XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Inverse analysis of constitutive models applied to steel fiber reinforced concrete

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Abstract. Steel fiber reinforced concrete (SFRC) structures are increasingly being inserted in engineering, mainly due to the good properties of the material to the tensile stresses. To represent the behavior of these structures, there are several constitutive models in the literature, in which the stress x strain diagram of the material is derived from laboratory tests, such as the flexural test EN 14651. The simplicity of obtaining the stress x strain diagram of these constitutive models is one of its main advantages, however, they do not always represent the real behavior of the material. In this paper, the modeling strategies developed to simulate the EN 14651 three point bending test are presented, where the constitutive models of 2010 fib Model Code (MC 2010) and 2008 Instrucction de hormigón structural (EHE 08) are inserted. Through a quasi - static analysis of the Abaqus software, the divergences and similarities between the two models and existents experimental results are discussed. An adaptation of the stress x strain diagram is suggested through an inverse analysis, where it was possible to perceive that the complex mechanical behavior of the SFRC needs adjustments to be better represented.

Keywords: Steel Fiber Reinforced Concrete, constitutive models, mechanical behavior

1 Introduction

Steel fiber reinforced concrete (SFRC) is a composite material characterized mainly by a residual tensile strength after cracking. This resistance comes from the capacity of the fibers to transfer the stresses of the internal cracks in the concrete to the fibers.

In the last two decades, the insertion of SFRC in engineering has undergone an advance due to the publication of codes and guidelines for structural design, mainly in Europe (Huang et al. [1]), where the spanish code EHE-08 (*Instruccion de Hormigon Estructurale*) [2] and fib (*Fédération internationale du béton*) Model Code 2010 (MC 2010) [3] stands out. These two codes indicate a constitutive model of a stress *x* strain diagram, where the insertion data for the calculation of these parameters are based on the experimental results of the EN 14651 3-point bending test.

The idea of studying the representativeness of constitutive models has been discussed in recent years (Laranjeira et al. [4]; Blanco et al. [5]; Yang, Lin and Gravina [6]; Abbas, Syed and Cotsovos [7]) mainly due to the lack of an approach unified to select a suitable model for the design issue.

With this in mind, this paper compares the constitutive models of MC 2010 and EHE-08 with the experimental data of Blanco [8], in order to assess their representativeness in representing the mechanical behavior of SFRC. The Abaqus Finite Element software is used, and the modeling strategies are exposed. The divergences between the constitutive models and the experimental results are discussed. Finally, an inverse analysis is made, where the curve of the load - displacement diagram of the numerical analysis is adjusted to the curve of the experimental results.

2 Constitutive Models

2.1 fib Model Code (MC 2010)

The *Fédération Internationale du Béton* (FIB) brings a constitutive model, whose parameters are defined by means of residual tensile strengths, determined by performing the EN 14651. From the EN 14651 test, the load - CMOD diagram (Crack Mouth Opening Displacement) is derived, as shown in Fig. 1, where the resistances F_j can be seen in relation to the delimited $CMOD_j$.

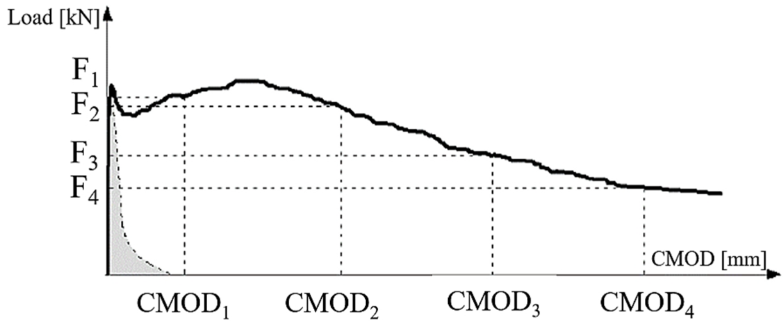
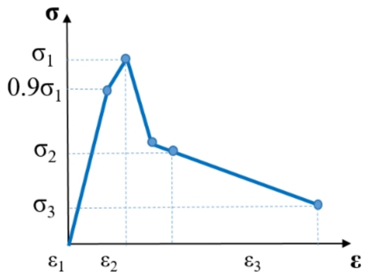


Figure 1. Typical load - CMOD diagram obtained from the EN 14651 test

The test consists of obtaining the residual tensile strengths (or loads, as in Fig. 1) in flexion, F_j, at predetermined distances, corresponding to CMOD = $CMOD_j$ (j = 1, 2, 3 and 4), where F_1 is the load corresponding to $CMOD_1$ = 0.5 mm, F_2 for $CMOD_2$ = 1.5 mm, F_3 for $CMOD_3$ = 2.5 mm and F_4 for $CMOD_4$ = 3.5 mm.

The constitutive model, as well as the equation referring to the strain and stresses in the diagram are shown in Fig. 2.



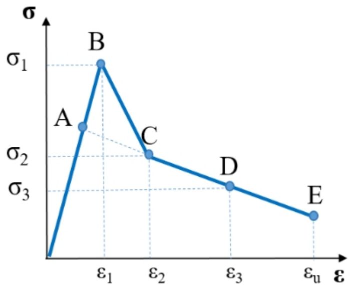
$$\sigma_1 = f_{ctm} = 0.3(fck)^{2/3}$$
$$\sigma_2 = f_{Fts} = 0.45 f_{R1}$$
$$\sigma_3 = f_{Ftu} = k[f_{Fts} - (w_u / CMOD_3)(f_{Fts} - 0.5 f_{R3} + 0.2 f_{R1})]$$
$$\varepsilon_1 = \varepsilon_{ELS} = CMOD1 / l_{cs}$$
$$\varepsilon_2 = \varepsilon_{ELU} = w_u / l_{cs}$$
$$\varepsilon_3 = \varepsilon_{Fu} [20‰ \text{ softening}; 10‰ \text{ hardening}]$$

Figure 2. Constitutive model of the fib Model Code

where:
f_{ctm} = average concrete tensile strength; f_{ck} = characteristic compressive strength measured at 28 days; f_{Fts} = residual tensile strength in the service limit state; f_{R1} residual tensile strength corresponding to CMOD1 = 0.5 mm; f_{Ftu} = residual tensile strength in the ultimate limit state; k = fiber orientation factor; wu = maximum permissible crack width; $CMOD_1$ = crack opening of 0.5 mm; $CMOD_3$ = crack opening at 2.5 mm; f_{R3} = residual tensile strength corresponding to CMOD3 = 2.5 mm; lcs = structural characteristic length; y = distance from the neutral line the base of the tensioned cross section; ε_{sls} = strain in the service limit state; ε_{slu} = strain in the ultimate limit state.

2.2 EHE-08

The Spanish standard EHE-08 contains in its text recommendations for the use of concrete with fibers, however, it does not specify the type of fiber to be used. The multilinear constitutive model is shown in Fig. 3. For more simplistic cases, the model can be used as a bilinear diagram (curve A-C-D-E). The diagram with the additional resistance provided by peak A-B-C allows a better approximation of the mechanical behavior of concrete with fiber.



$$\sigma_1 = f_{ct,d} = 0.6.f_{ct,fl,d}$$
$$\sigma_2 = f_{ctR1,d} = 0.45.f_{R,1,d}$$
$$\sigma_3 = f_{ctR3,d} = k_1(0.5f_{R,3,\mathrm{d}} - 0.2f_{R,1,d})$$
$$\varepsilon_1 = \sigma_1 / E_c; \quad \varepsilon_2 = \varepsilon_1 + 0.1‰; \quad \varepsilon_3 = 2.5 / l_{cs}$$
$$\varepsilon_u = [20‰ \text{ flexão; } 10‰ \text{ tração pura}]$$

Figure 3. Constitutive model of the EHE-08

where

$f_{ct,d}$ = concrete tensile strength; $f_{ct,fl,d}$ = concrete flexotration strength; $f_{ctR1,d}$ = residual strength to post-peak, related to strain 1; $f_{ctR3,d}$ = residual strength to post-pea, related to strain 3; $f_{R1,d}$ = tensile strength, related to CMOD$_1$; E_c = concrete modulus of elasticity; *lcs* = structural characteristic length.

3 Finite Element modeling

The present paper used the Abaqus 2017 software to model the problem computationally, where the Concrete Damaged Plasticity (CDP) model was used. The CDP represents the behavior of concrete and other fragile materials, such as rocks and mortars. One of the main advantages of CDP, according to Michał and Andrzej [9] is that the behavior of tensile and compression can be inserted separately, as shown in Fig. 4.

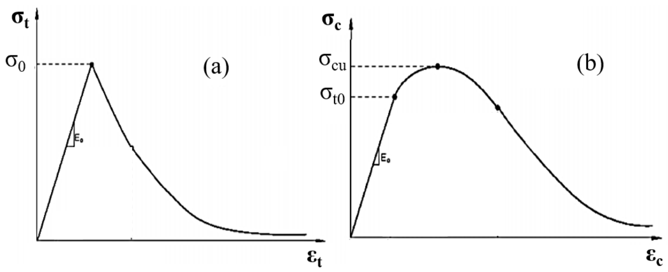


Figure 4. Concrete tensile (a) and compression (b) behavior, according to the CDP model

In addition to the insertion of the stress - strain diagram, it is necessary to provide Abaqus with other parameters. These are responsible for expanding the behavior of materials in uniaxial to multiaxial state, and are: dilatation angle, eccentricity, fb0 / fc0 ratio, parameter *K*, and viscosity (HAFEZOLGHORANI et al. [10]). For the present paper, Tab. 1 shows the values adopted for these variables, based on the suggestions of the Abaqus User guide [11], and also on works that used this model to simulate the CRFA, such as Othman and Marzouk [12], and Guler, Lale, Aydogan [13].

Table 1. Coefficients in constitutive relations

Dilatation angle (degrees)	Eccentricity	fbo/fco	K	Viscosity
30	0,1	1,16	0,667	0,0001

In addition, another modeling strategy was to use Abaqus/Explicit, through a quasi-static analysis. In the purely static analysis, there are some convergence problems when it comes to non-linear materials with large deformations. Some of these problems are solved, but the computational cost is enormous, as they need many increments in the analysis for this convergence to occur. In the Abaqus/Explicit dynamic analysis, the equilibrium equations are decoupled. That is, the solution algorithm does not use iterations as a convergence criterion, and a global set of equations does not need to be solved in each increment (THAI et al. [14]; OTHMAN and MARZOUK [12]; BITENCOURT et al. [15]). Thus, the computational cost of the explicit method is low in comparison with the other Abaqus algorithms, being indicated for problems with complex contact conditions, distorted meshes or non-linearities that result in large deformations.

With regard to quasi-static analysis, there is no standard formulation of what the correct external loading application rate is for the modeling to behave as quasi - static, and each problem must be analyzed individually. But one way to validate the analysis and prove that the inertial forces are irrelevant, is by measuring the kinetic energy of the problem. This should be small enough to be between 1-5% of the total internal energy, according to the Abaqus user guide [11]. Thus, Fig. 5 shows the kinetic energy / total internal energy ratio of the entire model, according to Thai et al. [14].

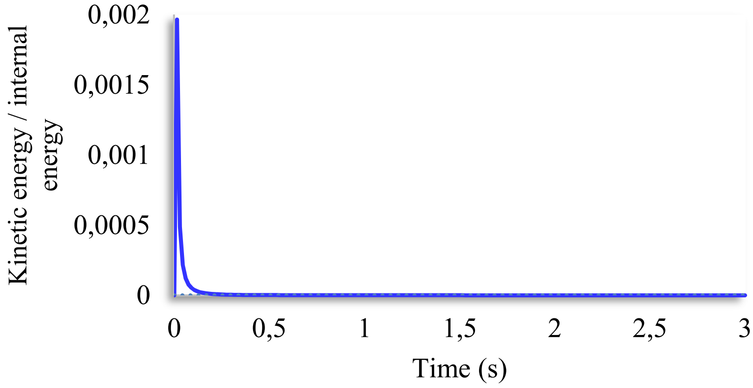


Figure 5. Typical load x CMOD diagram obtained from the EN 14651 test

In Fig. 5, it is possible to notice that at no point is the 5% limit exceeded, even with the analysis time being adopted as 3 seconds. Thus, the use of quasi-static analysis is proven, according to the proposed methodology.

4 Methodology

This paper initially aims at comparing the constitutive models of MC 2010 and EHE-08, in relation to their effectiveness in representing experimental results. It is important to note that the experimental results come from Blanco [8].

Thus, the 3-point bending test according to EN 14651 was modeled numerically, from the application of displacements at the central point of the beam, in a 2-D analysis.

After comparing the constitutive models, an inverse analysis is made, where an adjustment of the load - displacement curve is made, in order to approximate the constitutive model to the experimental results. This "inverse analysis" methodology is commonly used in structural engineering, with the intention of having a constitutive model that faithfully represents the experimental results in question. In Foster et al. [16], Mudadu et al. [17] and Buljak et al. [18] it can be seen that this type of analysis was useful so that the resulting numerical simulations were more accurate. In this paper, the necessary adjustments are discussed, and the adjusted constitutive model is compared with the original. The flowchart in figure 6 shows the methodology.

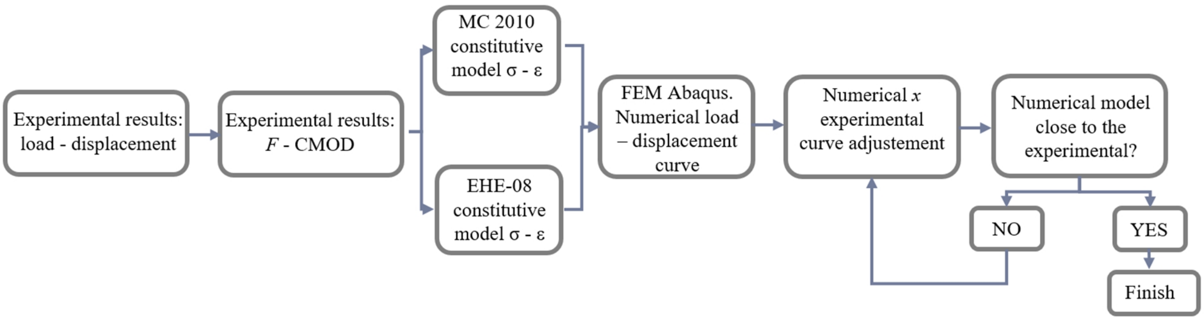


Figure 6. Methodology flowchart

5 Results

For simulations made using the Abaqus software, a 2-D model was used. The mesh was made with CPE3 elements, which are triangular elements with 3 nodes. The beam in question was bi-supported, with the displacement control simulation performed.

The load - displacement diagram of the experimental results in comparison with the constitutive models MC 2010 and EHE-08 can be seen in Fig. 7.

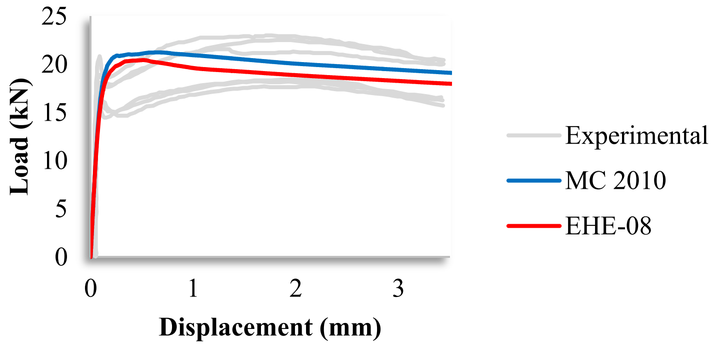


Figure 7. Comparison of the constitutive models MC 2010 and EHE-08 to the experimental results.

It is possible to notice that both the constitutive models, MC 2010 and EHE-08, proved to be similar to the experimental results. As the main divergence, it is noticed that both constitutive models represented the softening behavior of the material, while the experimental results have a slip hardening behavior. This experimental behavior is explained by the fiber dosage used by Blanco [8].

Despite the similarity of the values, the real behavior of the experimental results is only achieved through an inverse analysis, where the curves of the numerical results are adjusted to the experimental ones.

5.1 Inverse analysis

The inverse analysis made in the present paper refers to the adjustment of the force-displacement curve from numerical to experimental results. After adjusting the curve, the new stress and strain values are compared to the original constitutive model.

Regarding the adjustment protocol, the stress and strain values were modified, and the result of the load - displacement curve was evaluated in each change.

After some analysis it was found that the 3 insertion points of the stress - strain diagram, from the original constitutive model, were not sufficient to represent slip hardening. Even changing the values significantly, the format of the load - displacement response diagram remained unchanged, representing the softening behavior.

Thus, by means of linear interpolation, an intermediate point of stress and deformation was added. With that, the answer showed more similarity with the experimental curve, but still with important discrepancies. Following this line of thought, another point was added, and then, another point of tension and intermediate deformation. The insertion of these 3 new points was then decisive for the response to significantly resemble the experimental curve.

Thus, Fig. 8 shows the difference in the stress and strain values of the MC 2010 original constitutive model (yellow curve) and the adjusted model (blue curve). In Fig. 8 it is also possible to see the three intermediate points added, represented by the red marker. Still, Fig. 8 shows the comparison of the numerical and experimental result after the adjustment.

Similarly, figure 9 shows the comparison of the original and adjusted models for the EHE-08. Also, the result of the simulation, after adjustment.

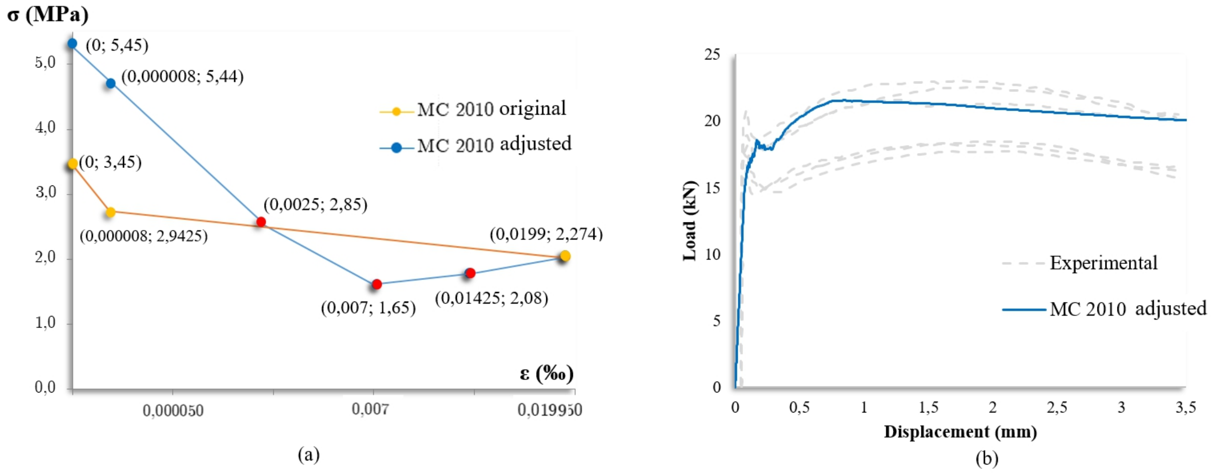


Figure 8. Original and adjusted MC 2010 constitutive model (a), and experimental result versus adjusted constitutive model (b)

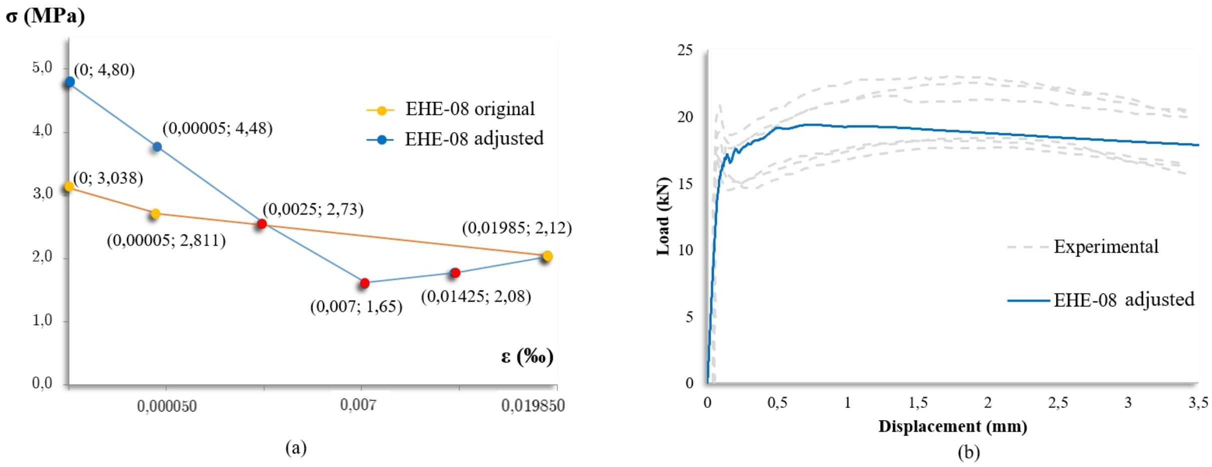


Figure 9. Original and adjusted EHE-08 constitutive model (a), and experimental result versus adjusted constitutive model (b)

As can be seen in figures 8 and 9, in both constitutive models, the stresses had to be adapted to higher values. The adjustment curve had to be started with a tension approximately 50% higher than the original model proposed. The strains remained the same.

The extra points added were necessary to induce the tension to return to increasing values after the sharp drop after peak. This strategy was crucial for the simulation to be able to capture the slip hardening behavior of the experimental results.

In order for the adjustment protocol to be as general as possible, the adjustment in the original models was made only for stresses. Of the 3 points added, 2 were repeated for both models. Thus, Tab. 2 shows the adjustments made, showing that the adjustment in stress 1 (σ_1) was the same for both models, stress 2 (σ_2) had a small difference, stress 3 (σ_3) and the strains remained unchanged.

Table 2. Adjusted/original model ratio

	σ_1	σ_2	σ_3	ε_1	ε_2	ε_3
fib Mode	1,58	1,85	1	1	1	1
EHE-08	1,58	1,60	1	1	1	1

In general, after adjustment, both constitutive models, Model Code 2010 and EHE-08 were able to better represent the mechanical behavior of the SFRC.

6 Conclusions

The present paper presented a comparison of the effectiveness in representing experimental results of two existing constitutive models, the fib Model Code 2010 and the EHE-08. As both models were unable to capture the real behavior of the results, a methodology was proposed to adapt these constitutive models to the experimental results of the SFRC. Through the load - displacement diagram it was possible to notice that the adjusted model better represents the mechanical behavior of the material, when compared to the simulation made with the original model.

This type of methodology can be useful when there are more numerical simulations, and the adopted constitutive model needs to be faithful to the experimental results. The results shown in this paper represent the incipient state of research around reliable numerical simulations for SFRC. However, this type of methodology has shown promise, especially in cases where a more generic and applicable adaptation protocol can be devised in other examples. In general, the modeling strategies adopted proved to be efficient.

Acknowledgements. The authors acknowledge the financial support given by CAPES.

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7 References

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