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An iterative MsCV method coupled to the high-resolution CPR approach via different solution smoothers for the simulation of oil-water flows in 2D petroleum reservoirs on unstructured grids

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Abstract. The use of very high-order schemes to solve the transport problem, together with multiscale methods applied to multi-phase flows in petroleum reservoirs has not been explored in literature. In this work, the Multiscale Control-Volume (MsCV) method is used to solve the elliptical pressure problem while the hyperbolic saturation equation is solved using the high-resolution nodal Correction Procedure via Reconstruction (CPR) approach. In order to properly couple the previous set of equations, we use the Implicit Pressure Explicit Saturation (IMPES) strategy and a velocity reconstruction operator based on the lowest order Raviart–Thomas shape functions. In addition, a hierarchical Multidimensional Limiting Process (MLP) is employed in the reconstruction stage of CPR approach to suppress numerical oscillations. To properly couple the MsCV method with the CPR approach an adequate velocity reconstruction throughout all control volumes (CVs) is necessary to assure the accuracy of the high-order method. Thus, the velocity field must present a proper degree of accuracy that, in general, is not handed by multi-scale methods. To deal with this issue and to remove the high-frequency components of the error, we have studied several smoothers. Hence, the aim of this paper is to describe the coupling of the MsCV method with CPR for the first time in literature and to analyze the behavior and efficiency of different smoothers applied to the elliptical problem in order to produce accurate velocity field and so proper two-phase flow results. Finally, through two representative problems, which were solved to evaluate the accuracy, efficiency, and shock-capturing capabilities of our new numerical methodology, we concluded that, for the same level o accuracy, our high-order proposed methodology reduces the computational effort.

Keywords: Oil–water displacements, Anisotropic and heterogeneous petroleum reservoirs, MsCV, CPR approach, Smoothers.

1 Introduction

Over the past few years, some multiscale methods have been proposed in order to overcome some limitations of upscaling methods. Some of these methods are Multiscale Finite Element Method [3], Multiscale Finite Volume Method [8], Multiscale Mimetic Methods [10], Mixed Multiscale Finite Volume [9], Multiscale Restriction-Smoothed Basis (MsRSB) [13]. These methods introduce basis functions that are able to project the discrete system of the fine-scale in the space of the coarse-scale, that system is solved and the multi-scale operator project the solution back to the original scale of the problem. Thus, the number of degrees of freedom is reduced, as well as the amount of information lost, when compared to the upscaling methods [5]. Souza [15] proposed a multiscale method capable of delivering accurate solutions for unstructured meshes and highly anisotropic and heterogeneous cases by coupling the MsRSB with a cell-center Multi-point Flux Approximation-Diamond type (MPFA-D) finite volume method. To obtain an accurate solution to the transport problem, it is necessary to obtain a conservative and precise velocity field, which is recovered from pressure distribution, especially when high-order methods are used to solve the hyperbolic problem. However, the velocity field handed by most multi-scale methods does not have the required accuracy. In order to increase the precision of the velocity field, some smoothers were used in an iterative fashion to ensure convergence to the fine-scale solution [2, 13]. On the other hand, to solve the Saturation equation a CPR scheme is employed to obtain stable, high-order accurate numerical solutions, with lower computational cost. The high-order shape functions used by the CPR method for approximating numerical solutions are usually chosen to be Lagrangian interpolants through Gauss-Lobatto-Legendre points [7]. Our goal in this work is to demonstrate the effectiveness of the proposed methodology for the solution of the oil and water displacement in 2-D petroleum reservoirs on unstructured grids via an appropriate combination of the MsCV scheme with a smoother in an iterative fashion, and the high-order CPR method using a sequential approach.

This paper is organized as follows. In Section 2, we present the mathematical model, describing the incompressible two-phase flow of oil and water in petroleum reservoirs. In Section 3, we describe the numerical formulation of the MPFA-D scheme, the reconstruction of the Darcy velocity inside of CVs, the CPR method, the MsCV scheme and the smoother technique. In Section 4, we solve two 2-D problems in order to verify the accuracy, efficiency and shock-capturing capability of the proposed methodology. Finally, we present conclusions in Section 5.

2 Mathematical Model

In this section, we briefly present the equations and assumptions adopted to describe the two phase flow (oil/water) in petroleum reservoir rocks. We assume that the fluid flow is isothermal and incompressible and, for the sake of simplicity and without lost of generality, we neglect capillary and gravitational effects. In addition, we use a segregated formulation in which the physical phenomena are governed by two PDEs, i.e., the pressure and saturation equations, respectively. The elliptic pressure equation is expressed by

$$\vec{\nabla} \cdot \vec{v} = Q, \quad \text{with,} \quad \vec{v}_i = -\lambda_i \underline{K} \vec{\nabla} p \quad \text{where,} \quad i = w, o \tag{1}$$

Here, the total fluid injection/production rate Q is denoted by $Q = Q_w + Q_o$, which is the sum of the sink/source terms of each phase and $\vec{v}$ is the total fluid velocity $\vec{v} = \vec{v}_w + \vec{v}_o$. With $\vec{v}_i$ being the Darcy's phase velocity, where p stands for the pressure, $\underline{K}$ is the absolute rock permeability tensor, and the phase mobility is $\lambda_i = k_{r_i}/\mu_i$, where the relative permeability of phase i, k_{r_i} is given by the Brooks and Corey model [16], and μ_i is the fluid viscosity.

The hyperbolic saturation equation can be written as [4, 16]

$$\phi \frac{\partial S_w}{\partial t} = -\vec{\nabla} \cdot \vec{F}(S_w) + Q_w, \quad \text{with,} \quad \vec{F}(S_w) = f_w(S_w)\vec{v} \tag{2}$$

where ϕ is the rock porosity, S_w the saturation of the water phase and $\vec{F}(S_w)$ is the flux function, with, $f_w = \lambda_w/(\lambda_w + \lambda_o)$ being the fractional flux function, and considering the assumption that the media is fully saturated $S_o + S_w = 1$.

2.1 Initial and Boundary Conditions

Typical initial and boundary conditions for the pressure, Eq. (1), and saturation, Eq. (2) , equations, respectively are given by

$$\begin{aligned} &p(\vec{x}, t) = g_D \quad \text{on} \quad \partial\Omega_D \times [0, t], \quad \vec{v} \cdot \vec{n} = g_N \quad \text{on} \quad \partial\Omega_N \times [0, t] \\ &S_w(\vec{x}, t) = \bar{S}_I \quad \text{on} \quad \partial\Omega_I \times [0, t], \quad S_w(\vec{x}, 0) = \bar{S}_w^0 \quad \text{on} \quad \Omega \times t^0 \end{aligned} \tag{3}$$

where Ω denotes the physical domain, $\partial\Omega_D$ and $\partial\Omega_N$ represent, respectively, Dirichlet and Neumann boundaries, $\partial\Omega_P$ and $\partial\Omega_I$ production and injection wells, g_D prescribed pressure, g_N prescribed flux, $\bar{S}_w$ is the prescribed saturation in an injection well and $\bar{S}_w^0$ is the initial saturation distribution at time t^0.

3 Numerical Formulation

3.1 Implicit formulation for the pressure equation using the MPFA-D method

In order to solve the pressure equation for unstructured grids with anisotropic and heterogeneous permeability, a MPFA-D method is adopted [4]. The MPFA-D scheme, uses multiple points to approximate the flux at the interface between neighboring cells. The MPFA-D is a completely cell-center formulation, which requires the values of pressure for each vertex, at the two edge's endpoints, that are obtained using a linear weighted combination of the pressure values in the center of the surrounding cells. By using the MPFA-D scheme, we can write the Eq. (1) in its discrete form, as

$$\sum_{IJ \in \partial\Omega_k} \vec{v}_{IJ} \cdot \vec{N}_{IJ} = \bar{Q}_k V_k \tag{4}$$

where the continuous flux on the edge IJ [4] is given by

$$\vec{v}_{IJ} \cdot \vec{N}_{IJ} \simeq \tau_{IJ}[p_{\hat{R}} - p_{\hat{L}} - \upsilon_{IJ}(p_J - p_I)] \tag{5}$$

In the previous equation p_I, and p_J are the pressures at the end points (in 2D) on the edge IJ, $p_{\hat{R}}$, and $p_{\hat{L}}$ are the CV pressures at the collocation points, to the left and to the right sides of the edge IJ, and with, τ_{IJ} and, υ_{IJ} being the scalar transmissibility and the non-dimensional tangential parameter, respectively.

3.2 Reconstruction of Darcy's velocities inside the CV of a quadrilateral mesh

By considering the Eq. (5) computed using the MPFA-D method, we can recover a full velocity field within the reference quadrilateral domain, via the lowest-order Raviart-Thomas, H-Div space RT_0 [7], in the following form

$$\vec{v}_{\mathbb{R}}(\vec{\xi}) = \sum_{IJ=1}^{4} \mathrm{F}_{IJ}\varphi_{IJ} \tag{6}$$

where $\mathrm{F}_{IJ} = \vec{v}_{IJ} \cdot \vec{N}_{IJ}$ is the flow rate through a Control Surface (CS) $\overrightarrow{IJ}$, and φ_{IJ} are the RT_0 shape functions, on standard bi-linear quadrilateral $\{\vec{\xi} = (\xi, \eta) | -1 \leq (\xi, \eta) \leq 1\}$. The velocity field from the reference to the physical domain is mapped back via the Piola transformation, $\vec{v}_{\mathbb{P}}(\vec{r}) = \mathcal{P}(\vec{\xi})\vec{v}_{\mathbb{R}}(\vec{\xi})$, in such a way that the flow rate is preserved.

3.3 The explicit saturation equation

In [18], the author introduces a new differential formulation to deal with hyperbolic equations in nodal differential form, the scheme is called CPR approach. This method is shown to be very simple and efficient, and, from a practical viewpoint, its implementation is relatively easy and of low computational cost. Equation (2) in the absence of source terms, and setting $\phi = 1$, without loss of generality can be written via the CPR approach in its semi-discrete form, as

$$\begin{aligned} \frac{\partial S^{P_n}_{w(i;j,k)}}{\partial t} &= -\Bigg(\Pi_{j,k}\left[\vec{\nabla} \cdot \vec{F}_w(S^{P_n}_i)\right] + \\ \frac{2}{|\mathcal{J}|_{i;j,k}} &\left\{ \begin{matrix} [\mathcal{F}^{\xi}(-1,\eta_k) - F^{\xi}_{w(i)}(-1,\eta_k)]g'_L(\xi_j) + [\mathcal{F}^{\xi}(1,\eta_k) - F^{\xi}_{w(i)}(1,\eta_k)]g'_R(\xi_j) + \\ [\mathcal{F}^{\eta}(\xi_j,-1) - F^{\eta}_{w(i)}(\xi_j,-1)]g'_L(\eta_k) + [\mathcal{F}^{\eta}(\xi_j,1) - F^{\eta}_{w(i)}(\xi_j,1)]g'_R(\eta_k) \end{matrix} \right\} \Bigg) \end{aligned} \tag{7}$$

where, $\mathcal{J}$ is the Jacobian matrix of the transformation, $\mathcal{F}$ is a Roe approximate Riemann solver with entropy fix, Π is a projection operator, and g' is the derivative of the Nodal Discontinuous Galerkin (DG) correction function g [6]. The integration in time is carried out using a third-order Runge-Kutta method and a hierarchical MLP is used in the reconstruction stage to prevent non physical oscillations [7].

3.4 Multiscale Control-Volume Method - MsCV

The MsCV scheme presented here can be seen as an extension of the classical MsRSB. The major advantage of the MsRSB over multiscale methods in the MsFV family is the possibility to work with both coarse and fine scale unstructured meshes. The MsCV extends the MsRSB, by replacing the linear TPFA, which is only consistent on k-orthogonal fine-scale grids, by the MPFA-D method, then, we devise a framework that correctly computes the prolongation operator on general grids, therefore allowing the simulation on unstructured grids on all scales. On the other hand, to mitigate spurious oscillations on the pressure field that can also impact the velocity field, we use a smoother in an iterative fashion in order to ensure convergence to the fine scale solution [1, 13, 17]. After properly calculating the prolongation (P_{op}) and restriction (R_{op}) operators, we can write the multiscale method as

$$P_{ms} = \underline{P}_{op} P_c = \underline{P}_{op} \underline{T}_c^{-1} \underline{Q}_c \equiv \underline{P}_{op} \left(\underline{R}_{op} \underline{T}_f \underline{P}_{op} \right)^{-1} \underline{R}_{op} \underline{Q}_f \tag{8}$$

where P_{ms} and P_c are the multiscale solution on the fine-scale and coarse scale, respectively, T the transmissibility and Q the source/skin term, here the subscript f and c indicate the values on the fine-scale and coarse-scale, respectively. For further details see [15].

3.5 Smoothers

Smoothers have the ability to remove high frequency components of the error, while maintaining low frequency components. To mitigate spurious oscillations on the pressure field that can also impact the velocity field, we use a smoother in an iterative fashion that allows convergence to the fine scale solution. In this work, the performance of four iterative solvers was analyzed [14]

1. Multiscale solver [2]
$$p_f^{k+1} = p_f^k + P_{op} T_c^{-1} \left(R_{op} r_f^k \right) + M^{-1} r_f^{km} \tag{9}$$
2. Modified Multiscale solver [12]
$$p_f^{k+1} = p_f^k + P_{op} \left(T_c^{-1} R_{op} \left(r_f^k - T_f \left(M^{-1} r^k \right) \right) \right) + M^{-1} r^k \tag{10}$$
3. Multilevel solver [11]
$$p_f^{n1} = p_f^k + M_f^{-1} r_f^k, \quad p_f^{n2} = p_f^{n1} + P_{op} M_c^{-1} R_{op} r_f^{n1}, \quad p_f^{k+1} = p_f^{n2} + M_f^{-1} r_f^{n2} \tag{11}$$
4. Richardson solver [11]
$$p_f^{k+1} = p_f^k + M_f^{-1} \left(Q_f - T_f p_f^k \right) \tag{12}$$

where p_f e p_c are the pressure in the fine scale and coarse scale, respectively, T is the transmissiblity matrix and M^{-1} is the smoother, and r^k referes to the residual.

The proposed smoother technique was implemented by using different testing methods [14]: Forward Gauss-Seidel (FGS), Backward Gauss-Seidel (BGS), Forward Successive Over Relaxation (FSOR), Backward Successive Over Relaxation (BSOR) and Incomplete LU Factorization, as will be discussed in section 4.

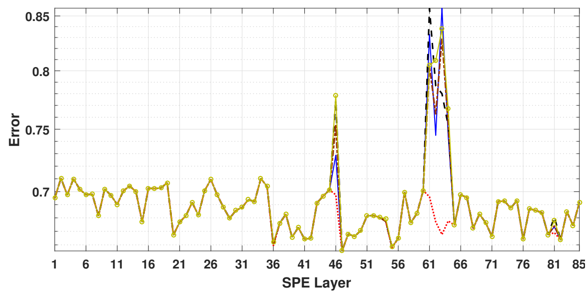
4 Numerical results

4.1 One-phase flow on the SPE 10 Data Set

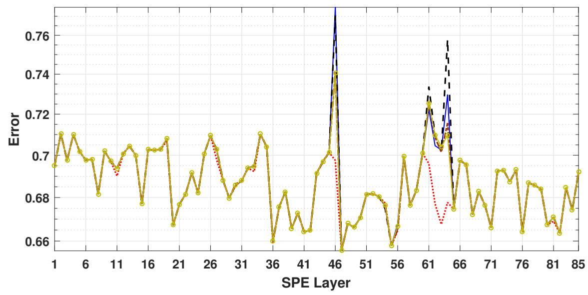
This test case was adapted from [15]. The aim of this study is to compare the reliability and accuracy of the smoother strategy on each of the 85 layers of the tenth SPE Comparative Solution Project (SPE10) using the corresponding permeability field. The SPE10 model describes a 1,200 x 2,200 x 170 (ft) field using a 60 x 220 x 85 quadrilateral structured grid [15]. However, we used a regular, and unstructured quadrilateral mesh on the fine scale, and a honey-comb coarse grid with 64 coarse cells. Then, each problem is solved using null flux conditions on the boundaries, pressure ($p_{\text{prod}} = 0$), and flux ($q_{\text{inj}} = 1$) are prescribed on the top-right corner and on the bottom-left corner of the domain, respectively. The Euclidean norm of the error for each SPE layer is shown in Fig.1, where the benefits of the previously mentioned iterative solvers are highlighted, using 15 iterations. Except for the Richardson solver, see Fig.1 (d), we can see that the methods provide a reasonably accurate solution, but the layers 46, 63 and 65 of the SPE10 model presented great discrepancies, where the errors stand up, see Fig.1 (a, b and c). The smoother obtained via the incomplete LU factorization managed to reduce errors even in the most problematic layers of the SPE10 model (Upper Ness formation), however, it can result in a significant increase in computational cost. In this work, we have adopted the BSOR method, due to its potential of delivering the same accuracy as the LU method, at a lower computational resource usage.

4.2 Water-oil flow on a highly heterogeneous and isotropic reservoir

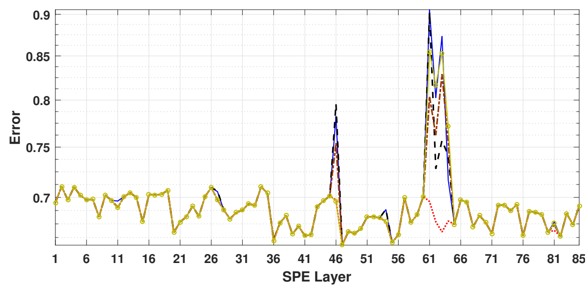
This test case was adapted from [15]. In this example, we solve an interesting test case that deals with the incompressible two-phase flow of oil and water in a reservoir medium considering a highly heterogeneous and



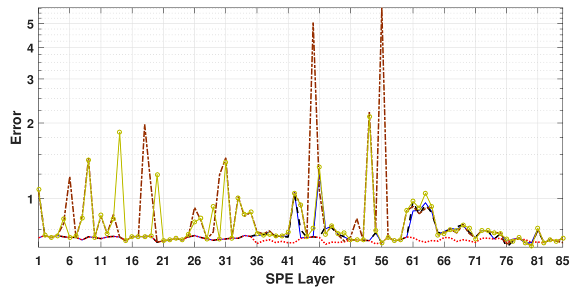
(a) Multiscale solver.



(b) Modified multiscale solver.

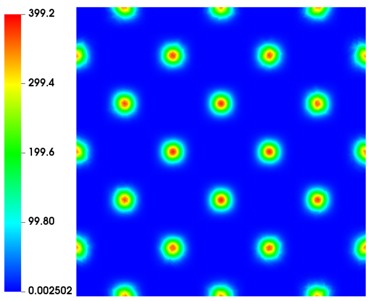


(c) Multilevel solver.

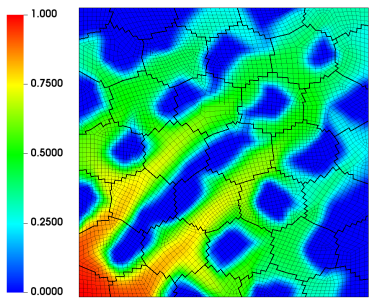


(d) Richardson solver.

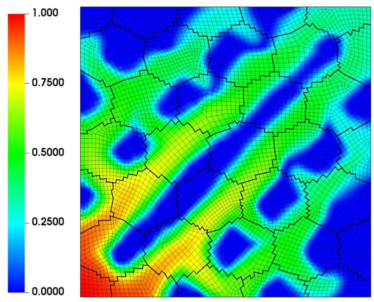
Figure 1. One-phase flow on the SPE 10 Data Set: Impact of the smoother strategy on the velocity field: Euclidean norm of the error for each SPE layer (15 iterations).



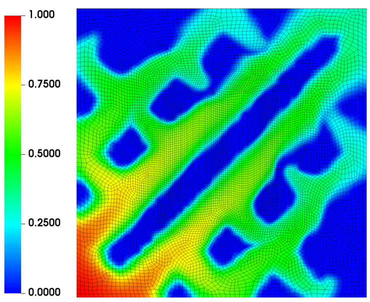
(a) Permeability field.



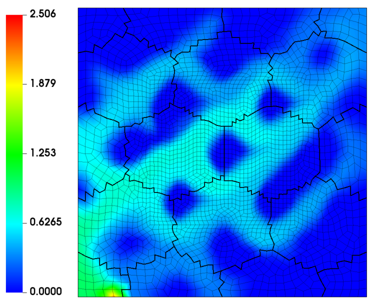
(b) MsCV-FOU-P0



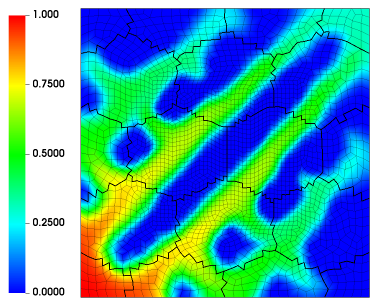
(c) i-MsCV(BSOR)-FOU-P0



(d) Reference, 11,180 CVs



(e) MsCV-CPR-P1



(f) i-MsCV(BSOR)-CPR-P1

Figure 2. Saturation profiles at PVI = 0.4 on a quarter five-spot problem computed on highly heterogeneous and isotropic reservoir, with CFL = 0.45: Saturation profiles without smoother strategy (Middle), and Saturation profiles using the BSOR smoother strategy (Right).

Table 1. Average CPU-times for different grid sizes and polynomial approximation.

Method	Fine-scale	Coarse-scale	CPU-time(min)
i-MsCV-FOU-P0	5,380 CVs	35 Cells	679.6
i-MsCV-CPR-P1	2,795 CVs	18 Cells	461.0

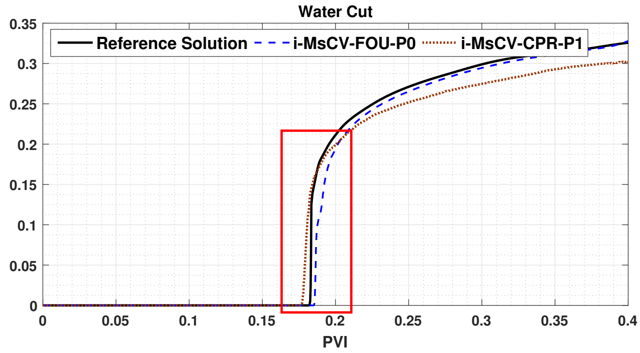


Figure 3. Water cut curves on highly heterogeneous and isotropic reservoir.

isotropic permeability field, which is given by the following expression

$$K(x, y) = \epsilon^{2\cos(6\pi x)\cos(6\pi y)} I \tag{13}$$

with I being the identity matrix, and $\epsilon = 5 \times 10^{-2}$, as illustrated in Fig. 2 (a). The computational domain consists in an unitary square $\Omega = [0 \;\; 1] \times [0 \;\; 1]$ in a $1/4$ five spot configuration in which flux is prescribed ($Q_{\text{inj}} = 1$) at the injector well and pressure is prescribed ($p_{\text{prod}} = 0$) at the producer well. Water saturation in the injection well is set to $S_w = 1$, in a reservoir initially saturated by oil ($\bar{S}_w^0 = 0$). We use this test case to assess the quality of the water saturation solution after an iterative multiscale smoother technique. As mentioned previously, the i-MsCV smoother technique drastically improves the total velocity field, which is recovered from pressure distribution, in order to compute a correct saturation solution, as presented in Figs. 2 (c and f). In the case of an MsCV computation without a smoother strategy, we can see anomalous behavior caused by an inaccurate evaluation of the velocity field required, for both low, and high order discretizations, as depicted in Figs. 2 (b), and (e), respectively. Comparing the FOU and the CPR methods in terms of resolution of the saturation front and computational efficiency for the similar level of accuracy, we observe that the CPR scheme has significant computational advantages over the FOU method, i.e., the CPR method shows an excellent improvement in resolution and reductions by around to 30% in computational effort, when compared to the FOU scheme, see Figs. 2 (c and f), and Tab. 1. On the other hand, as depicted in the inner region of the red rectangle, in Fig. 3, the solution obtained with the CPR method predicts the water breakthrough time accurately similar to the FOU approach, but the CPR scheme suffers from loss of accuracy after the breakthrough. We are working nowadays in the development of some strategies to overcome the problem of the CVs associated with production well, to make the proposed methodology more accurate and efficient.

5 Conclusions

In this paper, we have proposed a new methodology to simulate the incompressible two-phase flow of oil and water in non-homogeneous and non-isotropic petroleum reservoirs in 2D domains using unstructured quadrilateral meshes. In our new methodology, the two governing PDEs were solved using the classical IMPES formulation in which, the elliptic pressure and the hyperbolic saturation equations were approximated by the i-MsCV scheme and the CPR method, respectively. As the velocity field must present a proper degree of accuracy that, in general, is not handed by multiscale methods, we have studied several smoothers, to successfully remove the high-frequency components of the error and the residual. In order to evaluate our numerical formulation, we have solved two problems found in literature. Our results are very promising, and in the near future, we intend to extend this methodology for 3-D and more general compositional flows via h-p adaptive strategies.

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Authorship statement

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