºC. The perimeter surface over the threads GF, FE and AD is exposed to contact with the soil foundation, in which temperature changes from 100 ºC at the point G to 20 ºC at the point D. Over the line BD, the dam surface is in contact with the mass of water. Along the line CD, temperature is maintained by the order of 20 ºC, and along the line BC temperature changes from 20ºC at the point C to 100 ºC at the point B. At the initial time, t = 0 (ZERO), the massive dam temperature presents itself about 200 ºC.

The sound mass of the studied specimen presents thermal diffusivity $\alpha = 8{,}1\text{x}10^{-7}$ m²/s, Gambale and Guedes [6], corresponding to a thermal conductivity by 1,75 W/(m. °C), specific heat by 880 J/(Kg. °C) and specific mass about 24,0 kN/m³.

In numerical simulation cases involving heat diffusion analysis, especially if the material, as concrete, presents low values for the thermal diffusivity parameters, it may arise deficiencies regarding the accuracy of the results, characterized by instability of numerical nature, Madureira et al [2]. An effective solution for this kind of disturbance is to take care to get a suitable relation involving the discretization by time and the mesh discretization of the problem spatial domain. For that analysis involving an unidimensional modelling it is suitable to adopt a value between 0 and 0.5 for the β parameter of Eq. 8, Lima and Makino [7]. In the other hand, for two-dimensional modelling, the authors of this paper found out that such a parameter must take a value between 0 and 0.25. Once such proposal has been adopted and if it is considered the discretization that is practiced for the problem spatial domain, the adoption of time instants of the phenomenon observation about 1, 12, 60, 120, 168, 240 and 300.

In a similar proceeding adopted by Madureira et al [8], although the problem that is being analyzed in this paper is, indeed, over the three-dimensional mode, by adopting a special device, it may be treated such a case as if it were over two-dimensional type.

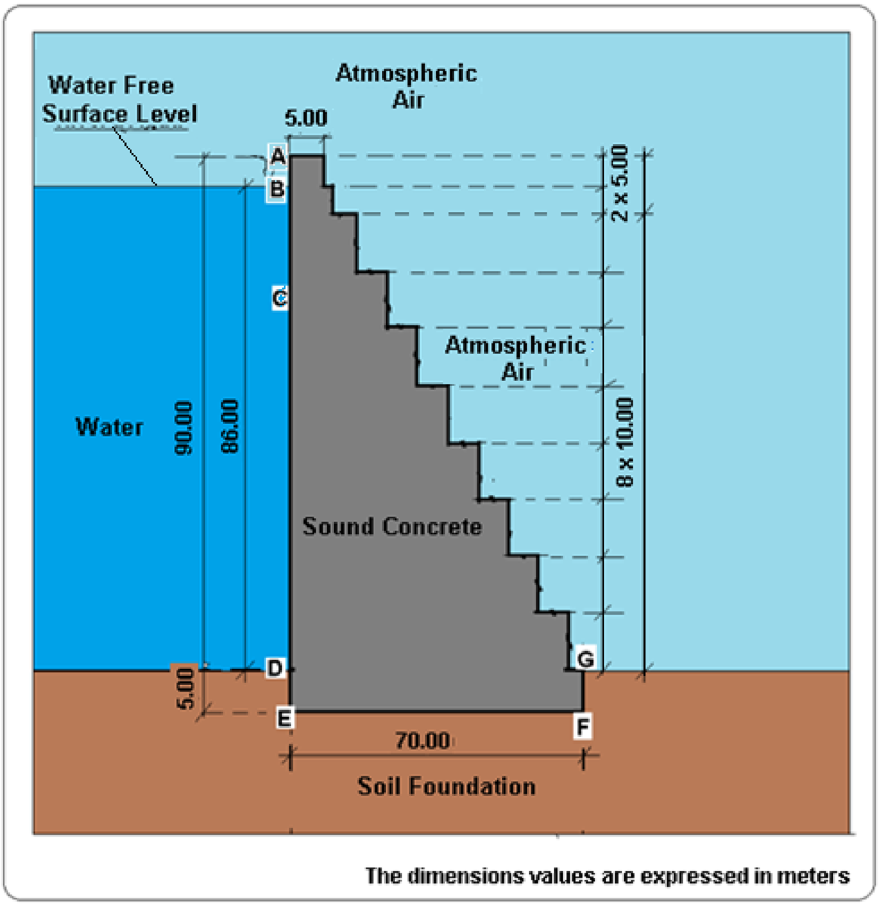

Figure 4. Studied specimens

6 Results

Figure 5 illustrates the evolution of the temperature fields by time over the dam body, which is the object of the thermal diffusion analysis proposed in this paper.

It is observed that Figure 5.a represents the instant of time in which, hypothetically, the phenomenon of thermal diffusion is started to be monitored, and shows the temperature distribution, so only, in the solid mass of concrete, omitting such information for the dam section contour.

On the other hand, it can be perceived that at the end of one month, counted from the date when the diffusion monitoring in analysis began, the day in which the temperature field presents the distribution illustrated in Figure 5.b, the thermal status of the dam section contour, reflecting, the reality of the boundary conditions of the problem that is analyzed, as defined in section number 5 of this work.

It is possible to constate, in fact, that at the end of 12 months, Fig. 5.c, the phenomenon already exhibits some evolution, although according discreet amount, as well as it is possible to indicate the tendency induced by the establishment of thermal gradients to promote heat flow through solid bodies from the regions at higher temperature levels, as in the case of the inner mass of the dam body, toward to those regions at cooler thermal level, as in the case of the dam contour perimeter.

By examining the set of all the thermal fields showed in Fig. 5, one can concatenate a clear idea of the temperature progress over time, even, corroborating the statement formulated in the previous paragraph of this work that the heat flow is processing from the inner mass of the dam forward to the boundary perimeter surrounding it.

By performing an analysis of the temperature fields of Fig. 5, in sequence, it is perceived that, from an instant of observation to its subsequently immediate, the thermal fields present distribution, notoriously, distinct even in the elapsed period from ten years to twenty years.

The behavior that is reported in the previous paragraph of this paper induces to the conviction that, up to twenty years, the heat diffusion phenomenon is yet in transient regimen, and from such date, the thermal diffusion hit, in practice, the stationary condition.

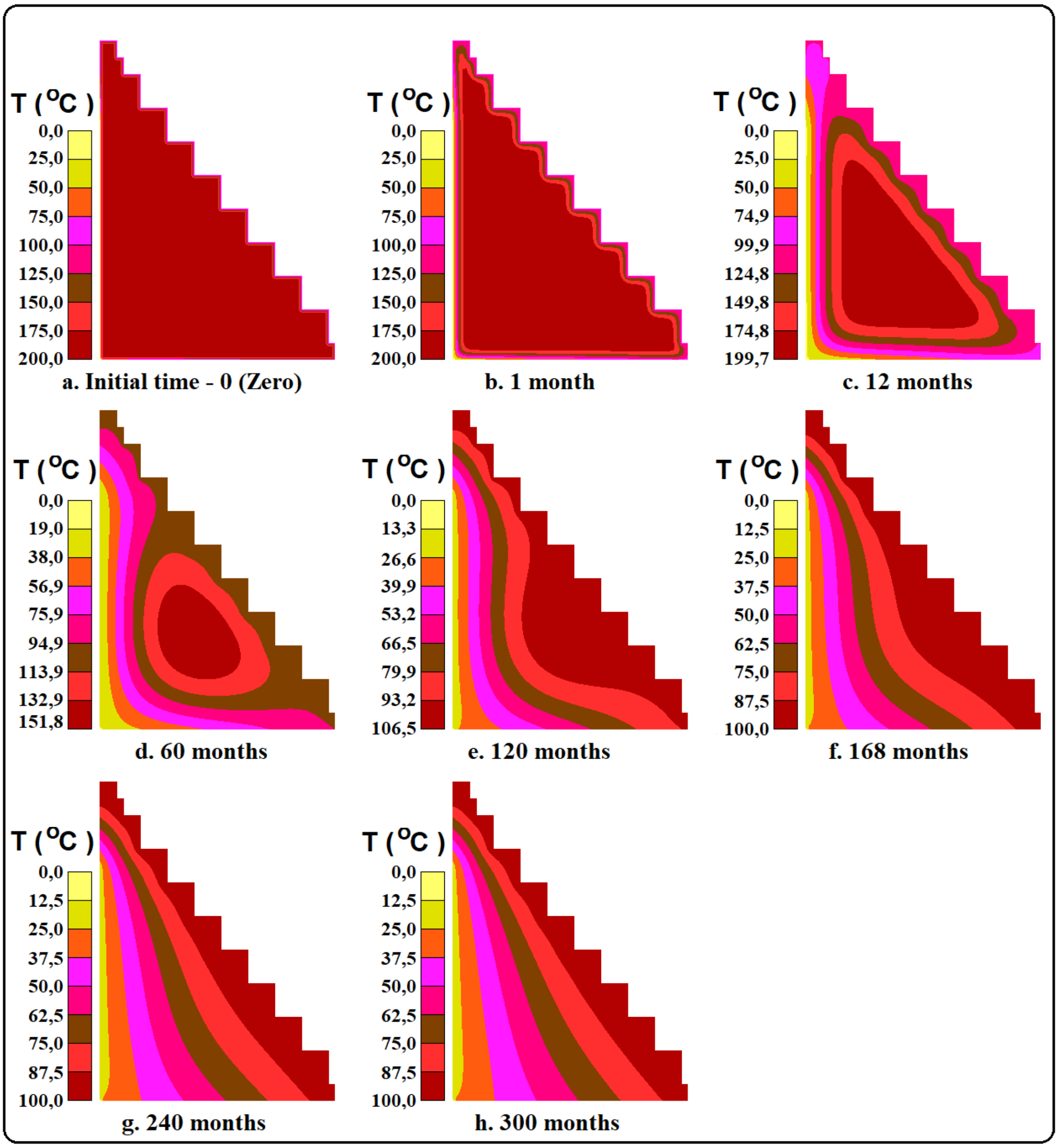

Figure 5. Dam temperature progress by time

7 Conclusions

The thermal diffusion promoted the heat flow from the regions at higher temperature levels, as in the case from the inner mass of the dam body, toward those regions at cooler thermal level, as in the case of the dam contour perimeter.

Due to thermal conductivity small value of the concrete, the heat diffusion phenomenon has endured itself over, at least, twenty years, even at the absence of a heat energy source and at the end of such period of time, the thermal diffusion hit, in practice, the stationary condition

From the obtained results of the numeric simulation performed in this work, the thermal diffusion process has developed itself according the expected trend.

Thus, the proceedings and the computational support that was applied to perform the analysis object of this paper showed themselves suitable.

Acknowledgements

This report is part of a research work on the numerical simulation of the mechanical performance of slabs supported by the Pró-Reitoria de Pesquisa of the Universidade Federal do Rio Grande do Norte – UFRN. This support is gratefully acknowledged.

Authorship statement

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] Holman, J.P.. Heat Transfer, McGraw-Hill Book Company, 1972.
[2] Madureira, E.L. Medeiros, E.M. e SILVA, A.L.A. Difusão Térmica em Estrutura de Contenção de Concreto. 58º Congresso Brasileiro do Concreto. Belo Horizonte. MG. 2016.
[3] Kreyszig, E.. Matemática Superior. Volume 3. 2. ed. Rio de Janeiro. Livros Técnicos e Científicos, Editora S.A., 1978.
[4] Farlow, S.J.. Partial Differential Equations for Scientists and Engineers, John Wiley and Sons, 1982.
[5] Madureira, E.L. and Silva, A.L.A. (2013): Project1 – Programa para visualização de campos de tensões resultantes de analises não lineares de modelos bidimensionais de elementos finitos. Versão 1.0, Rio Grande do Norte: DEC/UFRN.
[6] Gambale, E.A. and Guedes, Q.M.. Difusividade Térmica do Concreto. Concreto Massa, Estrutural, Projetado e Compactado a Rolo - Ensaios e Propriedades. Ed PINI, 1997.
[7] Lima, K.L. e Makino, M.. Método das Diferenças Finitas Aplicado à Geotermia rasa em solos de pastagem em Marabá, Revista Científica da Universidade Federal do Pará. Edição n. 2, 2001.
[8] Madureira, E. L. Medeiros, E.M. Silva, A. L. A. Spínola, G.B. Thermal Diffusion over a Massive Portland Cement Concrete Structure. Ibero-Latin American Congress on Computational Methods in Engineering. Natal. RN. 2019