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# Classical transient pipe flow analysis with SPH method

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**Abstract.** The main purpose of this paper is to analyze the applicability of the Smoothed Particle Hydrodynamics (SPH) method in water hammer modeling, a transient pipe flow problem. A reservoir-pipe-valve system with steady friction losses was chosen to implement the method. In order to suppress the limitations of the method due to boundary conditions and numerical instability, numerical corrections were applied to the SPH. The applied corrections are found in the kernel function (CSPH) and in the use of artificial viscosity. Furthermore, the method is validated by comparing the numerical results obtained with SPH and the numerical solution obtained by the Method of Characteristics (MOC).

**Keywords:** Water Hammer, SPH, MOC, Transient Pipe Flow, Steady Friction.

## 1 Introduction

Associated with water hammer, a transient condition in which rapid disturbances occur in the pipe flow (such as pump trip off, control valve adjustments, and accidental events) changes in pressure and velocity occur inside the pipe. According to Wichowski [1], it is important to consider the transient condition during the operation, maintenance, and design of water distribution systems. Due to its complexity, computational models are regularly used to overcome the transient problem.

Smoothed Particle Hydrodynamics (SPH) is a particulate, Lagrangian, and meshless numerical method commonly used in simulations of complex problems. Since the discretization of the problem domain is particulate, the SPH is used to estimate the value of a specific property of a particle that uses the same properties of neighboring particles. To this end, interpolation of the kernel function is taken into account. Still, the particulate method transforms partial differential equations into ordinary differential equations, facilitating a simulation and analysis of the transient pipe flow. Therefore, this paper aims to apply the SPH method to the water hammer problem. The case study is based on the transient flow in a reservoir-pipe-valve system with steady friction losses on the pipe walls, which can be treated as a one-dimensional problem. The application of the SPH method is validated by comparison with the numerical solution obtained by the well-known Method of Characteristics (MOC).

## 2 Water Hammer

The water hammer is a phenomenon that occurs during the transient pipe flow due to the rapid closure of a valve, power failure of pump stations, or any other form of abrupt flow interruption. When the phenomenon occurs, pressure peaks are formed inside the pipe that can exceed the maximum pressure allowed by the pipe material, which can lead to failure or rupture. The equations that govern the problem are described in the section 2.1.

### 2.1 Classical water hammer equations

According to Chaudhry [2] and Wylie and Streeter [3], the transient pipe flow problem is governed by a pair of partial non-linear differential equations, which represent the laws of momentum conservation - eq. (1) – and of mass conservation - eq. (2) - with $dx/dt = c \pm V$.

$$\frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x} + \rho c^2 \frac{\partial V}{\partial x} = 0, \tag{1}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{fV|V|}{2D} = 0, \tag{2}$$

in which $P$ is the internal pipe pressure, $\rho$ is the fluid density, $c$ is the the pressure wave speed, $V$ is the flow average velocity, $f$ is the friction factor, $x$ is the pipe length axis and $t$ is time.

Equations. (1) and (2) are written in their Eulerian forms, according to their convective terms. To obtain these same equations without the convective terms (Lagrangean form), it needs to use the total derivatives of $P$ and $V$. Thus, the rewritten EDPs are showed in eq. (3) and (4):

$$\frac{DP}{Dt} = -\rho c^2 \frac{\partial V}{\partial x}, \tag{3}$$

$$\frac{DV}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{fV|V|}{2D}. \tag{4}$$

Also, as the numerical value of $c$ is such $V \ll c$, the ratio $dx/dt = c$ can be assumed.

### 2.2 Valve closure

For a valve with discharge to the atmosphere, the volumetric flow ($Q$) and internal pressure ($P$) in it are related by the orifice equation, according to eq. (5):

$$Q = (C_d A)\sqrt{2P/\rho}, \tag{5}$$

in which $C_d$ is the discharge coefficient, $A$ is the effective flow area, $P$ the hydraulic pressure in the valve and ρ the density of the fluid.

According to Soares [4], the volumetric flow in a generic time ($Q_T$) and in the initial condition ($Q_o$) can be related by eq. (6):

$$Q_T = (Q_o \tau)\sqrt{\frac{P_T}{P_o}}, \tag{6}$$

in which $\tau$ is the opening valve coefficient, written in eq. (7).

$$\tau = \frac{C_{dP} A_P}{C_{do} A_o}. \tag{7}$$

### 2.3 Constant-level reservoir

For a large reservoir, the water column can be considered constant. Thus, the pressure at the reservoir outlet is considered as the pressure of the water column.

## 3 Smoothed Particle Method

In this topic, the SPH equations are presented and applied to the transient pipe flow. As the water hammer problem can be addressed as one-dimensional (1D) problem, the equations and corrections are written in 1D.

### 3.1 Classical SPH equations

The SPH function can be described accordingly to eq. (8).

$$f(x_i) = \int_\Omega f(\boldsymbol{x_j})\delta(\boldsymbol{x_i} - \boldsymbol{x_j})d\boldsymbol{x_j} \approx \int_\Omega f(\boldsymbol{x_j})W(\boldsymbol{x_i} - \boldsymbol{x_j}, h)d\boldsymbol{x_j}, \tag{8}$$

$$\delta(\boldsymbol{x_i} - \boldsymbol{x_j}) = \begin{cases} 1, & if \quad \boldsymbol{x_i} = \boldsymbol{x_j} \\ 0, & if \quad \boldsymbol{x_i} \neq \boldsymbol{x_j} \end{cases}, \tag{9}$$

in which $\boldsymbol{x}$ is the position vector, $\delta$ is the Dirac Delta function, $\Omega$ is the compact domain, $W$ is the kernel function (a approximation function for the Dirac Delta) and $h$ is the smoothing length.

The chosen kernel function $W$, besides being an even function, must also satisfy the following conditions:

I) Normalization condition: $\int_\Omega W(\boldsymbol{x_i} - \boldsymbol{x_j}, h) d\boldsymbol{x_j} = 1$;

II) Delta function condition: $\lim_{h \to 0} W(\boldsymbol{x_i} - \boldsymbol{x_j}, h) = \delta(\boldsymbol{x_i} - \boldsymbol{x_j})$;

III) Compact condition: $W(\boldsymbol{x_i} - \boldsymbol{x_j}, h) = 0$ when $|\boldsymbol{x_i} - \boldsymbol{x_j}| > kh$;

in which $k$ is a constant related to the smoothing length $h$ and defines the kernel effective (not null) region, with the effective region also called compact domain.

Monaghan [5] recommends the use of the Gaussian function for better SPH physical representation, as it is sufficiently smooth, stable, and precise. However, as the function never actually reaches zero, its use would create very long domain support, requiring greater computational effort. This way, the cubic spline function - eq. (10) - was proposed to represent the kernel function, with a compact support domain [6].

$$W(r,h) = \alpha_d \begin{cases} \frac{1}{4}[(2-r)^3 - 4(1-r)^3] & 0 \leq r < 1 \\ \frac{1}{4}(2-r)^3, & 1 \leq r < 2 \\ 0, & r \geq 2 \end{cases}, \tag{10}$$

so that $r$ is equal to $|\boldsymbol{x_i} - \boldsymbol{x_j}|/h$ and $\alpha_d$ is $2/(3h)$ for the 1D case.

## 3.2 Artificial viscosity

During the SPH simulations, it is common to appear oscillations in the results, since the water hammer governing pair of equations are hyperbolic PDEs. Thus, a term to suppress the numerical oscillations presented needs to be used. In this study, an explicit dissipative term is added to the momentum equation, which is called artificial viscosity [7]. Thus, eq. (4) will take the form presented in eq. (11):

$$\frac{DV}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{fV|V|}{2D} - \frac{\partial(\rho\Pi)}{\partial x}. \tag{11}$$

According to Monaghan & Gingold [7], artificial viscosity can be given by eq. (12):

$$\left(\frac{\partial(\rho\Pi)}{\partial x}\right)_i = \sum_{j=1}^{N} m_j \Pi_{ij} \frac{dW_{ij}}{dx}, \tag{12}$$

$$\Pi_{ij} = \begin{cases} \frac{\beta\mu_{ij}^2 - \alpha c \mu_{ij}}{\rho}, & (V_i - V_j)(x_i - x_j) < 0 \\ 0, & (V_i - V_j)(x_i - x_j) \geq 0 \end{cases}, \tag{13}$$

$$\mu_{ij} = \frac{\bar{h}_{ij}(V_i - V_j)(x_i - x_j)}{(x_i - x_j)^2 + \eta \bar{h}_{ij}^2}, \tag{14}$$

in which $\bar{h}_{ij} = (h_i + h_j)/2$ is the average smoothing length of the particles $i$ and $j$, $m$ is the mass, $c$ is the wave speed, the terms $\alpha$ and $\beta$ are constants to be adjusted for the problem, and $\eta = 0.01$ are used to avoid division by zero in eq. (14).

## 3.3 Kernel correction

The kernel function correction presented, also known as Corrective Smoothed Particle Hydrodynamics (CSPH), is a generalization of the classic SPH [9]. This method was proposed by Chen et. al. [10], by using Taylor series expansion in the kernel function and then applying the particle approximation of the water hammer equations. The CSPH method solves the lack of particles in the contour regions - problem illustrated in Fig. 1.b -, due to the effective region extension defined by $kh$, where there are no particles. Numerical results obtained by Chen et. al. [10] demonstrate that, in addition to solving the lack of particles in the contour regions, CSPH improves the solution precision not only in these regions but also within the general domain.

So, when expanding in the Taylor series a function $f(x)$ applied to the SPH and disregarding the derivative terms of equal to or greater than order 2, eq. (15) is obtained. Since the particles are volumetric and have mass, the particle $j$ differential volume $dx$ can be rewritten as $m_j/\rho_j$.

$$f(x_i) \cong \frac{\int_\Omega f(x)W_i(x)dx}{\int_\Omega W_i(x)dx}. \tag{15}$$

Similarly, the first-order derived function $\frac{df(x)}{dx}$ can be approximated by eq. (16):

$$\frac{df(x_i)}{dx} \cong \frac{\int_\Omega [f(x)-f(x_i)]\frac{dW_i(x)}{dx}dx}{\int_\Omega [x-x_i]\frac{dW_i(x)}{dx}dx}, \tag{16}$$

in which $W_i(\boldsymbol{x}) = W(\boldsymbol{x} - \boldsymbol{x_i}, h)$.

Equations (15) e (16) are corrections applied to classical SPH and are the base equations of CSPH.

## 3.4 Particle approach

For the transient pipe flow, Fig. 1.a illustrates the particle distribution through the pipe with the kernel function settled in the analysis particle. The region defined by the influence radius ($kh$) is taken as the support domain Ω. In this way, $N$ will be the number of particles within that domain.

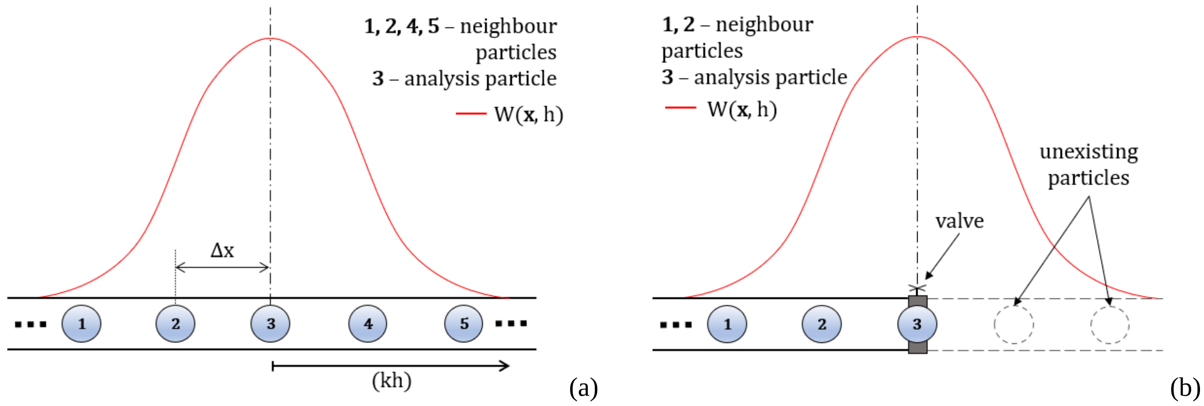


Figure 1. Kernel function and particle arrangement inside the pipeline (a) and close to the valve (b).

In this way, eq. (15) and (16), when discretized for the presented conditions, can be rewritten according to eq. (17) and (18):

$$f_i \cong \frac{\sum_{j=1}^{N} f_j W_{ij}\, m_j/\rho_j}{\sum_{j=1}^{N} W_{ij}\, m_j/\rho_j}, \tag{17}$$

$$\left(\frac{df}{dx}\right)_i \cong \frac{\sum_{j=1}^{N}[f_j-f_i]\frac{dW_{i,j}}{dx} m_j/\rho_j}{\sum_{j=1}^{N}[x_j-x_i]\frac{dW_{i,j}}{dx} m_j/\rho_j}. \tag{18}$$

Thus, the pair of equations that govern the transient problem and all the corrections presented can be rewritten as eq. (19) and (20):

$$\left(\frac{DP}{Dt}\right)_i \cong -\rho c^2 \left(\frac{\partial V}{\partial x}\right)_i, \tag{19}$$

$$\left(\frac{DV}{Dt}\right)_i \cong -\frac{1}{\rho}\left(\frac{\partial P}{\partial x}\right)_i - \frac{fV_i|V_i|}{2D} - \left(\frac{\partial(\rho\Pi)}{\partial x}\right)_i, \tag{20}$$

in which the expression $\Delta x/\Delta t = c$ is valid, $\Delta x$ is the spacing between fluid particles in the pipe and Δt is the time step.

### 3.5 Time integration

SPH solves the water hammer spatial evolution by turning the PDEs equations into temporal ODEs. Thus, the temporal evolution was performed using the first-order Euler integration method, given by eq. (21):

$$f_{t+\Delta t} = f_t + \Delta t \frac{Df}{Dt}. \tag{21}$$

## 4 Computational Simulations

Computational simulations were performed according to the data presented in this topic to obtain the time evolution of pressure and velocity of a given particle.

### 4.1 Fixed input data

For the simulations, a large reservoir with pressure at the base $P_R = 1 \times 10^6$ Pa connected to a pipe of $L = 20$ m in length, diameter $D = 800$ mm and friction factor $f = 0.02$ were considered. The volumetric flow is $Q = 0.5$ m³/s of a fluid of $\rho = 1 \times 10^3$ kg/m³ under $g = 9.81$ m/s², and the wave velocity adopted for the problem was $c = 1025$ m/s, based on physical conditions.

Regarding the SPH, $\Delta x = 0.1$ m, $h = 1\ \Delta x$ and $k = 1.0$ were adopted. It is also considered that in the gradual valve closure the valve starts to be closed in $t = 0$ and its closure occurs linearly, that is, $\tau$ varies linearly from 1 (fully open) to 0 (fully closed).

### 4.2 Particle distribution

Particles were distributed so that the first particle coincided with the reservoir outlet and the last one with the valve, with these two being the boundary conditions applied according to sections 2.2 and 2.3. The other particles were distributed evenly spaced along the pipe, with its total amount equal to 201 particles to guarantee the established $\Delta x$.

## 5 Results and Discussions

To verify the applicability of SPH to the problem of the hydraulic transient, the artificial viscosity effects and the gradual and rapid valve closure were chosen for analysis, performed for a maximum time of 0.3 s. The outlet reservoir velocity (particle 1) and the valve pressure (particle 201) were analyzed. In this sense, the valve closure analysis uses the values of $\alpha = 0.6$ and $\beta = 0$, simulating the instantaneously and gradually (closure occurs in 0.05 s) valve closure situations. For the artificial viscosity analysis, the instantaneous closure condition was fixed and situations were simulated in which $\alpha = 0.6$ and $\beta = 0.0$ and $\alpha = \beta = 0.0$ (neglecting artificial viscosity).

### 5.1 Gradual valve closure

The pressure in particle 201 and the velocity in particle 1 results are shown in Fig. 2. The running times considering instantaneous valve closure were 513.8 s (SPH) and 82.5 s (MOC); for gradual closure, 558.7 s (SPH) and 85.3 s (MOC).

From observing Fig. 2, the obtained results of both pressure (a) and velocity (b) from SPH are very close to the analytical solution obtained by MOC. As time grows, there is a numerical effect of dispersion, seen in through the smoothing of the rectangular pressure and velocity curves ends. Hou et. al. [11] also observed the same effect on their results, with this numerical dispersion being an effect intrinsic to the use of artificial viscosity.

It is also observed that the small extrapolation of the 2 MPa value in the first pressure rise does not constitute a physical result, being a numerical effect from the SPH method, and was corrected in [11].

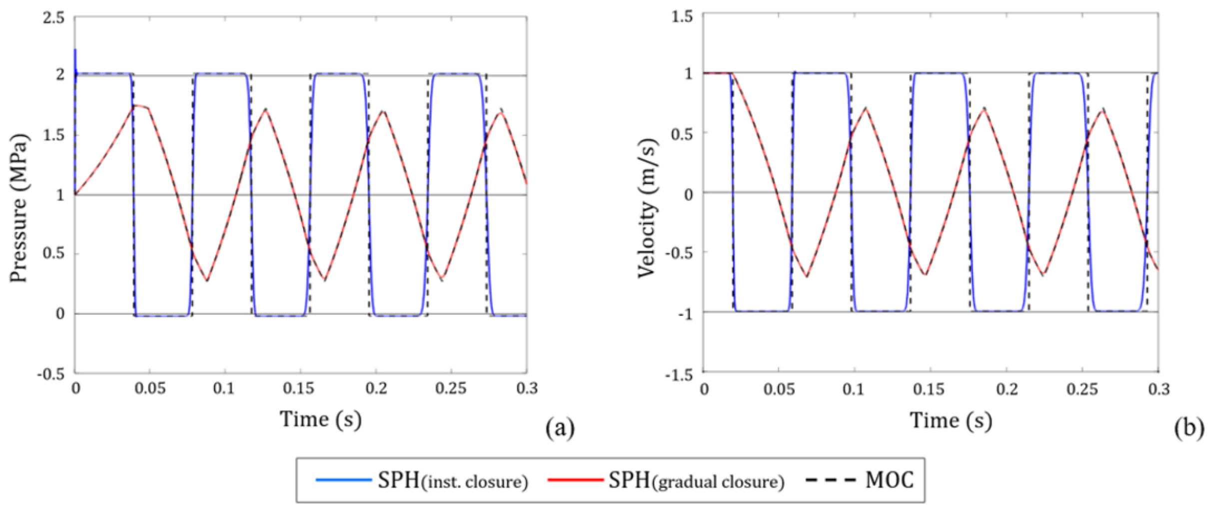


Figure 2. Time evolution of pressure in particle 201 (a) and flow velocity in particle 1 (b) for the gradual and rapid valve closure analysis.

## 5.2 Artificial viscosity

The pressure in particle 201 and the velocity in particle 1 results are shown in Fig. 3. The running times with artificial viscosity were 513.8 s (SPH) and 82.5 s (MOC), while for simulation without artificial viscosity they were 559.6 s (SPH) and 84.3 s (MOC).

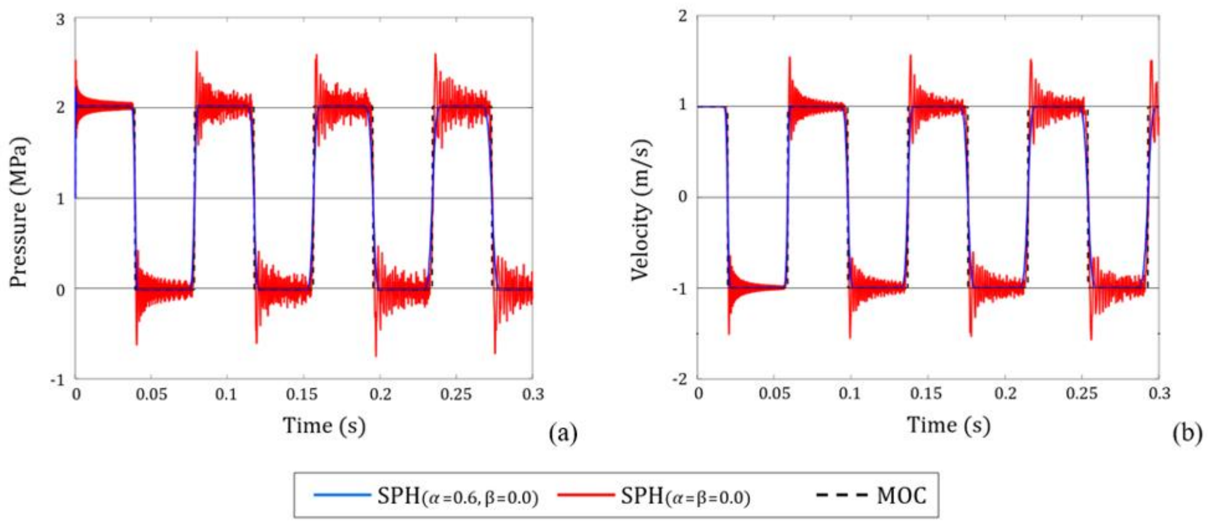


Figure 3. Time evolution of pressure in particle 201 (a) and flow velocity in particle 1 (b) through the artificial viscosity effect analysis.

From what can be seen in Fig. 3 (a) and Fig. 3 (b), when neglecting the artificial viscosity condition ($\alpha = \beta = 0.0$), numerical oscillations appear due to the physical result discontinuity, since the drop in pressure from 2 MPa to 0 happens almost instantaneously.

The literature brings some advisable values for $\alpha$ and $\beta$. However, the simulated values used in this paper show good results, which are very close to the numerical solution of MOC.

## 6 Pseudocode

The computational steps for the water hammer solution are as follow:

***Step 1*** set $p = 1$
***Step 2*** while ($p \leq N_t$) do Steps 3 – 9
  ***Step 3*** set valve boundary conditions
  ***Step 4*** For ($i = 1, 2, \ldots, N_x$) do Steps 5 – 7
    ***Step 5*** solve eq. (18) setting $f = V$
    ***Step 6*** calculate $\left(\frac{DP}{Dt}\right)_i$ with eq. (19)
    ***Step 7*** solve eq. (21) setting $f = P$
  ***Step 8*** set reservoir boundary conditions
  ***Step 9*** For ($i = 1, 2, \ldots, N_x$) do Steps 10 – 12
    ***Step 10*** solve eq. (18) setting $f = P$
    ***Step 11*** calculate $\left(\frac{DV}{Dt}\right)_i$ with eq. (20)
    ***Step 12*** solve eq. (21) setting $f = V$
***Step 13*** Output (vector $P$ and $V$)

Since $N_t$ and $N_x$ are, respectively, the number of time steps taken and the number of particles defined within the pipeline and, on ***Step 11***, the previous time step velocities are used to calculate the friction contribution.

## 7 Conclusion

The SPH method can be considered a good method for transient pipe flow computational simulation. Despite its limitations, such as the necessary corrections to be inserted, the SPH presents satisfactory results when compared with the MOC ones. However, when analyzing the SPH simulation effort with the MOC, it is observed that the mesh method has advantage, with the time required for SPH simulation being almost 6.5 times greater than the time required for MOC simulation.

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**Authorship statement.** The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors.

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