

# Computation of stay cable forces in Segmental Construction of Concrete Cable-Stayed Bridges

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**Abstract.** Concrete deck cable-stayed bridges are complex, highly hyper-static nonlinear structures. Design of such structures must consider not only these aspects but also time-dependent effects (such as creep, shrinkage and varying stiffness) of the concrete deck. Determination of the stay cables installation forces is highly influenced by these effects, as well as by the construction method. Since these types of bridges are mostly constructed in phases, these effects become even more pronounced. This work presents a methodology, using computational methods, to compute stay cable forces in such bridges taking into account the construction method and time-dependent effects. We also assess the distribution of internal (i.e., cross-sectional) forces of the deck as influenced by these effects. The methodology will be applied to a case study consisted of the cable-stayed railway bridge over the Ayrton Senna and Hélio Smidt freeways that give access to the André Franco Montoro International Airport in the city of Guarulhos, São Paulo, Brazil. A thorough discussion of results, and comparisons between techniques, are provided.

**Keywords:** Cable-stayed Bridges, Installation Forces of Stay Cables, Cantilever Construction Method.

## 1 Introduction

In cable-stayed bridges, the stay cable forces help in the control of the geometry and ensure a better distribution of stresses and moments on the deck. To correct construction deviations and reach the final geometry of the deck, cable tensioning is required (Martins, Simões and Negrão [1]). Also, the tension of the stay cables must be controlled in order to do not reach the limit strain and to avoid excessive deformations of the deck and the tower.

A number of conventional methods are available in the literature to determine the installation forces on the cables. Among them, the most known are (i) the method of the simply supported deck, in which the deck is articulated along its length at the connections with the stay cables (for a first approximation of the problem), (ii) the method in which the deck is considered as a continuous beam on rigid supports with the dead load applied, and (iii) the so-called “zero displacement method” (Wang, Tseng and Yang [2]), which determines forces on stay cables that would cancel the vertical displacements of the deck due to the permanent loads. In general, these methods are based on the final configuration of the structure and do not take into account the construction phases (Khalil, Dilger and Ghali [3]; Somja and De Ville De Goyet [4]; Arici et al. [5]).

Currently, most cable-stayed bridges are built through the cantilever method (Almeida [6]). Accordingly, the deck is built in phases, in a segmented way such that it is often consisted of concrete with different ages. In these cases, for determination of the installation forces on the cables, it is important to consider methods that deal with the construction schedule. Among these, there is the “unit load method” (Janjic, Pircher, M. and Pircher, H.[7]), the method of the “forward process analysis and the backward process analysis” (Wang, Tang and Zheng [8]), and the “zero displacement method” along the construction, presented by Ytza [9] and Arici et al. [5]. Most of these methods cover the possibility of considering second order and time-dependent effects (such as creep, shrinkage and stiffening of the concrete), except the backward analysis presented by Wang, Tang and Zheng [8].

The objective of this work is to use and assess some of these methods to determine the stay cable forces in

segmental construction of concrete cable-stayed bridges. The investigation is based in a special case-study, the cable-stayed railway bridge over the Ayrton Senna and Hélio Smidt freeways that give access to the André Franco Montoro International Airport in the city of Guarulhos, São Paulo, Brazil. It is a curved bridge, with one part as cable-stayed and another as a box girder segmental bridge. A thorough discussion of results, and comparisons between techniques, are provided.

## 2 Determination of installation forces on stay cables of cable-stayed bridges

For large span cables, the influence of the geometric non-linearity due the cables' catenary must be taken into account (Granata et al. [12]). The cable, when balanced, assumes the shape of a catenary since it is unable to present null transverse displacements when subjected to its self-weight load. The catenary shape, however, varies depending on the cable's axial forces. The apparent stiffness of the cable depends on the pre-installed force (or pre-elongation) and on its length. A widely used engineering practice is considering the stay cable as a beam and adopt the so-called Dischinger's elastic modulus ( $E^*$ ). As presented by Torneri [13], the Dischinger's modulus can be calculated from the stay cable's steel elastic modulus ( $E$ ), specific weight ( $\gamma$ ), the horizontal projection of the stay cable length ( $L$ ) and the average of the cable strain ( $\sigma$ ), as follows:

$$E^* = \frac{E}{1 + \frac{\gamma^2 L^2 E}{12\sigma^3}} \quad (1)$$

The tensioning of the cable may be modelled by applying an equivalent temperature change to the cable. Accordingly, the variation in the stay cable length can be calculated by multiplying the fictitious temperature variation ( $\Delta T$ ) by the stay cable length ( $l$ ) and its coefficient of the thermal expansion ( $\alpha$ ):

$$\Delta l = \alpha \cdot l \cdot \Delta T \quad (2)$$

$$\frac{\Delta l}{l} = \alpha \cdot \Delta T \quad (3)$$

We then assume that the cable is linear elastic such that the length variation of the beam element ( $\Delta l$ ) due the axial force  $F$ , for a stay cable with cross-sectional area  $A$  and Dischinger's modulus of elasticity  $E^*$ , may be computed through:

$$\frac{\Delta l}{l} = \frac{F}{E^* \cdot A} \quad (4)$$

By equating equations (3) e (4), it results:

$$\alpha \cdot \Delta T = \frac{F}{E^* \cdot A} \quad (5)$$

from which the stay cable force ( $F$ ) for a given temperature variation ( $\Delta T$ ) follows:

$$F = \Delta T \cdot \alpha \cdot E^* \cdot A \quad (6)$$

For determination of the installation forces, the zero-displacement method throughout the construction process, as presented by Ytza (2009) [9] and Arici et al. [5], considers the displacements during the construction stages and allows to take into account second order as well as time-dependent effects, such as the creep and shrinkage of the concrete and the relaxation of the steel. The two variants of the method will be shortly presented in the next subsections.

### 2.1 The zero-displacement method throughout the construction process according to Ytza [9]

In the study presented by Ytza [9], for each construction phase, the deck's displacement  $\delta_i$  at the control point  $i$  of the last segment is obtained considering its self-weight and all other loads during the construction, such as the scaffolding truss, accidental loads, among others. For each stage being constructed, it is necessary to compute the equivalent temperature to be applied to the cables in order to nullify the vertical displacement at the

control point  $i$  due to the tensioning phase of the stay cable  $j$ . This can be solved using the displacement  $\delta_{ij}$  at  $i$  generated by a  $100^\circ\text{C}$  tensioning of cable  $j$ . Let  $(\Delta T_j)$  be the vector of cables' tensions in terms of equivalent temperature, it has:

$$\delta_i + \delta_{ij} \cdot \Delta T_j = 0 \quad (7)$$

$$\Delta T_j = \frac{-\delta_i}{\delta_{ij}} \quad (8)$$

Then, the stay cable force is obtained through equation 6. A flow-chart of the method is represented in Figure 1.

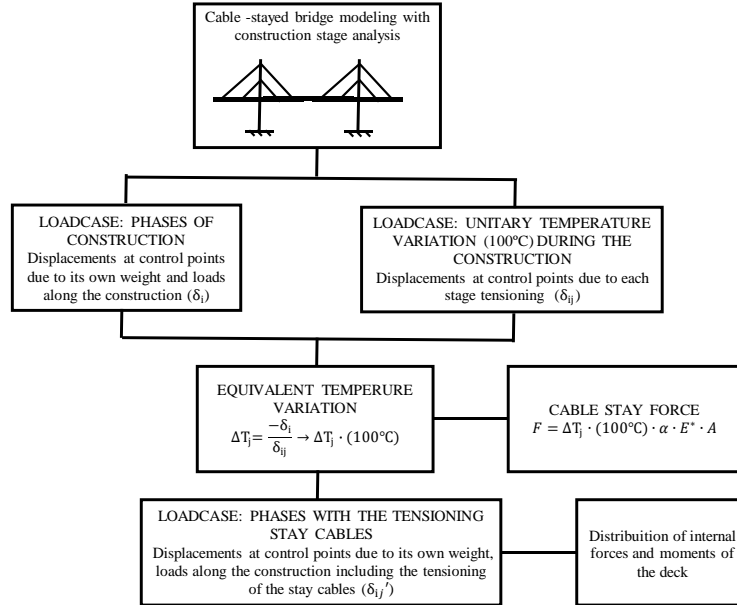


Figure 1. Flow-chart from Ytza [9] method.

This method is valid for flat (plane) models, for rectilinear structures, or curved bridges with one (central) plane of cables, where the control points are the central points in the deck between the anchorages of the cables. For curved bridges, with two planes of cables, it can be used by setting the central deck points between the two planes as the control points for a first approximation. To obtain the results for each cable of a curved bridge with two planes of cables, equations presented by Arici et al. [5] can be applied to find the force corresponding to the tensioning stage of the stay cable for each one in the construction phases.

## 2.2 The zero-displacement method during the construction process according to Arici et al. [5]

The determination of the stay cable forces by Arici et al. [5] uses the construction stage analysis. The so-called influence matrix  $[D]_{ixj}$  is obtained from the displacement  $\delta_{ij}$  corresponding to the vertical displacement of the control point  $i$  due to  $100^\circ\text{C}$  temperature variation loading during the construction stage  $j$ , according to equation 9:

$$[D] = \begin{pmatrix} \delta_{11} & \cdots & \delta_{1j} \\ \vdots & \ddots & \vdots \\ \delta_{i1} & \cdots & \delta_{ij} \end{pmatrix} \quad (9)$$

Let  $[e]$  be the vector of cables' tensions imposed in terms of temperature, and  $[d^*]$  the vector of displacements due to the loads applied at each stage, including the weight loads and the construction loads at the control points. The vector of total displacements  $[d]$  at each stage at the control points is given by:

$$[d] = [D][e] + [d^*] \quad (10)$$

By enforcing the total displacements in the control points to be zero, like a generalization of the zero-

displacement method, one has:

$$[D][e] + [d^*] = 0 \quad (11)$$

The vector of stay cable tensions  $[e]$ , for each stage, can be found by inverting matrix  $[D]$  and applying equation 12:

$$[e] = [D]^{-1}[d^*] \quad (12)$$

A flowchart of the method is illustrated in Figure 2. Arici et al. [5] optimizes the displacements on the deck and the tower adjusting the forces at the stay cables along the construction. The structure's influence matrix is used to achieve bending a moment pattern similar to a continuous beam in each of the phases and also in its final configuration. As presented by Arici et al. [5], at each stage, the new and previous cables are tensioned, and at the end of the construction process, all the stay cables are further adjusted to their final configuration.

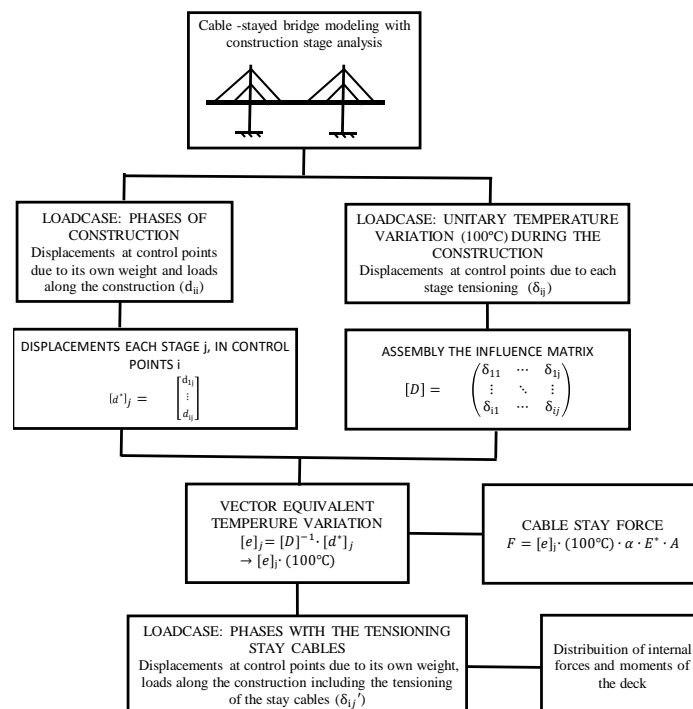


Figure 2. Flow-chart from Arici [5] method for the determination stay cable force.

### 3 Case study: Cable-stayed railway bridge over the Ayrton Senna and Hélio Smidt freeways

The cable-stayed railway bridge over the Ayrton Senna and Hélio Smidt freeways, which give access to the André Franco Montoro International Airport in the city of Guarulhos, São Paulo, Brazil, and belongs to line 13 Jade of the São Paulo Metro, is studied here (Figure 3). It is a curved bridge with 690m extension, composed by a 180m central span, two main spans with 175m each (where the first half of the span is cable-stayed and the second half is a box girder segmental bridge) and complemented with two adjacent lateral 80m spans. The two cross-sectional cells have a variable height from 7,5m to 3,5m in the box segmental parts of the main spans and a constant height of 3,5m in the cable-stayed parts (Oyamada et al. [14]). The cable-stayed part has two lateral cable planes with a semi-harp arrangement, each with 48 stay cables that are anchored on the top of two 45m-high towers, but spaced by 1,5m in the tower resulting in different inclinations, which vary from 26 to 64 degrees. The stay cables are composed of 15.7mm strands (with 7 wires) of CP190RB steel (f<sub>ptk</sub>=1900MPa), and are lined with polyethylene pipes for protection. The concrete used was C45 (f<sub>ck</sub>=45 MPa) for the deck and C35 (f<sub>ck</sub>=35 MPa) for the tower. The cross section, in turn, is formed by two cells with a constant width of 12m, with each track being designed as a slab with a width of 2,80m and thickness of 0,41m to support the two rails (Figure 4).



Figure 3. Cable-stayed railway bridge over the Ayrton Senna and Hélio Smidt freeways (Oyamada et al. [14]).

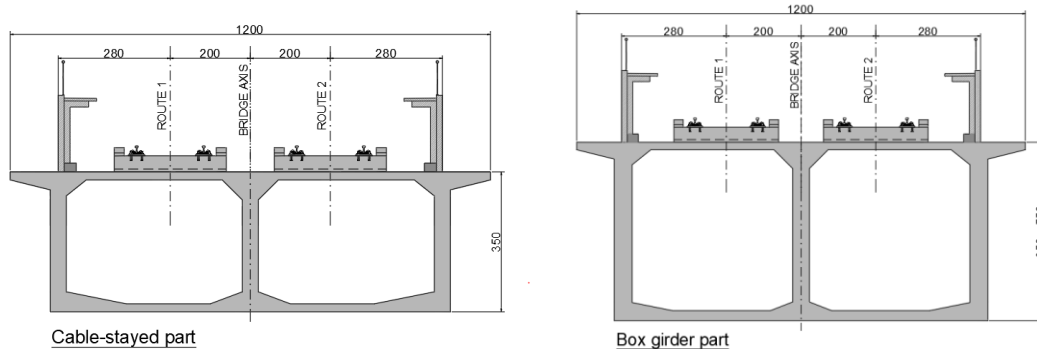


Figure 4. Typical Cross Section.

### 3.1 Construction phases

The knowledge of the stresses and deformation levels as the function of the different construction stages is essential so that the corresponding ultimate limits are not exceeded and thereby both the strength and stability of the structure is guaranteed throughout its construction phases. The construction schedule is shown below. Firstly, the meso and infrastructure were built, including the piles cap, columns and temporary supports. Secondly, the starting deck segments were built on temporary supports. Before advancing the scaffolding truss and doing the stay cables tensioning, the first phase of the tower was built, including the transversal beam. Then, the first deck segments were built by the cantilever method. The whole scheme is illustrated in Figure 5a.

The segments in the cantilever method were sequentially added on each side of the towers and the supporting columns through advancement of the trusses supported on the previously executed segments. In the cable-stayed span, the stay cables were anchored and tensioned. After execution of the fifth segment, on each side, plates of the walkway, trays and reinforcement of the permanent track were launched, and without this stage, the work could not proceed. The end transversal beam and the above deck segments were built. Concomitantly, the second part of the tower was built (Figure 5b). The rest of the segments of the box girder segmental bridge, except for the closings and the segments 6 to 13 of the cable-stayed part, were built, and the stay cables tensioned. The trusses of the last segments between the cable-stayed part were kept in place, and 200 kN counterweights were added in the center of the segments 12 and 13 of the central span (Figure 5c). The end closing segments were built. Subsequently, plates of the walkway, trays, and reinforcement of the permanent track were launched in the other sections. In the end supports, a length of 5,20m was filled with 130m<sup>3</sup> of concrete. Later, the unbalanced segments were added, which were used to reduce the deformations imposed on the tower due effects of creep and shrinkage of the concrete. Then, the rest of the closing segments were added with the removal of the counterweights and the truss half supported by each previously segments (Figure 5d). In the end, the temporary supports and truss were removed. The bridge was ready to receive the permanent track of the railway and the aerial network services (Figure 5e).

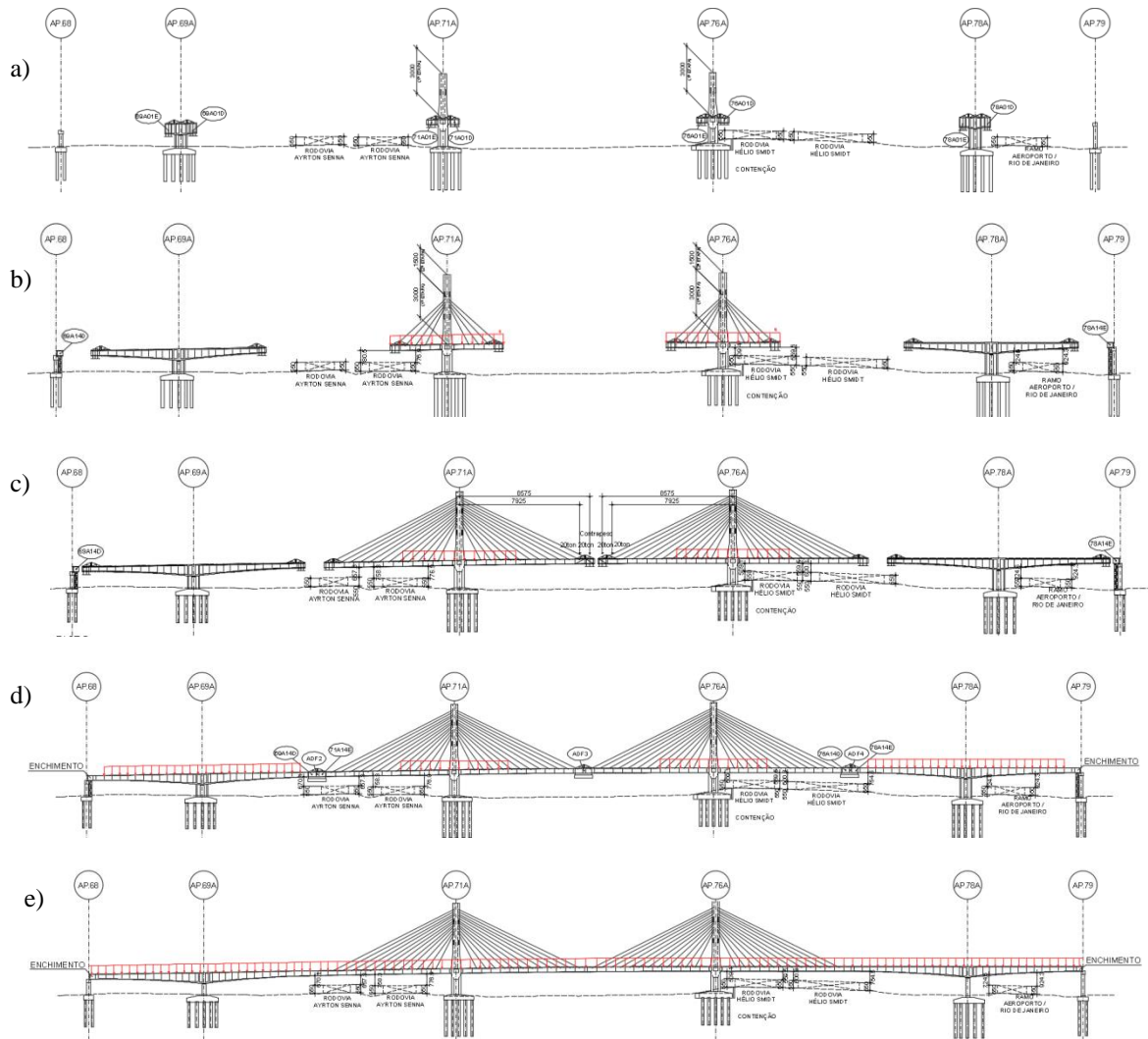


Figure 5. a) First stage of the tower and first deck segment building. b) Construction until the fifth segment, and building of the second part of the tower. c) The box girder segmental bridge, except for the closings and the segments 6 a 13 of the cable-stayed part built. d) Closing segments added. e) Final structure.

### 3.2 Numerical analysis

The computational model devised for analysis of the bridge is a 3D elastic finite element model with beam elements in the software SAP2000 v.14, considering the construction phasing. The deck was considered fixed on the tower, because of the monolithical connection with the first segments due to the construction method. The stay cables are modeled as articulated beams at both ends. The columns of the box girder segmental part of the bridge were connect to the deck by elastomeric metallic support devices, thus, they were considered articulated at the top, and during the construction phases, before the execution of the closing segments between the cable-stayed part and the box girder segmental part, they are in addition on temporary columns to maintain the stability of the structure during the construction. As a beam model was adopted, with one beam axis for the deck, considered as rigid cross section, the connection of the stay cables on the deck and tower was made through rigid bars, that is, bars of very high inertia, which are used to transfer the axial forces from the cables to the deck and tower. The deck was considered as rigid cross section. The deck on the box girder segmental part with the variable-height cross-section from 7,5m to 3,5m was considered through using 19 different cross-sections along the discretized deck. The time-dependent effects (creep, shrinkage and varying stiffness) of the concrete deck were considered according to the materials applied and constructive sequence following the FIB model MC1990 [15].



## 4 Results

Applying the Ytza [9] method, the installation stay cable forces obtained are depicted in Figure 6. For this case, the deck control points are the middle points between the stay cables in the section. So, the two lateral cables planes are considered with the same installation force.

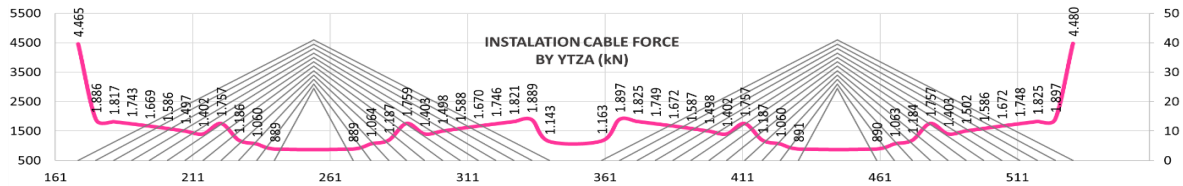


Figure 6. Installation stay cable forces by Ytza [9] (kN).

Applying the Arici [5] method, in turn, the deck control points are at the anchorage of each stay cable. Using the influence matrix, the values each cable planes obtained at the moment of the stay cables tensioning are indicated in Figure 7, and the forces during the phases 1 to 15 are plotted in Figure 8.

The resulting displacements at the final stage for both methods are shown in Figure 9, whereas the bending moment envelopes and the final moments are presented in Figure 10.

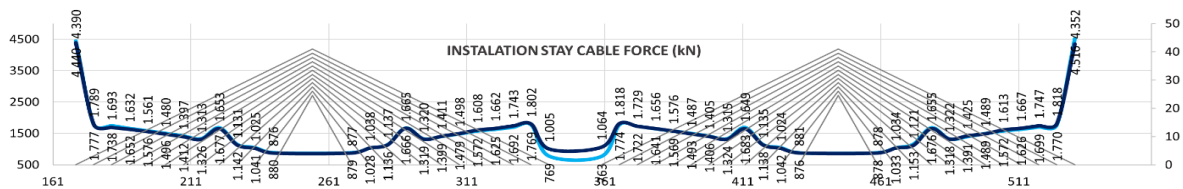


Figure 7. Installation stay cable forces (kN).

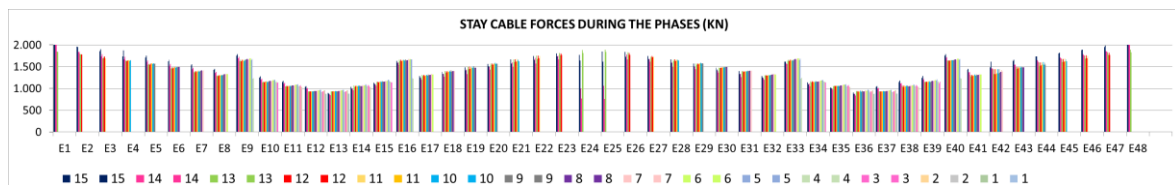


Figure 8. Stay cable forces during the phases (kN).

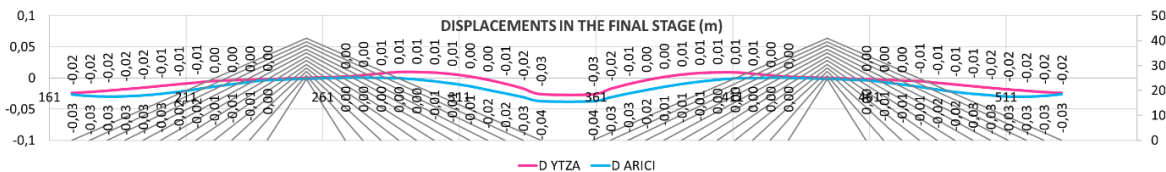


Figure 9. Displacements at the final configuration (m).

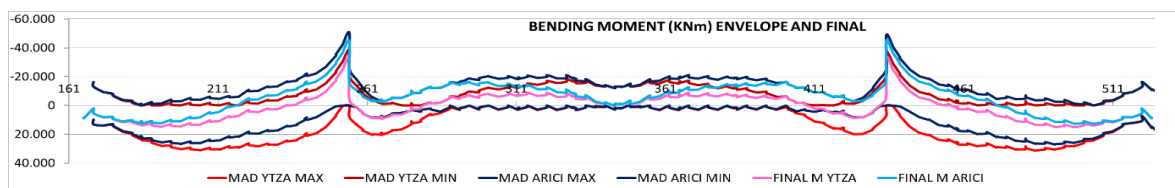


Figure 10. Bending moment envelope and at the final configuration(kN/m).

## 5 Conclusions

Among the methods of determination of stay cable forces studied in this work, the easiest to be used considering the construction stages is the one proposed by Ytza [9]. However, for bridges with large curvatures and two cable planes, such as the one studied here, it is more appropriate to find the tensioning forces on each stay cable in the tensioning phase through the influence matrix, as proposed by Arici [5]. The tensioning forces on the stay cables do not show major changes for each of the stages before the closing segments with the girder bridge part, as the deck returns to its reference line in each tensioning stage. Thus, the forces remain more or less constant throughout the phases. Analyzing the displacements and the moment distribution, it follows that for the case studied here it would not be necessary to adjust the previous cables, as proposed by Arici [5]. It should be noted that this is an exploratory study with the objective to assess the methods on the definition of the stay cable axial forces considering the constructive stages. In this study, the prestressed cables of the deck were not considered, despite being a prestressed concrete deck bridge. Like the construction phases, the prestressing cables may also influence the stay cable forces, since they have an influence on the deck's displacements and stresses, which would change the stay cables forces. This is a matter of future research.

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