XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Finite element model for pretensioned concrete beams using partial interaction theory

João Batista M. Sousa Jr.[1], Evandro Parente Jr[1], Eduardo F. Morais[1]

[1]*Laboratório de Mecânica Computacional e Visualização*
DEECC, Universidade Federal do Ceará
Campus do Pici, bloco 728, Fortaleza, CE, Brazil
joaobatistasousajr@ufc.br, evandro@ufc.br

Abstract. Pretensioned beams are an efficient load-carrying structural system, and are usually precast. Tendons are pretensioned on a pretensioning bed, and the concrete is cast in contact with these tendons. After curing, the tendons are cut resulting in a self-equilibrating system with improved mechanical properties. The bond and slip between tendon and concrete may be simulated in different ways depending on the purposes of the model. Recent works have employed partial interaction assumptions on the modelling and simulation of pretensioned concrete beams. Analytical results have been provided giving some insight as to how a numerical modeling by the finite element method (FEM) should be developed. in this paper, a FE formulation for the linear analysis of pretensioned concrete beams is described, developed and tested. The bond-slip relationship is taken identical to the form which is used in partial interaction composite beam analysis, frequently associated to steel-concrete or other bimaterial beams. The relative merits of the ensuing formulations, the ocurrence of locking and the precision of the numerical schemes are addressed with respect to analytical solutions.

Keywords: pretensioned concrete, prestressed oncrete, partial interaction

1 Introduction

Pretensioned concrete is regarded as a very efficient solution for girders and has been employed massively by the precast element industry. This technique involves the early tensioning of tendons, either of steel or a composite material, prior to concrete pouring. When a certain desired resistance is attained the tendons are cut and there is normal stress transfer between tendon and concrete, resulting in an more efficient behavior under flexure, for service as well as ultimate, see Figure 1. Worldwide, pretensioning industry employs long stressing beds which allow a sound productivity with a set of straight tendons which transfer the tensioning force simultaneously to many concrete elements.

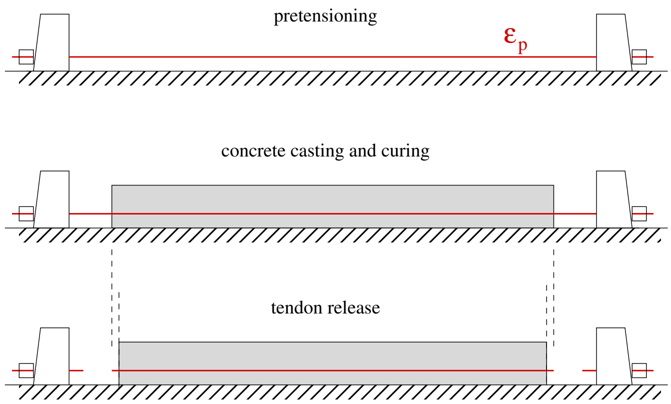

Figure 1. Single pretensioned concrete beam production

The structural analysis of pretensioned beams means the evaluation of displacements, section forces, as well as the distribution of the final stress and strain of the tendons after the transfer of force between the elements, eventually including time effects. Due to elastic shortening and the need of a certain length to transfer the force from tendon to concrete, with influence on bending and transverse force values, it is important to evaluate these values precisely, especially near both beam ends where the tendon and concrete stresses are reduced.

Several models have been proposed to evaluate the bond-slip relationship between steel and concrete, obtain transfer lengths, prestressing losses and identify which are the most influential parameters. According to Deng et al. [1], the two most widely accepted models are the bond–radial expansion model and the bond–slip model. The first employs rigorous analytical or numerical schemes in three dimensions, while the latter tries to find a closed-form equation for the assessment of the interaction between concrete and tendon, and is simpler to employ. Usually, practice codes provide approximate formulas to evaluate these transfer lengths, see Martí-Vargas and Hale [2] or Dang et al. [3]. Research on behavior of pretensioned elements, including bond-slip and transfer length evaluation, is still active, including analytical and numerical studies which take advantage of large sets of experimental tests, see Deng et al. [1], Oh et al. [4], Martí-Vargas et al. [5], Pellegrino et al. [6], Mohandoss et al. [7].

The theory of partial shear connection in composite beams has grown since its first applications, based on the analytical developments of which is known in the field as the Newmark model. In its early developments this scheme was based on Euler-Bernoulli beam theory, with a linear slips-connection forces per unit length. The top and bottom beam components share the vertical displacements and derivatives (e.g. rotation and curvature) leading to simple ordinary differential equations, which render the two axial and transverse displacements, from which the slip is evaluated.

Based on this theory, a large amount of analytical and numerical work followed. Further developments and extensions of the basic formulation include the consideration of Timoshenko beam assumptions by Girhammar and Gopu [8], Xu and Wu [9], higher order beam theories by Uddin et al. [10], nonlinear geometric models, dynamics and stability byGirhammar et al. [11], Ranzi et al. [12], Nijgh and Veljkovic [13]. The generic multilayered case has also been subject of analytical and numeric studies, see Sousa and da Silva [14] and Sousa Jr. [15]. Numerical solutions mainly by the finite element method have been developed for many situations of composite beams, see for example Sousa Jr. et al. [16].

Recently, Bai and Davidson [17] connected pretensioning and partial-interaction theory by recognizing the applicability of the partial composite beam models for the analytical analysis of pretensioned beams. Based upon a linear mechanical model, they presented closed-form expressions for displacements, normal forces and bending moments of a simply supported pretensioned beam and showed the suitability of the formulation for several related problems, such as transfer length evaluation and short-term loss of prestressing. Moreover, the authors stated that reinforced concrete beams could be analyzed by their formulation. Later, Sha and Davidson [18] extended the theory for beams prestressed with FRP tendons including some nonlinear force-slip relations.

The purpose of the present paper is to apply partially connected beam theory to develop, implement and test displacement-based finite elements able to simulate the behaviour of pretensioned beams under linear elastic assumptions. As in these structures it is common to have uncracked concrete sections, the linear elastic hypothesis is adequate for the determination of the behaviour of pretensioned beams under service loads.

The FE formulation departs from the Principle of Virtual Work. Proper interpolation schemes are analyzed and the merits of the formulations are discussed. The slip locking phenomenon is shown in terms of the erroneous slip distributions which appear due to load application. Although not carried out in the present paper, the basic formulation may be easily extended to cope with nonlinear bond-slip models.

2 Basic assumptions

In the development of the numerical formulation the following assumptions are made: (i) plane sections remain plane, and the reference axis goes through the centroid of the concrete section. Therefore, the axial displacement is

$$u(x, y) = u_c(x) - y\,\theta(x) \tag{1}$$

where u_c is the axial displacement of the reference axis and θ its rotation; while vertical displacement is the same for every section

$$v(x, y) = v(x) \tag{2}$$

(iii) the pretensioning operation is performed by an imposed initial strain ε_0 constant along the tendon; (iv) the tendon force comprises the initial prestressing force N_p and the incremental force due to displacement after release:

$$N_s = N_p + \Delta N_s = N_p + E_s\,A_s\,u_s' \tag{3}$$

(v) linear elastic properties for the concrete ($\sigma_c = E_c\varepsilon_c$) and for the tendon ($\sigma_s = E_s\,\varepsilon_s$); (vi) the kinematic hypothesis for the slip between the tendon and the surrounding concrete takes into account the axial displacements and section rotation through

$$s = u_s - u_c - e\,v' \tag{4}$$

where e is the tendon eccentricity, positive downwards; (vii) Linear relation between tangential force per unit length and the slip

$$F = Ks, \tag{5}$$

with the total slip force related to the distributed force $\frac{\mathrm{d}S}{\mathrm{d}x} = F$.

These assumptions are quite similar in structure to the ones used in partial interaction composite beam analysis. The independent variables are the concrete axial displacement u_c, the tendon axial displacement u_s and the reference axis displacement v, see Figure 2.

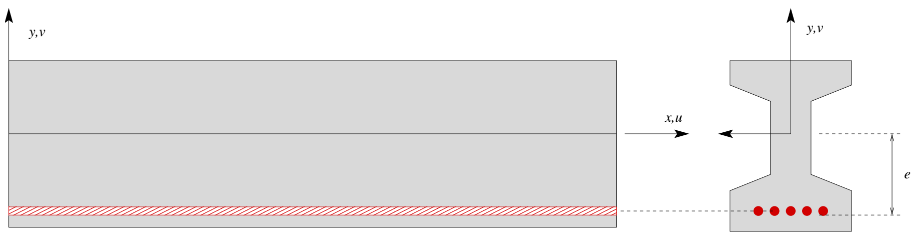

Figure 2. Pretensioned concrete beam

3 FE formulation

Under Euler-Bernoulli assumptions, following the continuity requirements, the interpolation schemes may be defined as lagrangian for the axial displacements and hermitian cubic for the transverse displacements. The implementation allows usage of different degrees of freedom for the axial interpolant, and is has been shown in the study of composite beams that linear interpolation gives rise to the appearance of the so-called slip locking phenomena, as will be shown later.

The generalized strains are the axial concrete and tendon strains, the curvature and the slips between each tendon and the surrounding concrete layer.

$$\epsilon^T = \left\{ \begin{array}{cccc} \epsilon_c & \epsilon_s & \kappa & s \end{array} \right\} \tag{6}$$

The generalized strains are obtained from the displacement field through

$$\epsilon = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_x & 0 \\ 0 & 0 & \partial_{xx} \\ -1 & 1 & e\partial_x \end{bmatrix} \left\{ \begin{array}{c} u_c \\ u_s \\ v \end{array} \right\} = \boldsymbol{\partial}\,\mathbf{u}(x) \tag{7}$$

It is easy to realize that the basic degrees of freedom of this displacement-based formulation are the displacements u_c and u_s, as well as the transverse displacements v and rotations $\theta = v'$. Previous works have used bond-slip degrees of freedom in the numerical analysis Limkatanyu and Spacone [19, 20], but here the interpolation is performed not on the slip but rather on the individual components:

$$u_c = \mathbf{N_u}^T\mathbf{u_c} \qquad u_s = \mathbf{N_u}^T\mathbf{u_s} \qquad v = \mathbf{N_v}^T\mathbf{v} \tag{8}$$

where vectors $\mathbf{u_c}$, $\mathbf{u_s}$ collect the nodal values of concrete and tendon axial displacements, and vector $\mathbf{v}$ the transverse displacements and its derivatives at the end points. The element degrees of freedom are

$$\mathbf{u}_e^T = \left\{ \begin{array}{cccccccc} u_{c1} \ldots u_{cn_u} & u_{s1} \ldots u_{sn_u} & v_1 & \theta_1 & v_2 & \theta_2 \end{array} \right\} \tag{9}$$

which may be easily rearranged in a sequence more adequate to globalmatrix and vector assembly.

Introducing the interpolation schemes into the expression for the generalized strains, the linear strain-displacement matrix for the element is obtained as

$$\boldsymbol{\epsilon} = \boldsymbol{\partial}\mathbf{N}\mathbf{u} = \mathbf{B}\mathbf{u} \tag{10}$$

Matrix $\mathbf{B}$ depends on the axial interpolation chosen. Minimum potential energy as well as virtual work may be used to obtain the finite element stiffness and load vector. The second is chosen as it is more general and extensible for the nonlinear cases. The internal virtual work for a single finite element is given as

$$\delta U = \int_{V_c} \delta\epsilon_c \sigma_c \mathrm{d}V_c + \int_{V_s} \delta\epsilon_s \sigma_s \, \mathrm{d}V_s + \int_{\ell_s} \delta s F_s \mathrm{d}x \tag{11}$$

Expressing concrete strains

$$\epsilon_c(x, y) = \epsilon_{c,0}(x) - y\kappa(x) \tag{12}$$

and noting that the tendon stresses may be written as

$$\sigma_s = E_s \left(u_s' + \epsilon_0 \right) \tag{13}$$

The generalized strains and stresses may be collected for the internal virtual work

$$\delta U = \int_0^\ell \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma} \mathrm{d}x \tag{14}$$

where the generalized strains are defined as

$$\boldsymbol{\varepsilon}^T = \left\{ \begin{array}{cccc} \varepsilon_c & \varepsilon_s & \kappa & s \end{array} \right\} \tag{15}$$

and the generalized stresses

$$\boldsymbol{\sigma}^T = \left\{ \begin{array}{cccc} N_c & N_s & M & S \end{array} \right\} \tag{16}$$

The linear material behaviour of the materials leads to the generalized constitutive relation

$$\sigma = \mathbf{C}\varepsilon + \sigma_0 \tag{17}$$

where matrix $\mathbf{C}$ is given by

$$\boldsymbol{C} = \begin{bmatrix} E_c A_c & 0 & 0 & 0 \\ 0 & E_s A_s & 0 & 0 \\ 0 & & E_c I_c & 0 \\ 0 & 0 & 0 & k \end{bmatrix} \tag{18}$$

and centroidal axis are assumed for the concrete section. The initial stress vector is due to the pretensioning strain

$$\sigma_0^T = \left\{ \begin{array}{cccc} E_s A_s \varepsilon_0 & 0 & 0 & 0 \end{array} \right\} \tag{19}$$

and does not depend on the element displacements, contributing to the right-hand side of the equations.

Introducing the displacement interpolation into the virtual work expression the internal force $\mathbf{g}_e$ along with the element stiffness $\mathbf{k}_e$ matrix may be obtained

$$\mathbf{k}_e = \int_0^L \mathbf{B}^T\mathbf{C}\mathbf{B}\mathrm{d}x \tag{20}$$

The element load vector has the influence of the transverse load $q(x)$ and is derived from the expression for the external virtual work:

$$\delta W_{ext} = \int_0^\ell \delta v q(x)\mathrm{d}x \tag{21}$$

It may be noticed that the initial prestressing might as well be considered as an external applied load.

4 Application –Test data by Kim, reported by Bai and Davidson

In order to check their analtical developments, Bai and Davidson [17] compared results from several tests of pretensioned concrete beams by Oh and Kim [21]. For all the beams, the span length was 3 m, with rectangular cross sections with 200 mm height and variable widths (from 112.7 to 191.4 mm) . Two different prestressing tendons, with 12.7 and 15.2 mm diameter grade 1860 MPa were employed, with varying eccentricities. Elastic modulus for the prestressing strand was taken as 196.5 GPa. Strands were tensioned to 75% of their ultimate state, with initial prestress equal to 0.0071. Single and double-strand specimens were tested.

In the analysis the concrete modulus of most of the specimens was taken as 27.4 GPa . The specimens were placed on the ground before prestress release and subjected to self weight after release. For single strand specimens two identical tests were carried out, and one test for each double strand configuration. In this paper the beam M13-H-C4 was chosen to present detailed results of the analytical and numerical analyses. It is a single-strand beam with 6 mm eccentric tendon, transverse load of 531.2 N/m and the connection stiffness K is taken as 2.785×10^8 N/m^2. Regarding the FE solution, two different elements are employed. Both of them use cubic hermitian interpolation for the transverse displacement, but the element labeled as PTL uses linear interpolation for the tendon and beam axial displacements, while element PTQ uses quadratic functions. The goal is to confirm whether the slip locking happens for this kind of problem as it is of the same nature of the classical steel-concrete composite beam. Four and eight-element meshes were employed for the beam (no symmetry employed).

The results for slip, curvature, transverse displacement, normal force on tendon and moment on concrete section are plotted in Figure 3. The initial tendon force is also shown, illustrating the loss of tendon force due to the transmission of stress between steel and concrete.

5 Conclusions

In this paper, a displacement-based finite element formulation for the numerical simulation of pretensioned composite beams was developed, implemented and tested. The formulation is based on the kinematical hypotheses of partially connected composite beams. Different finite elements were implemented with these assumptions and good results were obtained in the simulation of the linear problem, allowing the determination of various important features of the problem solution, such as the transfer length determination, the stress loss due to axial shortening and the relative slip between steel and concrete. Slip locking was noticed in elements with linear interpolation of axial displacements, so the quadratic interpolation for these variables is recommended. Although not carried out in this paper, the extension for nonlinear material properties or bond-slip equations is straightforward.

Acknowledgements. The authors wish to thank CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) and FUNCAP (Fundação Cearense de Amparo à Pesquisa) for the financial support to this work..

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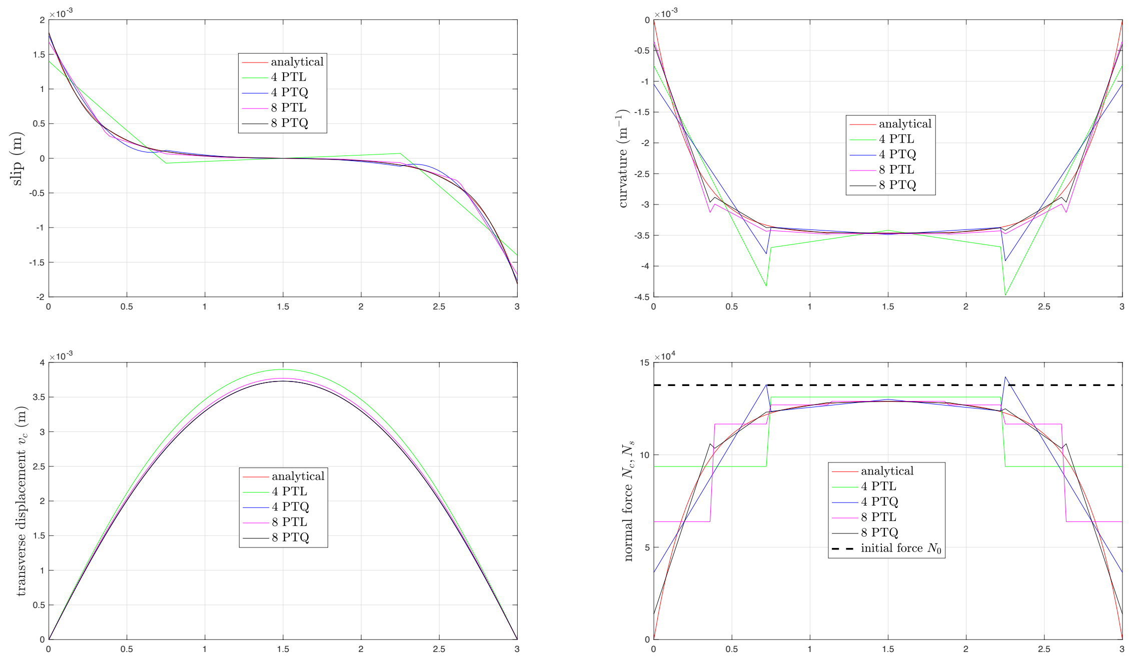

Figure 3. results for slip

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