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A new approach for modeling of fluid flow in naturally fractured porous media

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Abstract. It is important to consider the distinct hydraulic properties of the discontinuities and porous matrix in the numerical simulation of fluid flow in fractured porous medium. In this sense, many researchers have been proposed numerical models in the context of discrete fracture approaches. In general, numerical models with a discrete fracture representation present a large number of degrees of freedom to adequately represent the discontinuities, and as a consequence, requires high computational cost to solve the problem, even with a small number of fractures. The present work introduces an approach based on the use of coupling finite elements to consider the effect of the fractures in the simulation of steady-state fluid flow in fractured porous media. This scheme allows the independent discretization of fractures and matrix (overlapping non-matching meshes) without increasing the total number of degrees of freedom of the problem. Two examples are performed to demonstrate the versatility of the proposed model.

Keywords: naturally fractured reservoir, porous medium, coupling finite elements.

1 Introduction

Petroleum is a natural occurrence of hydrocarbons and inorganic impurities, usually found in liquid or gas phases in a petroleum system (Fanchi and Christiansen [1]; and Alyafei [2]). In general, conventional resources are the most used petroleum system; however, as the demand for hydrocarbons has increased, unconventional resources gained particular importance on petroleum industry (Chengzao et al. [3]; Vedachalam et al. [4]; and Zhang et al. [5]).

Reservoir rock is equally important for both petroleum systems, and consists of many pore spaces filled with oil, gas, or water. The presence of fractures in reservoir rocks can strongly influences the fluid pressure distribution since the discontinuities usually increase the permeability of the reservoirs. Thus, it is important to understand the fluid behavior in a fractured porous medium to petroleum industry. As consequence, many numerical techniques have been proposed to obtain the response of the system accurately.

The numerical approaches available in literature for modeling fluid flow in naturally fractured porous medium can be classified in three main groups (Warren and Root [6]): continuum models, dual porosity models, and discrete fracture models. The continuum models replace the fractured porous medium by an equivalent continuum model adopting average properties (Jackson et al. [7]). This kind of approach is usually applied in the simulation of large-scale problems. The dual porosity models treat the domain with a simplistic representation of complex fracture network, and the hydraulic properties of both fracture and porous medium are considered (Moinfar et al. [8]; and Zimmerman et al. [9]). Discrete models have been proposed for a more realistic representation of the fractures, since the fractures are represented individually in the porous domain. Despite the advantages of the explicit representation of the fractures, several models require that fracture and matrix elements share the same nodes in the interface of domains, which can be feasible for a problem with a single fracture, but cumbersome for a fracture network. This drawback can be solved using non-matching meshes. However, the classical models usually introduce a significant number of degrees of freedom and requires special integration schemes (Jiang and Younis [10]; Zhang et al. [11]; and Yang et al. [12]).

This work proposes a new scheme approach based on the use of coupling finite elements (Bitencourt et al. [13]) to consider the effect of the fractures in the simulation of steady-state fluid flow in fractured porous media. The formulation is based on standard element shape functions to avoid particular integration procedures and without increasing the total number of degrees of freedom of the problem.

2 Governing equations

2.1 Fluid flow in porous medium

In this work is considered a 2D fractured saturated porous medium with an incompressible single-phase fluid. In the absence of body forces and sinks (or sources), the continuity equation for the steady flow of an incompressible fluid phase over a fixed porous medium Ω_m is given by:

$$\nabla . \mathrm{v}_m = 0 \tag{1}$$

The representative element volume fluid average velocity (v_m) in the equation above is described in this work by the Darcy's law:

$$\mathrm{v}_{\mathbf{m}} = -\frac{k_{\mathrm{m}}}{\mu} \nabla p_{\mathrm{m}} \tag{2}$$

where, k_m, μ and p_m are the permeability, fluid viscosity and pressure, respectively. Finally, the standard form (Bear [14]) can be written by inserting equation (2) into equation (1):

$$\nabla \cdot \left(\frac{k_{\mathrm{m}}}{\mu} \nabla \mathrm{p}_{\mathrm{m}} \right) = 0 \tag{3}$$

To solve the latter equation, the following boundary conditions are considered: $\mathrm{p_m} = \bar{p} \in \Gamma_p$ and $\mathrm{q_m}. n_\Gamma = \bar{q} \in \Gamma_q$, where $\mathrm{q_m}$ is the fluid flux and $\Gamma = \Gamma_p \cup \Gamma_q$ is the boundary of the problem domain.

2.1.1 Weak form and finite element discretization

The weak form of the equation (3) can be obtained by multiplying the test function w (which satisfies the essential boundary conditions), and integrating over the domain Ω_m:

$$\int_{\Omega_m} w \left[\nabla \cdot \left(\frac{k_{\mathrm{m}}}{\mu} \nabla p_{\mathrm{m}} \right) \right] d\Omega_m = 0 \tag{4}$$

The governing equations can be finally written, by applying the Divergence theorem and integrating by parts the equation (4):

$$-\int_{\Omega_m} \nabla w \frac{k_{\mathrm{m}}}{\mu} \nabla \mathrm{p}_{\mathrm{m}} d\Omega_m + \int_{\Gamma_q} w\bar{q} d\Gamma = 0 \tag{5}$$

Herein, the equation (5) can be discretized in finite elements. Therefore, the pressure field in the porous domain can be approximated by:

$$\mathrm{p}_{\mathrm{m}} = \mathrm{N}_{\mathrm{m}} \bar{\mathrm{p}}_{\mathrm{m}} \tag{6}$$

where, N_{m} and $\bar{\mathrm{p}}_{\mathrm{m}}$ are the shape functions and nodal pressures, respectively. In this work three-noded triangular finite elements are used in the discretization of the porous matrix domain.

By replacing the equation (6) into equation (5), the following discretization equations are obtained:

$$\mathrm{K}_{\mathrm{m}} \mathrm{p}_{m} = \mathrm{q}_{\mathrm{m}} \tag{7}$$

where, K_{m} is the permeability matrix of the porous medium, which can be written as:

$$\mathrm{K}_{\mathrm{m}} = \int_{\Omega_m} \mathrm{B}_{\mathrm{m}}{}^{\mathrm{T}} \frac{k_{\mathrm{m}}}{\mu} \mathrm{B}_{\mathrm{m}} d\Omega_m \tag{8}$$

B_{m} stands for the partial derivatives of the shape functions of the element, and

$$\mathrm{q}_{\mathrm{m}} = \int_{\Omega_m} \mathrm{N}_{\mathrm{m}}{}^{\mathrm{T}} \bar{q} d\Gamma \tag{9}$$

2.2 Fluid flow in fracture domain

Considering the same hypotheses and procedures developed for fluid flow in porous medium, and the discretization of the fractures in 1D finite elements, the permeability matrix for the discontinuity can be written as:

$$\mathrm{K}_f = \int_{\Gamma_f} \mathrm{B}_f{}^{\mathrm{T}} \frac{k_{\mathrm{f}}}{\mu} \mathrm{B}_f d\Gamma \tag{10}$$

where B_{f} stands for the partial derivatives of the shape functions of the 1D finite elements, and k_{f} is the permeability of the fracture, for which a cubic law is employed in this work.

3 Coupling non-matching meshes

The equations (8) and (10) were developed independently. A strategy based on the use of coupling finite element is proposed in this work to consider the effect of the fractures in the simulation of steady-state fluid flow in fractured porous media

Fig. 1 (a) illustrates a problem of porous Ω_m and fracture Γ_f domains, and boundary surface Γ. Initially, porous domain and fractures are discretized in finite elements (Fig. 1(b)). Then, coupling finite elements, are inserted to establish the connection between the meshes (Fig. 1(c)) of the porous matrix and fractures, discretized initially in a totally independent way. In this work the porous matrix is discretized using three-noded triangular elements and the fractures are discretized using two-noded linear elements. So, four-noded triangular coupling elements (CFEs) are employed. A coupling element has the same nodes of an element of the porous matrix (base element), and an additional node (coupling node), herein designed by $\mathrm{C}_{\mathrm{node}}$, which is the node of the fracture that belongs to the domain of the base element. It should be highlighted that the additional node will not require more degrees of freedom in the global system of equations and it can also be located anywhere inside the element, including along its boundaries.

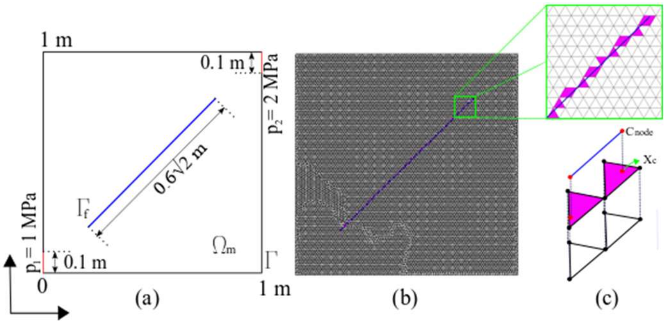

Figure 1. Case 1: (a) Geometry and boundary condition, (b) mesh discretization and (c) scheme of coupling node.

The vector that stores the pressure components of the CFEs can be written as:

$$p^{c^T} = [p_1 \quad p_2 \quad p_3 \quad p_{C_{node}}] \tag{11}$$

Therefore, a relative pressure $[\![p]\!]$ (or pressure drop) can be defined as the difference between the pressure of coupling node (C_{node}), and the pressure evaluated using the shape functions of the CFE at material point (X_c) that corresponds to the coordinates of the C_{node}:

$$[\![p]\!] = \mathrm{B^c p^c} = p_{C_{node}} - \sum_{i=1}^{3} \mathrm{N_i(X_c)}\, p_i \tag{12}$$

where,

$$B^c = [-\mathrm{N_1(X_c)} \quad -\mathrm{N_2(X_c)} \quad -\mathrm{N_3(X_c)} \quad 1] \tag{13}$$

Following the analogy between mechanical and hydraulic problems presented by Segura and Carol [15], the internal fluid flow vector can be defined as:

$$\mathrm{Q_c^{int}} = \mathrm{B^{c^T} C B^c} p^C \tag{14}$$

and the permeability matrix of the CFE can be obtained by deriving the internal fluid flow vector with respect of pressure:

$$\mathrm{K_c} = \mathrm{B^{c^T} C B^c} \tag{15}$$

where C is a constant penalty factor enforcing a null pressure drop, i.e. the compatibility between the porous matrix and fracture meshes.

After the final configuration, the permeability matrix of the problem can be written as:

$$K = A_{e=1}^{n_{\Omega m}} (\mathrm{K_m})_{\Omega_m} + A_{e=1}^{n_\Gamma} (\mathrm{K_f})_{\Gamma_f} + A_{e=1}^{n_C} (\mathrm{K_c})_C \tag{16}$$

where A stands for the finite element assembly operator. The first and second terms of equation (16) are related to the domain of matrix and fractures, respectively and the third term is tied to introduction of CFEs.

4 Results

4.1 Case 1: diagonal fracture

The example carried out in this subsection was firstly studied by Zeng et al. [16] and was presented in Fig. 1. The single diagonal fracture is located at $y = x, x \in (0.2, 0.8)$ embedded in the porous domain Ω_m $(0,1)^2$. A specific pressure is applied on both bottom left (0.10 m), and upper right side (0.10 m), with following values $p_1 = 1\, MPa$ and $p_2 = 2\, MPa$. As reported by the authors [16], the permeability of the matrix and fracture are: $k_m = 10^{-12}\, m^2$ and $k_f = 10^{-8}\, m^2$, while the fluid viscosity of $\mu = 10^{-9}\, MPa.s$ is used.

Fig. 2(a) illustrates gradient pressure and as expected, the neighboring fracture domain influences the pressure distribution, which is a consequence of highly permeability of fracture over porous matrix permeability, since the fluid flow for preferential path of fracture. To a better visualization of this behavior, the pressure profiles along the lines y=0.2 m and y=0.5 m for both CFEs and the reference solution (phase field based discrete fracture model - PFDFM) are presented in Fig. 2(b) and (c), respectively. The results present very good concordance, and the pressure field has a continuum behavior; it means that overlapping meshes were coupled correctly. Furthermore, it is possible to analyze in Fig. 2 (b) that the non-uniformity of pressure distribution tends to increase next to fracture endpoint. On the other hand, for the fluid flow out of fracture there is a slight drop of pressure. This behavior is a consequence of different permeabilities in the whole domain, then it turns to increase slowly. Additionally, for the line y=0.5 m the pressure around fracture remains almost constant, which means the fluid flow is higher in that region (see Fig. 2 (c)).

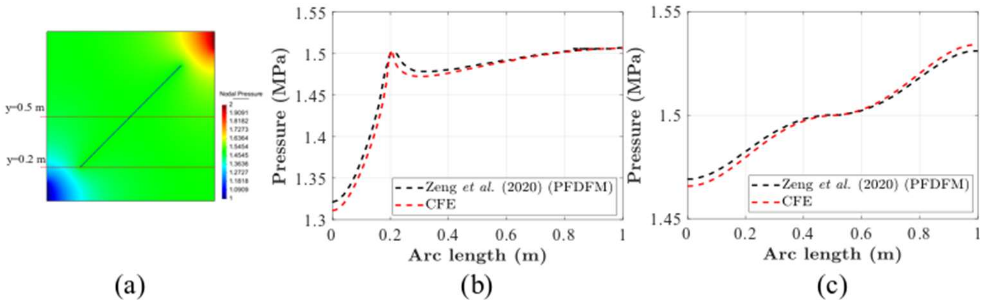

Figure 2. Results of (a) pressure field and pressure profiles at (b) y=0.2 m and (c) y=0.5 m.

4.2 Case 2: fractured network

With the purpose to validate the efficiency of numerical simulation using CFE, a benchmark presented by Flemisch, et al. [17] was simulated, using three different mesh refinements. The example consists of a regular fracture network embedded in the porous medium domain $\Omega(0,1)^2$. Top and bottom boundary faces are impermeable (no flow), while the left boundary face has a constant fluid flow, $q = 1 m/s$, and the right boundary face is applied a pressure of $p = 1 MPa$. The matrix permeability was stated at $k_m = 1$, fracture permeability was stated at $k_f = 10^4$, and for fluid viscosity was employed $\mu = 10^{-9} MPa.s$.

Fig. 3(a) illustrates the geometry and boundary conditions. Fig. 3(b), (c) and (d) present the distinct mesh refinements studied. The number of degrees of freedom (DOFs) for each mesh are: 194 DOFs for coarse, 415 DOFs for intermediate and 1442 DOFs for highly refined mesh. It is important to mention that the reference solution (mimetic finite differences - MDF) proposed by Flemisch et al. [17] uses 2.352.280 DOFs.

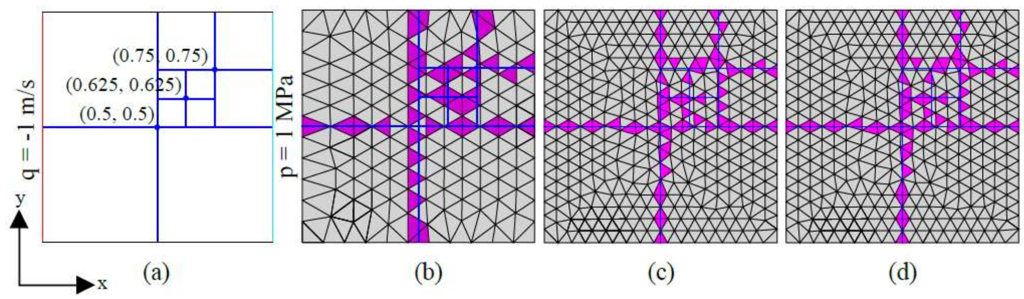

Figure 3. Case 2: (a) Geometry and boundary conditions and mesh discretization of (b) 0.09 m (coarse), (c) 0.06 m (intermediate) and (d) 0.03 m (highly-refined).

The solution of pressure gradient shows that the mesh refinement influences the fluid behavior in the left face of matrix domain (Fig. 4). As consequence, a desired discretization can guarantee the pressure field behavior of fracture and porous medium, accurately, with reduced computational cost.

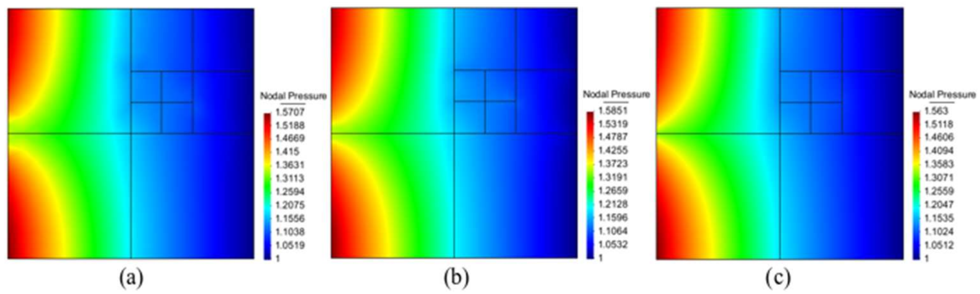

Figure 4. Pressure gradient of (a) coarse, (b) intermediate and (c) highly-refined mesh.

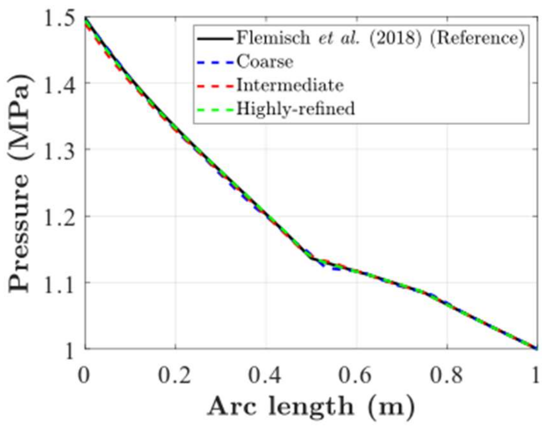

Figure 5. Pressure profile at y=0.7 m.

The pressure distributions of the three cases studied computed along the line y=0.7 m are plotted and compared with reference solution, which show good agreement, especially for the intermediate and highly-refined meshes, instead coarse mesh that presented an outstanding behavior at x=0.5 m (see Fig. 5). The relative error of pressure at this coordinate using the three cases in comparison with MDF method is 0.43%, 0.33% and 0.21% for coarse, intermediate and highly-refined meshes, respectively.

5 Conclusions

The present work proposes a new approach based on the use of coupling finite elements to consider the effect of the fractures in the simulation of steady-state fluid flow in fractured porous media.

For this purpose, in the first example, a porous matrix with one embedded diagonal fracture was numerically analyzed. Both meshes are completely independents, using bar elements to represent the fracture and triangular elements to discretize the porous medium, which were properly coupled via CFEs. The results validated the coupling approach and show that the pressure field has a continuum behavior between both domains, showing good agreement with the reference solution. In second example, a porous domain with vertical and horizontal fractures was studied, by assuming different mesh refinements. A good agreement between numerical results and reference solution was observed.

Thereafter, it is possible to conclude that the technique is able to couple overlapping non-matching meshes of two different domains successfully without increasing the total number of degrees of freedom of the problem.

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