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Parametric analysis of prestressed beam variation force with nonlinear feature through nonlinear finite element analysis

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Abstract. Prestressing concrete has been used as a good options to beams and plates mainly those structures which has enough strength to support large loads and distance. Parameters like fields strain and tension must be considered in the mathematical models to design structures and predicts its behavior. In this article the main goal is to analyze the relationship between loads and displacement of prestressing concrete structures with bound. The approach taking into account the methods of nonlinear finite elements to describe the equations of tensor fields regarding the strain/tension tensor fields. In order to analysis physical nonlinearities constitutive models are assumed. On the other hand the geometrical nonlinearities are analyzed by means of a beam supported in both ends with rectangular transversal section. The strain of the beam is given by the Euler-Bernoulli theory. In addition the prestressing tendon is modeled as truss element. With analytical techniques based on nodal equilibrium forces are employed to predict its mechanical behavior. A huge amount of results coming from literature is performed in order to get news insights regarding the displacements in cables. The equilibrium conditions are obtained by means of the equations regarding the nodal displacements. Thus the tangent stiffness matrix is assembled. Finally the article is closed by comparing the numerical with experimental results.

Keywords: Prestressed Concrete, Finite elements, Nonlinear Analysis.

1 Introduction

In structural elements subject to bending deformation, longitudinal reinforcement is considered in order to mitigate the effects due to tensile. One of its main features is that they are requested from the moment of concrete deformation. Thus from such deformation the steel tendons start to resist the tensile efforts. This kind of structures is known as passive reinforcement, that is, they start work after concrete loose its strength.

The reinforcement structures embedding into concrete that start to work before it loose its strength is known as prestressing structures. This happen because the prestressng steel, which is tensile stresses by means of appropriate device, grows the capacity of concrete to support large loads even before its deformation. For this reason the steel designed in this way in the interior of concrete is called active reinforcement. Thus, according to ([4]), one concluded that the main goal of the prestressing concrete is reduce the deflection and the tensile cracks at concrete by introducing additional normal compressed tension to cancel ordinary loads in which that structures are subjected. It is well known that in regions where the structure are subjected to tensile tension the strength contribution of concrete is poor, 10 times less if compared to compressive strength. This shows that the contribution of concrete regarding the safety performance of structure is small in such regions, [4]. On the other hand regarding the types of prestressing it can be divided into bounds and execution. Regarding bound, it can be designed at initial of process which consist of bound between steel and concrete before application of concrete and the other option is to consider the prestressing of steel after hardened concrete. Generally the prestressing strength is transfer due to the anchor process between the prestressing concrete and the equipment of prestressing. The prestressing method without bound have been one of the mainly methodologies employed in civil engineering projects where the use prestressing is necessary, for instance, building constructions, [9, 10], due to its good performance regarding execution process. However from numerical simulation point of view, the prestressing methodology without bound introduce additional difficulty in order to get the equilibrium equations once there is no complete adherence between the concrete and the prestressing tendon. Because of this, few numerical simulation have been employed if compared to experiments which have been conducting in order to get a better knowledge about the mechanical behavior of strucutes subject to prestressing forces, [1, 2, 5, 7, 11–14].

In order to get a better insight about the behavior of strucutes subject to prestressing forces, in the last years many numerical models have been employed. In [6] the authors adopt numerical analysis based on incremental deformation method to analyses the concrete behavior of both service and ultimate loads by using nonlinear models to predict the deformation of concrete upon the prestressed steel.

In the last years, many issues regarding the stability of structures have been considered in literature, between that we quoted a few examples: deformation analysis of prestressed concrete with tendon projected to outside structure can be seen in [1, 3, 5–7, 9–15]; bending analysis of beam with high eccentricity can be founded in [2]; problems boundness prestressing can be founded in [3]; second order effects were studied in[6]; combined effects due to axial and bending loads can be founded in [8].

2 Mathematical modeling

At present work the relationship between stress/strain tension to a prestressed concrete beam with bound are analyzed. In order to get the equations the beam is modeled using the assumptions of Euler-Bernoulli theory. Next the prestressing steel are molded as truss element the same being truth to reinforced steel. Using material models well established in the literature we achieves the equilibrium equations of the structure. Next the nonlinear theory of Galerkin method are employed to get the tangent stiffness matrix and using a numerical method proposed in (zienkiewicz) the numerical solution is obtained. A typical structure element at present work is described in Figure 1,below By assumptions the shear deformation are neglected and the transversal section remain plans after

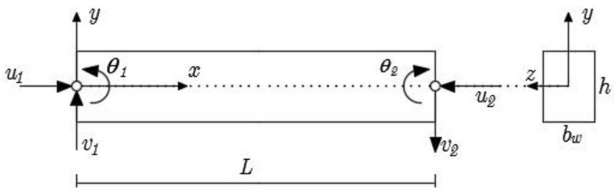

Figure 1. Beam geometry

deformation. The element contains two nodes and six degree of freedom, each of them presents three displacement being one axia, one transversal and other rotation.

The use of axial displacement is justified by the presence of axial forces due to the prestressed steel. Thus the field displacement are given by

$$u = a_0 + a_1 x, \tag{1}$$

$$v = b_0 + b_1 x + b_2 x^2 + b_3 x^3, \tag{2}$$

where u, v stands, respectively, the axial and vertical displacement of structure. The coefficients, $a_i, b_j, i = 0, 1, j = 0, \ldots, 3$ are obtained using the boundary conditions, which means that them must satisfy the following conditions

$$a_0 + a_1 x_i^e = u_i^e, \tag{3}$$

$$a_0 + a_1 x_j^e = u_j^e, \tag{4}$$

where in equations (3)-(4), u_i^e represents the axial displacement of node i connected to element e. Solving the system (3)-(4) we obtain the following formula to axial displacement of element e,

$$u^e(x) = N_1^e(x) d_i^e + N_2^e(x) d_j^e. \tag{5}$$

In (5), L_{ij}^e stands the length of element e given from the nodes i and j, and the shape functions $N_i^e, i = 1, 2$ are given by:

$$N_1 = 1 - \frac{x}{L_{ij}},\; N_2 = \frac{x}{L_{ij}}, \tag{6}$$

In order to calculate the vertical displacement it is necessary to consider the compatibility equations at nodes i e j, in a similar way to axial displacement, we can find the coefficient $b_j, j = 0, \ldots, 3$. Thus the vertical displacement can be expressed as below,

$$v^e(x) = \sum_{k=1}^{4} N^e_{ik}(x) d^e_{ik}, \tag{7}$$

where

$$d^e_i = \left[v^e_i \; \theta^e_i \; v^e_j \; \theta^e_j \right]^T, \tag{8}$$

and

$$N^e_i = \left[N^e_{i1} \; N^e_{i2} \; N^e_{i3} \; N^e_{i4} \right], \tag{9}$$

whit

$$N^e_{i1}(x) = \frac{1}{(L^e_{ij})^3} \left(2x^3 - 3x^2 (L^e_{ij}) + (L^e_{ij})^3 \right), \tag{10}$$

$$N^e_{i2}(x) = \frac{1}{(L^e_{ij})^3} \left(x^3 L^e_{ij} - 2x^2 (L^e_{ij})^2 + x L^e_{ij} \right), \tag{11}$$

$$N^e_{i3}(x) = \frac{1}{(L^e_{ij})^3} \left(-2x^3 + 3x^2 L^e_{ij} \right), \tag{12}$$

$$N^e_{i4}(x) = \frac{1}{(L^e_{ij})^3} \left(x^3 L^e_{ij} - x^2 (L^e_{ij})^2 \right). \tag{13}$$

The total displacement fields U^e_{ij}, taking into account the axial displacement (5), vertial and rotational (7), can be write as

$$U^e_i = \sum_{k=1}^{6} N^e_{ik}(x) d^e_{ik}. \tag{14}$$

For $k = 1, 2$, N^e_{ik} is given by (6) and for $k = 3 \ldots, 4$, N^e_{ik} is given by (9).

By assuming the usual theory of deformation in beams, the relationship stress/strain is given by

$$\epsilon_x(x, y) = y \frac{du(x)}{dx}, \tag{15}$$

where $u(\cdot)$ represents the axial displacement of structure and y is distance of bottom to the neutral axis of structure. From the beam deformed configuration is possible write its axial displacement fields,

$$u = y \frac{dv(x)}{dx}, \tag{16}$$

where $v(\cdot)$ stands the vertical displacement of structure. Replacing (16) in (15) we have that the axial deformation is write as a function of vertical displacement fields,

$$\epsilon_x(x, y) = -y \frac{d^2 v(x)}{dx^2}. \tag{17}$$

As a way to grow the precision of the curve stress/displacement the following second order model is proposed [19],

$$\epsilon_x(x, y) = \frac{du}{dx} - \frac{y}{2} \left(\frac{dv}{dx} \right)^2 - y \frac{d^2 v}{dx^2}. \tag{18}$$

The equation given in (18) is the motivation to define generalized moment and shear as

$$m(x) = \frac{EI}{2} \left(\frac{dv}{dx} \right)^2 + EI \frac{d^2 v}{dx^2}, \tag{19}$$

$$V(x) = EI \frac{dv}{dx} \frac{d^2 v}{dx^2} + EI \frac{d^3 v}{dx^3}, \tag{20}$$

while the axial force is given by

$$F(x) = AE \frac{du}{dx}. \tag{21}$$

2.1 Internal forces of concrete

This section concerns with the construction of tangent stiffness matrix regarding the nodal equilibrium of forces. In order to do one use the expression (7) which is replaced in (19) and proceeding to derivation we obtain

$$\delta m^e(x,d^e) = EI\sum_{l=1}^{4}\left(\sum_{l_1=1}^{4} N^e_{l_1,x}(x)d^e_{l_1}\right)N^e_{l,x}(x)\delta d^e_l + EI\sum_{l=1}^{4} N^e_{l,xx}(x)\delta d^e_l. \tag{22}$$

In the same way the linearization of $V(x)$ is calculated using the equation (7), and replacing it in (20) we obtain

$$\begin{aligned}\delta V^e(x,d^e) &= EI\left[\sum_{l=1}^{4} N^e_{l,x}(x)\left(\sum_{l_1=1}^{4} N^e_{l_1,xx}(x)d^e_{l_1}\right) + \sum_{l=1}^{4} N^e_{l,xx}(x)\left(\sum_{l_1=1}^{4} N^e_{l_1,x}(x)d^e_{l_1}\right)\right]\delta d^e_l \\ &\quad + EI\sum_{l=1}^{4} N^e_{l,xxx}(x)\delta d^e_l.\end{aligned} \tag{23}$$

Regarding the axial displacement the equation (5) are used to obtain the expression

$$F^e(x) = \frac{EA}{L^e_{ij}}(\delta u^e_j - \delta u^e_i). \tag{24}$$

In (24) u^e_i stands the displacement of node i with respect the element e.
Using the equations (22), (23) and (24) the stiffness matrix of concrete K_c is assembled, which is given by

$$K^i_c(1,l) = \begin{cases} -\frac{EA}{L^e_{ij}}, \text{se } l=1 \\ 0, \text{caso contrário}, \end{cases} \quad ; \quad K^j_c(4,l) = \begin{cases} \frac{EA}{L^e_{ij}}, \text{se } l=1 \\ 0, \text{caso contrário}. \end{cases} \tag{25}$$

The terms regarding the shear forces and moment we have

$$^eK^i_c(2,l) = EI\sum_{l_1}^{4} N^e_{l_1,xx}(x_i)N^e_{l,x}(x_i)d^e_{l_1} + EI\sum_{l_1}^{4} N^e_{l_1,x}(x_i)d_{l_1}, \tag{26}$$

$$^eK^i_c(3,l) = EI\sum_{l_1=1}^{4} N^e_{l_1,x}(x_i)N^e_{l,x}(x_i)d^e_{l_1} + EI\sum_{l_1=1}^{4} N^e_{l,xx}(x_i), \tag{27}$$

$$^eK^j_c(5,l) = EI\sum_{l_1}^{4} N^e_{l_1,xx}(x_j)N^e_{l,x}(x_j)d^e_{l_1} + EI\sum_{l_1}^{4} N^e_{l_1,x}(x_j)d_{l_1}, \tag{28}$$

$$^eK^j_c(6,l) = EI\sum_{l_1=1}^{4} N^e_{l_1,x}(x_j)N^e_{l,x}(x_j)d^e_{l_1} + EI\sum_{l_1=1}^{4} N^e_{l,xx}(x_j), \tag{29}$$

In equations (26)-(29), the indexes i, j stands the nodes associates to element e.

2.2 Prestressing tension

The modeling of prestressing forces acting in an arbitrary transversal section of structure will be done as a truss element with two degree of freedom in both direction axial and perpendicular. In order to do that considers the displacement fields and deformation similar to axial displacement. Thus strain due to prestressing forces is defined as

$$\varepsilon_{xp} = \frac{d\hat{u}^e_{ij}}{dx} = \frac{d^e_j - d^e_i}{L^e_{ij}}. \tag{30}$$

and stresses at tendon is given by

$$T^e_p = AE\left(\frac{\tilde{d}^e_j - \tilde{d}^e_i}{L^e_{ij}}\right). \tag{31}$$

The nodal displacement $(\tilde{d}_i^e, \tilde{d}_j^e)$ in the local coordinate system is given by

$$\begin{bmatrix} d_i^e \\ \tilde{d}_j^e \end{bmatrix} = \begin{bmatrix} \cos\alpha_{ij} & \sin\alpha_{ij} & 0 & 0 \\ 0 & 0 & \cos\alpha_{ij} & \sin\alpha_{ij} \end{bmatrix} \begin{bmatrix} \delta u_i^e \\ \delta v_i^e \\ \delta u_j^e \\ \delta v_j^e \end{bmatrix} \tag{32}$$

Once that the displacements are obtained from equation 32 the nodal forces can be calculated due to prestressed tendon,

$$\begin{bmatrix} F_{ip}^e \\ V_{ip}^e \\ 0 \\ F_{jp}^e \\ V_{jp}^e \\ 0 \end{bmatrix} := \frac{A^e E_p}{L_{ij}^e} \begin{bmatrix} \cos(\alpha_{ij})^2 & \cos(\alpha_{ij})\sin(\alpha_{ij}) & 0 & -\cos(\alpha_{ij})^2 & -\cos(\alpha_{ij})\sin(\alpha_{ij}) & 0 \\ \cos(\alpha_{ij})\sin(\alpha_{ij}) & \sin(\alpha_{ij})^2 & 0 & -\cos(\alpha_{ij})\sin(\alpha_{ij}) & -\sin(\alpha_{ij})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \cos(\alpha_{ij})^2 & -\cos(\alpha_{ij})\sin(\alpha_{ij}) & 0 & \cos(\alpha_{ij})^2 & \cos(\alpha_{ij})\sin(\alpha_{ij}) & 0 \\ -\cos(\alpha_{ij})\sin(\alpha_{ij}) & -\sin(\alpha_{ij})^2 & 0 & \cos(\alpha_{ij})\sin(\alpha_{ij}) & \sin(\alpha_{ij})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_i^e \\ v_i^e \\ 0 \\ u_j^e \\ v_j^e \\ 0 \end{bmatrix} \quad (3$$

2.3 Analysis of Physical Nonlinearities

The nonlinear physics is analyzed by assuming the following assumptions: the transversal section remain plane during the bending deformation; there is perfect coupling due to bound between both concrete, passive and active reinforcement; the relationship stress/strain to concrete is given like [8], that is,

$$\sigma_c^{concreto} = \begin{cases} \sigma_c' \left[\frac{2\epsilon_c}{\epsilon_0} - \left(\frac{\epsilon_c}{\epsilon_0}\right)^2 \right], & \text{se } \epsilon_c \leq \epsilon_0 \\ \sigma_c' \left[1 - 0.15 \left(\frac{\epsilon_c - \epsilon_0}{\epsilon_u - \epsilon_0} \right) \right], & \text{se } \epsilon_0 \leq \epsilon_c \leq \epsilon_u, \end{cases} \tag{34}$$

where σ_c' strength tensile compression of concrete; ϵ_0 is the strain associated with σ_c'; ϵ_u is the ultimate tension of concrete.

To active reinforcement, the model adopted in this work, to stress/strain is proposed in[16],

$$\sigma_p = E_p \epsilon_p \left[Q + \frac{1 - Q}{\left(1 + \left(\frac{\epsilon_p E_p}{K \sigma_{py}} \right)^R \right)^{\frac{1}{R}}} \right]. \tag{35}$$

In equation (35), E_p is the modulus of elasticity of active reinforcement; σ_{py} is the yield tension of the prestressing steel; $K,\ Q$ e R are parameters obtained empirically whose values are $K = 1.0618,\ Q = 0.01174$ e $R = 7.344$.

2.4 Solving the equilibrium equations

In order to get the solutions of equations the Newton-Raphson method will be employed. First, we note that the second order effect will not be treated in the present work.

The equilibrium equations in each element is given by

$$F^e = K^e u + G^e, \tag{36}$$

where the tangent stiffness matrix of element e, K^e is defined as a resultant of concrete, prestressing tendon and reinforcement steel tangent stiffness matrix. The modulus of elasticity are calculated using the constitutive models of materials showed in equations 34, 35.

After applied the boundary conditions the following system of equations are obtained

$$K_{LL} u_L + K_{LP} u_P = f_L, \tag{37}$$

$$K_{PL} u_L + K_{PP} u_P = f_P. \tag{38}$$

In system equations (37)-(38), u_L represents the degree of freedom without restrictions and u_p are the prescribed nodes. Once that u_p and f_L, are knowns variables, the system (37)-(38) is solved in the following way

$$u_L = K_{LL}^{-1}(f_L - K_{LP}u_P), \quad (39)$$
$$f_P = K_{PL}u_L + K_{PP}u_P. \quad (40)$$

As (37) is nonlinear, we can rewrite it in the form

$$\Psi(u_L) = f_L - K_{LL}(u_L)u_L - K_{LP}(u_L)u_P = 0, \quad (41)$$

in such case $u_{L,n}^1 = u_n$ and the interactive process is defined as

$$du_{L,n}^i = K_{LL,T}(u_{L,n}^i)^{-1}(f_L(\mu_n)) - K_{LP}(u_{L,n}^i)u_P, \quad (42)$$
$$\Delta u_{L,n}^i = \sum_{k=1}^{i} du_{L,n}^k, \quad (43)$$
$$u_n^{i+1} = u_n^i + \Delta u_{L,n}^i. \quad (44)$$

3 Results

Using experimental results given in [17] the load/displacement curve given by the numerical model regarding to central span of structure are validate by Figure 2. In this example was modeled a beam supported in its both ends with rectangular section built with six elements, seven nodes and three degree of freedom in each node. The prestressing steel has yield tension of $f_{pyk} = 1710Mpa$ and ultimate tension of $f_{ptk} = 1900Mpa$. The peak tensile of concrete was estimated in $\epsilon_0 = 22,46\%$ and the ultimate tensile $\epsilon_u = 31,0\%$. For the concrete under tensile, the values adopted are $\epsilon_{cr} = 1.0\times10^{-5}$ and $\epsilon_{tu} = 4,0\times10^{-4}$. The Figure 2 shows the profile calculated by the algorithm proposed in the present work which can see in equatios (42)-(44). According to (42) the equilibrium equations are parametrized by external forces $f_L(\mu_n)$ in which together with infinitesimal displacement $du_{L,nk}$ set the parametric relationship which represents the second order effects.

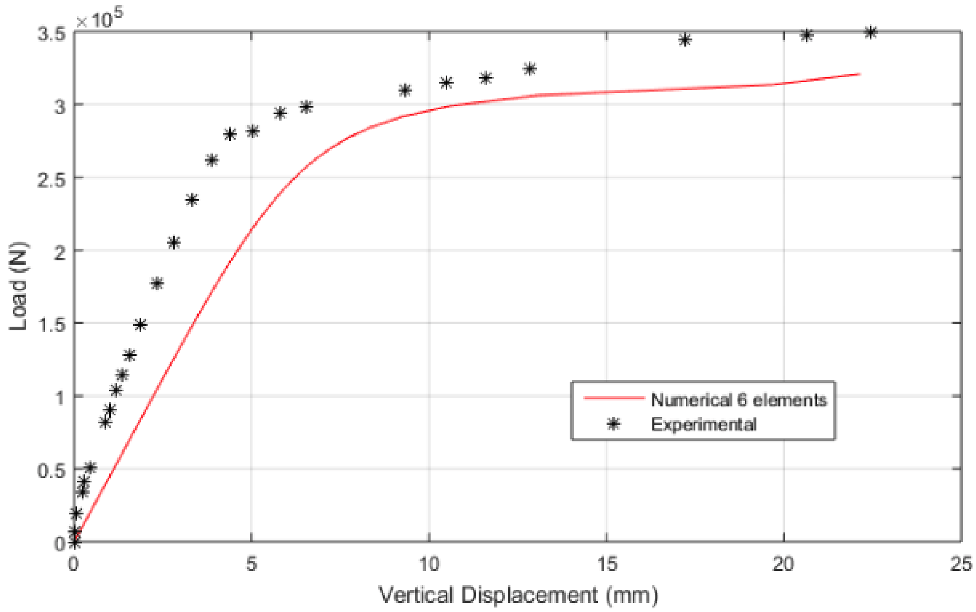

Figure 2. Load-dispalcement curve

4 Conclusions

The methodology employed in the present work presents a good performance. The numerical results when compared with the experimentals shows that the procedures adopted allows us to get a better understanding regarding how to built the tangent stiffness matrix of the structure. Taking into account the physical and material nonlinearities, due to the concrete and prestressing, the procedures adopted shows that the model has a good performance. It's worth mentioning that the methodology proposed in the present work to calculate the tangent stiffness matrix is new and constitute the most difficult part of the work. For future works we intended to consider structures like-frames which implies the second order increasing. To handle with this kind of problem the bifurcation theory will be used. For being a consolidate theory expected that analysis can identify the critical loads in which the structure goes from elastic to plastic regime. Finally, the authors pointed out the the numerical results obtained in the present work are in agreement with experimental result which are presented in Figure 2 , which leads the authors to conclude that the implementation of numerical models are correct.

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