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MODELING AND SIMULATION OF A TANK SYSTEM USING PREDICTIVE CONTROLLERS

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Abstract. The study and application of PID controllers -Proportional, Integral and Derivative- represents a great advance in productivity to the automation, being this one the most used for control of dynamic systems. However, despite the robustness and ease of use of PID, we have more efficient options for nonlinear processes or with dominant delay time. Among them, it should be emphasized that studies are developed on predictive controllers based on model or MPC-Model Predictive Control-. The present work aims to simulate a tank system with the objective of studying and evaluating the applicability of MPC implemented in PLC - programmable logic controller. The PLC was chosen as one of the main industrial automation tools, showing robustness to applications in adverse environmental conditions and is widespread in the sector. Allied to the PLC, a supervisory software was developed that is responsible for receiving information via the MODBUS protocol and storing it in a database. Such automation tools as PLC and supervisory software stand out, for their ease, simplicity and robustness. The information obtained by the process sensors will be used to construct the dynamic model of the system. Among the steps, it was initiated by the instrumentation of the system, construction of the sensor, calibration and connection to the PLC. The logical controller, in addition to sending information for presentation and storage, will be the tool used to interpret the process signals and then execute the predictive control algorithms. Thus, MPC presents dependency on the model for its construction and performs iterative optimizations based on constraints that can be imposed on the cost function. Therefore, it is observed that the construction of its algorithm is more complex when compared to the PID. It then becomes an advantage to develop process simulations and use for MPC controller tuning. The construction of the simulated model will allow to adjust parameters of the predictive controller and the cost function of the optimization algorithm and after we can check that MPC was able to run in PLC and with results based on control performance index.

Keywords: Automation, Control, PLC, MPC

1 Introduction

The constant adavances in computational systems allow the use of complex automation tools. Among the devices that aid on the automtion process, as described by [1], the programmable logic computer - PLC - come up as a special one. It was designed, during the 60's to work in industrial environment, as replacement for rele logic, and acording to [2], its use grows exponencially.

The referred deviced has evolved from simply replacing the rele logic up to the execution of PID - Proportional, Integral and Derivative -, algorithms, for instance. Actually, can be supported by the norm IEC 61131-3 [3].

In reference if control, most utilized in linear control is PID. According to the study presented by [4], among 11000 controllers implemented in paper factories, oil refineries and boms, 97% are regulated by structures based on PIDs. The factors that contribute for this widespread utilization, second [5] and [6], are the ease aplication, simplicity, robustness and reliability. However, in accord to [7], there are limitations for the control structure previous presented, as treating non linearities. The possible dificulties in this controller are associated, also, to the transport delay, offline sintony, variations of the reference signal or multivariable systems.

Therefore, other techniques are necessary, out of the scope of this structure, as an attempt to treat the problems listed above. Amid the available tools, the MPC - model predictive control - stands out. Described by [8] as one of the methods of control for systems with reference tracking, based on time domain, available for using.

Thus, this work aims compare a classic control, represented by PID, with a MPC technique. This will be similar to that presented by [9], acrescing to the simulations techniques of instrumentations of process implemented in a didatic plant and feasibility analysis of MPC using Schneider TM218 PLC.

2 Literature Review

In this section will be presented the fundaments for the obtain MPC. Finally, the performance index of control systems will be presented, with values quantifying the efficiency of the proposed controllers.

2.1 Controle preditivo

In this section we will review the model-based predictive control, MPC -*Model Predictive Control-*. According to [10], the following basic concepts are required to understand the MPC controller:

1. Initially, the reference, known as *setpoint*, must be specified and may be a step or path to be followed.
2. Then, from the discrete model of the plant, determine the future outputs, depending on the state position and the current and past control signal, so the MPC makes explicit use of the model. The discrete time interval at which prediction is performed is called the *prediction horizon* represented by the controller N_p parameter.
3. Then, the actuator output actions that minimize the objective function are calculated. That is, they are related to the state and output equations of the current model and predicted with the current control signal, according to the equation 8. This is called by [11] the cost function and should be minimized. This step considers the value of r_w that regulates the use of the decision variable.

That is, the predictive control method through the control matrix has three basic tuning parameters. These are N_p, N_c and r_w. The first two parameters are respectively the prediction horizon and the control horizon. In turn, the r_w parameter determines the weight of the actuator to the process and depends on the design requirements, ie if it is not necessary to prevent actuation, r_w is adjusted to values close to zero, otherwise increasing it The value of this parameter is set to reduce actuator action.

State Space Model - After listing the concepts of predictive control, the steps for controller development will be listed, starting from the model to the control signal to be applied. Thus, according to [11], there are three main types of MPC controllers, each with its structure, advantages and disadvantages. For this work we will use the discrete model, in process state space, 1 and 2. Among the main advantages, [11] cites the simplicity of development and direct connection to linear quadratic regulators. Therefore, the model determined by the transfer function must be transformed to the discretized state space model. Details of this transformation can be found in the basic literature of the control study.

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

$$y(k) = Cx(k) \tag{2}$$

State Space Model with Integrator - Next, you need to change the model by adding an integrator to fit the purpose of the predictive controller. Thus, from the A, B, and C matrices, in 1 and 2, the state space model with integrator is constructed, also identified as a space model. state array, formed by the arrays A_e, B_e, and C_e, according to eq. 3. Therefore, we reconstruct the enlarged state space model by replacing A_m, B_m, and C_m with, respectively, A, B, and C in eq. 4 and 5.

$$A_e = \begin{bmatrix} A_m & o_m^T \\ C_m A_m & 1 \end{bmatrix}; B_e \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}; C_e = \begin{bmatrix} o_m & 1 \end{bmatrix} \tag{3}$$

Therefore, the augmented state space model is reconstructed by replacing A, B and C with, respectively, A_e, B_e and C_e resulting in ref ss1a and ref ss2a.

$$x(k+1) = A_e x(k) + B_e u(k) \tag{4}$$
$$y(k) = C_e x(k) \tag{5}$$

Predicting Output Variables - Based on the state space model (A, B, C) and the U input signal, the future of state variables is given by:

$$\begin{aligned}
x(k_i+1 \mid k_i) &= Ax(k_i) + B\Delta u(k_i) \\
x(k_i+2 \mid k_i) &= Ax(k_i+1 \mid 1) + B\Delta u(k_i) \\
&= A^2 x(k_i) + AB\Delta u(k_i) + B\Delta u(k_i+1) \\
&\vdots \\
x(k_i+N_p \mid k_i) &= A^{N_p} x(k_i) + A^{N_p-1} B\Delta u(k_i) + A^{N_p-2} B\Delta u(k_i+1) \\
&\quad + \cdots + A^{N_p-N_c} B\Delta u(k_i+N_c-1)
\end{aligned}$$

Since states are determined by $Deltau$, it is then possible to override these predicted values in 5 to determine future system output. Thus, the system output prediction, Y is a result of the following equation, 6:

$$\begin{aligned}
y(k_i+1 \mid k_i) &= CAx(k_i) + CB\Delta u(k_i) \\
y(k_i+2 \mid k_i) &= CA^2 x(k_i) + CB\Delta u(k_i) \\
&= CA^3 x(k_i) + CA^2 B\Delta u(k_i) + CB\Delta u(k_i+1) \\
&\vdots \\
y(k_i+N_p \mid k_i) &= CA^{N_p} x(k_i) + CA^{N_p-1} B\Delta u(k_i) + CA^{N_p-2} B\Delta u(k_i+1) \\
&\quad + \cdots + CA^{N_p-N_c} B\Delta u(k_i+N_c-1)
\end{aligned}$$

The left side of the equations described above can be separated into matrices. Thus, the system output prediction equation, as a function of input, states and prediction interval N_p. Being the first left-hand column identified by F and multiplied by the states. The second term, Φ, is a function of the input signal.

$$Y = Fx(k_i) + \Phi\Delta U \tag{6}$$

The arrays that form the rows and columns of F and ϕ in 7, are the result of the system's increased state space modeling. Therefore, the following steps for obtaining the increased state space model will be described below. The approximation revision of the state-space transformation function can be reviewed in cite Ogata2011, the discretization of the model in state variables, in [12], as well as the transformation to state array augmented in [**?**].

$$F = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N_p} \end{bmatrix}; \Phi = \begin{bmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ CA^2B & CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \cdots & CA^{N_p-N_c}B \end{bmatrix} \tag{7}$$

Optimization - According to [11] the cost function to be minimized is given by 8

$$J = (R_s - Y)^T(R_s - Y) + \Delta U^T \bar{R} \Delta U \tag{8}$$

Em que ${R_s}^T = [1\ 1\ ...\ 1]r(k_i)$ e $r(k_i)$ é o vetor coluna com valores de referencia multiplicado por um vetor linha de valor unitário e comprimento igual a *Np*.

Where ${R_s}^T = [1\ 1\ ...\ 1]r(k_i)$ and $r(k_i)$ is the column vector with reference values multiplied by a row vector of unit value and length. equal to *Np*.

The optimization of this cost function can be performed by equating the derivative of **??** to zero, having as a decision variable the control signal, aiming to minimize the difference between output and reference, given by $E = (R_s - Y)$.

$$\Delta U = (\Phi^T\Phi + \bar{R})^{-1}\Phi(\bar{R}_s r(k_i) - Fx(k_i)) \tag{9}$$

2.2 Performance Index

According to [13], a performance index is a quantitative measure of the performance and response of a system. The systems are intended to be adjusted so that they have extreme and commonly minimal values. Systems that achieve such conditions are said to be optimal control systems. The performance index used in this paper which are ISE, IAE, ITAE and ITSE. Actuator variance inversely relates the control signal variance measurement with the actuator life cycle, as presented by [14] the actuator.

2.3 Design of Model Predictive Control structure and tunning

Predictive controller development and tuning will be done according to the ΔU equation. Since the LTI system - linear and time invariant - will be treated, some portions of the control signal, which depend only on the system in question, become constant. Therefore, these factors will be called K_y and K_mpc. According to [11], the controller structure.

$$K_y = (\Phi^T\Phi + \bar{R})^{-1}(\Phi^T R_s) \tag{10}$$

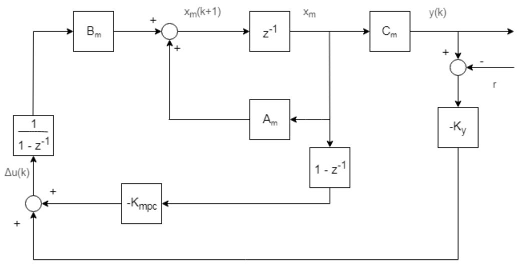

Figure 1. Diagrama P&ID do processo

$$K_{mpc} = (\Phi^T \Phi + \bar{R})^{-1} (\Phi^T F) \tag{11}$$

2.4 Simulations, results and performance

To compare the efficiency of the PID and MPC controllers, performance indexes will be used as described in 2.2. The tuning of the PID controller will be done by the SISOTOOL package, present in the MATLAB software, where the user is offered several tunings and has as input parameter the system and the purpose of the controller, be it for regular reference, path tracking or disturbance rejection. .

The tests to be performed aim to compare the performance of a PID controller structure and an MPC one. In turn, the MPC controller structure described in 2.1 will be tested in two configurations, the first one without the reference vector in the input, ie , the predictive controller response in this configuration will be corrective only. However, although not using in this configuration the prediction, as proposed in the name of the controller, the calculation of the control signal is a function of the optimization of the control signal in order to minimize the error having r_w as parameter;

Then, the reference for the proposed tests is defined, and they are the step input, according to [15] also referred to as change of *set-point* or reference. The other proposed test to be used here will be the ramp entry, also cited by [15] as reference reference.

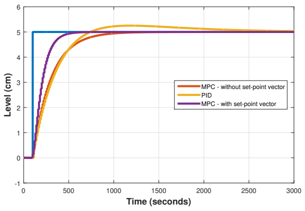

Figure 2. Closed loop step response

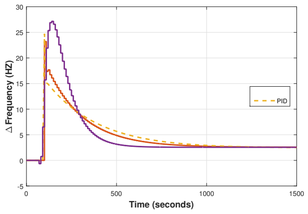

Figure 3. Actuator control effor in step response

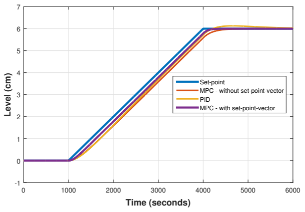

Figure 4. Closed loop ramp response

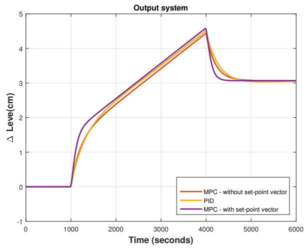

Figure 5. Actuator control effort in response to ramp input

	MPC - without setpoint vector	**PID**	**MPC - with setpoint vector**
IAE	1.0229×10^3	1.3045×10^3	0.5437577×10^3
ITAE	3.0301×10^5	7.4453×10^5	1.0211×10^5
ITSE	5.0860×10^5	6.2510×10^5	2.2462×10^5
ISE	2.6005×10^5	2.8432×10^3	1.5377×10^3

	MPC - without setpoint vector	**PID**	**MPC - with setpoint vector**
IAE	1.2274×10^3	1.1164×10^3	0.6252915×10^3
ITAE	3.3156×10^6	3.1581×10^5	1.6210×10^5
ITSE	1.2622×10^6	8.2788×10^5	3.2770×10^5
ISE	468.5322	328.1236	126.4741

2.5 Development of IEC Control Structures - 61131-3

Thus, it is observed that the predictive controller without reference vector, applied to first order systems, presents results comparable to PID controllers. Both authors [6] and [5] cite the ease of application of the PID controller, based on the simplicity of the algorithm and number of parameters, however in this paper we can observe that the MPC without the reference vector has the same amount. of input parameters than the PID - K_p, K_i and K_d. These parameters are K_y, K_mpc, determined by the model and equations 10, 11 and the gain of the C matrix, the latter obtained from the discretized state space matrix. [3] says that the PID is inserted in most controllers used in automation projects and as seen in this paper, the MPC based controller, previously described as MPC without reference vector can also be inserted in PLC's because the development The algorithm of the two controllers compared in this paragraph is performed with less than 10 lines of structured code - ST - in the IEC - 61131-3 standard.

In turn, the reference vector MPC controller has, according to error-based performance indices, the best results among all controllers. However there is an increase in computational development costs. In the first step, algorithms with the function of listing the reference vector must be developed, that is, from a path determined by the reference vector, the length algorithm N_p must go through the vector and send values to the MPC controller, resulting in R_s. In turn, upon receiving the reference vector, R_s from a given prediction window, the controller calculates at run time the value of K_y given by 10. Another portion of the controller is given by the calculation of K_mpc and is also calculated at runtime of the controller. It can be summarized that this technique has as input parameters the reference vector R_s. Other controller tuning criteria are the matrices used in the Φ and F prediction window, whose dimensions depend on N_p and N_c, directly influencing the computational cost. The values that determine the mode of execution of this controller are previously generated by algorithms involving matrix multiplication and inversion. As these are fixed parameters for each system, they can be generated externally to the controller.

Therefore, it is observed that the predictive control algorithms presented more efficient results when compared to a classical control technique, noting that the controller development cost was high. Thus, in addition to the study, development and analysis of simulations of three types of controllers, IEC 61131-3 standard control blocks can also be used in various controllers and industrial automation projects. It is suggested as an improvement of the results the addition of restrictions for the MPC controller, however

	MPC - without setpoint vector	**PID**	**MPC - with setpoint vector**
Step			
Average	1.2274×10^3	1.1164×10^3	0.6252915×10^3
Variance	3.3156×10^6	3.1581×10^5	1.6210×10^5
Rampa			
Average	1.2622×10^6	8.2788×10^5	3.2770×10^5
Variance	468.5322	328.1236	126.4741

it is necessary to invert matrices in the controller or external device that performs the inversion. Still as a suggestion of continuity is the carrying out of plant tests.

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