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Nonlinear analysis of three-point bending notched concrete beams via global-local Generalized Finite Element Method approach

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Abstract. The Generalized Finite Element Method (GFEM) was developed in order to overcome some limitations inherent to the Finite Element Method (FEM), related to the defects propagation, presence of large deformations or even in the description of high gradients of state variables. The GFEM enriches the space of the polynomial FEM solution with a priori known information based on the concept of Partition of Unit (PoU). Certain obstacles of nonlinear analysis can be mitigated with the GFEM, and damage and plasticity fronts can be accurately represented. In this context, the global-local approach to the GFEM (GFEM global-local) was proposed. The success of its application to problems of Linear Elastic Fracture Mechanics is already proven, but its extension to the simulation of collapse of structures made of quasi-brittle media is still a vast field to be researched. Here, a coupling strategy is presented based on the global-local GFEM to describe the deterioration process of quasi-brittle media, such as concrete, in the context of Continuous Damage Mechanics. The numerical solution used to enrich the global problem, represented by a coarse mesh, is obtained through physically nonlinear analysis performed only in the local region where damage propagation occurs, represented by constitutive models. The linear analysis of the global region is performed considering the incorporation of local damage through the global-local enrichment functions and damage state mapped from local problem. Numerical examples of three-point bending notched concrete beams have been presented to evaluate the performance of the proposed approach and the obtained results were compared with the experimental results and with the ones obtained with standard GFEM. Two constitutive models were applied to represent the concrete in the local region: smeared crack model and microplane model.

Keywords: Global-local strategy; Physically nonlinear analysis; Software `INSANE`

1 Introduction

The application of the GFEM to the nonlinear analysis and representation of the damage and plasticity fronts is well consolidated. In this context, the GFEM global-local was proposed by Duarte and Babuška [1] and widely studied by authors such as Kim *et.al* [2], Freitas *et.al* [3] and Kim and Duarte [4]. The latter, in particular, applied the global-local GFEM in the three-dimensional analysis of the propagation of cohesive cracks in concrete structures. Recently, Evangelista Jr. *et.al* [5] proposed the formulation and development of GFEM global-local strategy which incorporates a Continuum Damage Model that uses scalar damage variable for quasi-brittle materials to simulate failure in mode I and mixed-mode crack propagation.

In this paper, a global-local approach to the GFEM based on Kim and Duarte [4] is applied to describe the deterioration process of quasi-brittle media within the context of Continuous Damage Mechanics, by assuming two constitutive models to represent the concrete in the local region: smeared crack model of fixed direction with the Carreira and Chu [6, 7] stress-strain laws and microplane model of Leukart and Ramm [8]. The numerical solution used to enrich the global problem is obtained through physically nonlinear analysis performed only in the local region. With the damage of the local region incorporated into the global problem, through the global-local enrichment functions, the linear analysis is performed in the global region. This process is carried out in blocks of global-local analysis able to capture the evolution of the deterioration process and their influence on the global behavior of structures.

This GFEM global-local approach was implemented by Monteiro [9] in the computational system `INSANE` (*INteractive Structural ANalysis Environment*) (Gori *et.al* [10]), and in the section 2 the implemented formulation is summarized. Section 3 presents two sets of numerical examples (one with the smeared crack model and the other with de microplane model) of three-point bending notched concrete beams to evaluate the performance of the proposed approach. The obtained results were compared with the experimental results and with the ones obtained with standard GFEM.

2 Formulation

Each block of global-local analysis has three stages:

Stage 1. Initial and estimated linear global problem

For the initial linear global problem, step $k = 0$, the domain is defined by $\bar{\Omega}_G = \Omega_G \cup \partial\Omega_G$ in $\mathbb{R}^n$. The vector field $\mathbf{u}^0_{G,0}$ is the approximate solution of the weak form of the initial global problem:

$$\int_{\Omega_G} \boldsymbol{\sigma}(\mathbf{u}^0_{G,0}) : \boldsymbol{\varepsilon}(\mathbf{v}^0_{G,0})\, d\mathbf{x} + \int_{\partial\Omega^u_G} \mathbf{u}^0_{G,0} \cdot \mathbf{v}^0_{G,0}\, d\mathbf{s} = \int_{\partial\Omega^\sigma_G} \bar{\mathbf{t}} \cdot \mathbf{v}^0_{G,0}\, d\mathbf{s} + \int_{\partial\Omega^u_G} \bar{\mathbf{u}} \cdot \mathbf{v}^0_{G,0}\, d\mathbf{s}, \tag{1}$$

where $\mathbf{v}^0_{G,0}$ are the test functions of the initial global problem, $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{\varepsilon}$ is the strain tensor, $\bar{\mathbf{t}}$ is the prescribed stress vector, and $\bar{\mathbf{u}}$ is the prescribed displacement vector.

The solution $\mathbf{u}^0_{G,0}$ is obtained for the entire load (load factor $\lambda = 1$) and then it is adjusted according to the size of the displacement step P_{DG} (predefined for the global problem in the control node). The load factor is obtained by:

$$\lambda^0 = \frac{P_{DG}}{u^0_{G,0,DC}}, \tag{2}$$

where $u^0_{G,0,DC}$ is a displacement component of the control node, obtained from eq. (1).

To $k \geq 1$, in the estimated linear global problem, $\mathbf{u}^k_{G,0}$ is estimated by the following expression, adapted from Kim and Duarte[4]:

$$\mathbf{u}^k_{G,0} = \frac{(k+1)}{k}\mathbf{u}^{k-1}_G. \tag{3}$$

Stage 2. Nonlinear local problem

The local problem is solved incrementally-iteratively in the local domain Ω_L.The local displacement vector $\mathbf{u}^k_L$ is calculated by the following equation, which has boundary conditions from the initial global solution of the Stage 1.

$$\int_{\Omega_L} \boldsymbol{\sigma}(\mathbf{u}^k_L) : \boldsymbol{\varepsilon}(\mathbf{v}^k_L)\, d\mathbf{x} + \eta \int_{\partial\Omega_L \cap \partial\Omega^u_G} \mathbf{u}^k_L \cdot \mathbf{v}_L\, d\mathbf{s} = \int_{\partial\Omega_L \cap \Omega^\sigma_G} \bar{\mathbf{t}} \cdot \mathbf{v}^k_L\, d\mathbf{s} + \int_{\partial\Omega_L \setminus (\partial\Omega_L \cap \partial\Omega_G)} [\mathbf{t}(\mathbf{u}^k_G) + \eta\mathbf{u}^k_G] \cdot \mathbf{v}^k_L\, d\mathbf{s}, \tag{4}$$

where η is the penalty parameter, $\mathbf{v}^k_L$ are the test functions of the local problem, and $\mathbf{t}(\mathbf{u}^k_G)$ is the stress vector

In this stage of each block of global-local analysis, it is necessary to solve the problem from the beginning of the loading, up to the level of loading of the block. In order to adequately represent the problem, the number of local steps resolved at each block is increased. The number of total local steps (N_{PL}) solved in each block k is given by:

$$N_{PL} = P_{LI} + [(k+1)P_{LA}], \tag{5}$$

where P_{LI} is the number of initial local steps, and P_{LA} is the number of local steps added to each global-local block.

Stage 3. Enriched linear global problem

The constitutive relation is given by $\boldsymbol{\sigma} = \mathbf{C}^s : \boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon}$ is the strain tensor and $\mathbf{C}^s$ is the secant approximation of the constitutive tensor adopted in the balance of the global model and obtained considering the damage occurred in the local problem. In this stage, local solution $\mathbf{u}^k_L$ is applied as extrinsic basis for enriching the global problem:

$$\{\phi_J\}(x) = \mathcal{N}_J(x) \times \mathbf{u}^k_L, \tag{6}$$

where J is referred to nodal points, $\mathcal{N}_J$ is the PoU function of the initial global problem and $\mathbf{u}_L^k$ is the local solution, named global-local enrichment function. The global enriched problem is defined by:

$$\int_{\Omega_G} \boldsymbol{\sigma}(\mathbf{u}_G^k) : \boldsymbol{\varepsilon}(\mathbf{v}_G^k)\, d\mathbf{x} + \int_{\partial\Omega_G^u} \mathbf{u}_G^k \cdot \mathbf{v}_G^k\, d\mathbf{s} = \int_{\partial\Omega_G^\sigma} \bar{\mathbf{t}} \cdot \mathbf{v}_G^k\, d\mathbf{s} + \int_{\partial\Omega_G^u} \bar{\mathbf{u}} \cdot \mathbf{v}_G^k\, d\mathbf{s}, \tag{7}$$

The solution $\mathbf{u}_G^k$ is obtained for the entire load (load factor $\lambda = 1$). $\mathbf{u}_G^k$ is adjusted according to the size of the displacement step P_{DG} predefined for the global problem. The load factor λ_E^k is defined as:

$$\lambda_E^k = \frac{(k+1)P_{DG}}{u_{G,DC}^k}, \tag{8}$$

where $u_{G,DC}^k$ is a displacement component of the control node, obtained from eq.(7).

Figure 1 presents the solution algorithm of the proposed approach. k is the block, i is the local step and j is the local iteration.

begin
  execute();
  foreach *block* k **do**
    Solve Stage 1:
    if k*=0* **then**
      Solve linear equation system and get $\mathbf{u}_{G,0}^0$
    else
      Get the estimated solution $\mathbf{u}_{G,0}^k = \frac{(k+1)}{k}\mathbf{u}_G^{k-1}$
    end
    Transfer boundary condition from Stage 1 to Stage 2;
    Solve Stage 2:
    foreach *local step i=i+1* **do**
      repeat
        Assemble stiffness matrix $[K]_{j-1}^i$;
        Get the incremental displacement $\left\{\Delta U^P\right\}_j^i$ e $\left\{\Delta U^Q\right\}_j^i$;
        Get the load factor increment $\Delta\lambda_j^i$;
        Update the nodal displacement vector $\{U\}_j^i = \{U\}_{j-1}^i + \Delta\lambda_j^i\ \left\{\Delta U^P\right\}_j^i + \left\{\Delta U^Q\right\}_j^i$;
        Update the load factor $\lambda_j^i = \lambda_{j-1}^i + \Delta\lambda_j^i$;
        Get the vector of equivalent nodal internal forces $\{F\}_j^i$;
        Update the residual forces vector $\{Q\}_j^i = \lambda_j^i\ \{P\} - \{F\}_j^i$;
      until *convergence*;
    end
    Solve Stage 3:
    Enrich global problem with $\mathbf{u}_L^k$ obtained in 2;
    Solve the linear equation system and calculate $\mathbf{u}_G^k$;
  end
end

Figure 1. Solution algorithm to the nonlinear global-local approach.

3 Numerical Simulations

Based on the three-point bending tests in notched concrete beams performed by Petersson [11], Fig. 2 illustrates geometry, loading (P$= 800,0$ N) and boundary conditions.

In the initial global problem, it is adopted Young's modulus $E_0 = 30000,0$ MPa and Poisson ratio $\nu = 0,20$. In the local problems the parameters adopted for smeared crack and microplane constitutive models are presented in the sections 3.1 and 3.2, respectively.

The beam is analyzed with the mesh shown in Fig. 3. There are 101 four-noded quadrilateral elements, totalizing 132 nodes, and 64 elements in the local mesh, with 12 global nodes enriched with only the local numerical solution. The eight white nodes are enriched only with the polynomial functions P1 or P2, whose approximation functions, with monomials expressed in coordinate x, are defined by eqs.(9) and (10), respectively:

$$P1 \rightarrow \phi_j^T(x) = \left[\mathcal{N}_j(x) \quad \mathcal{N}_j(x)\left(\frac{x - x_j}{h_j}\right)\right]; \tag{9}$$

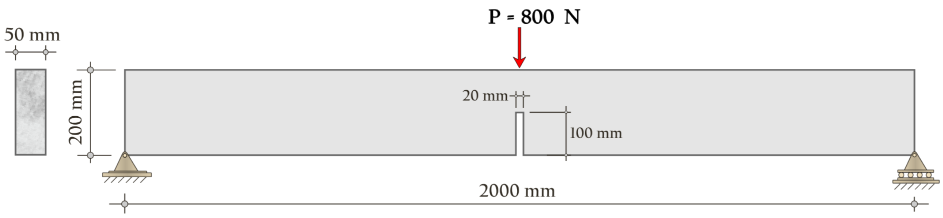

Figure 2. Three-point bending test.

$$P2 \rightarrow \phi_j^T(x) = \left[\mathcal{N}_j(x) \quad \mathcal{N}_j(x)\left(\frac{x-x_j}{h_j}\right) \quad \mathcal{N}_j(x)\left(\frac{x-x_j}{h_j}\right)^2 \right]. \tag{10}$$

In the global mesh, point A corresponds to the node whose vertical displacement is considered in the composition of the equilibrium paths. In the local mesh point B is adopted as a control node in the nonlinear analysis by the displacement control method (Batoz and Dhatt [12]).

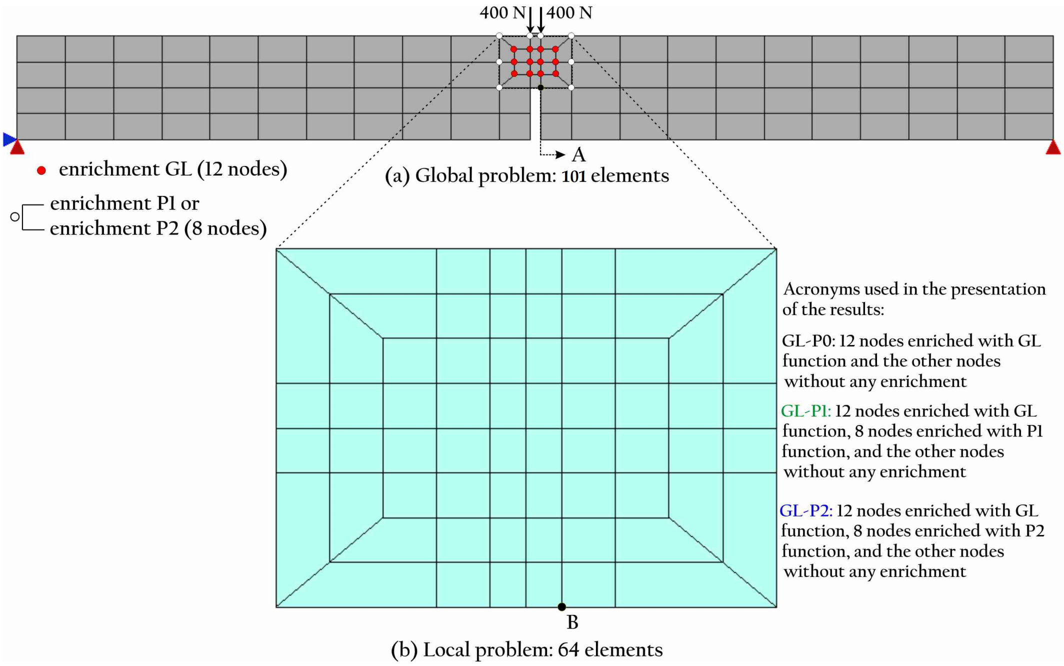

Figure 3. Global and local problems.

The nonlinear analysis of the local problem is performed with secant approximation to the constitutive tensor and tolerance to convergence equals to $1 \times 10^{-4}(\times 100\%) = 0,010\%$ in relation to the norm of incremental displacements vector. In local mesh, penalty parameter is $\eta = 3 \times 10^{12}$ and there are 4×4 Gauss points per element (same number in the global mesh). The following parameters were adopted: 50 global steps, 20 initial local steps, 2 local steps added to each block of analysis, and global displacement step of $0,020$ mm.

3.1 Smeared Crack Model

Smeared Crack Model is based on monitoring the deterioration of the physical properties of the material, and the crack evolution process is described by the gradual decay of stresses with increased strains. The parameters obtained experimentally by Petersson [11] presented a range of values of $2,50$ N/mm^2 to $3,90$ N/mm^2 to the tensile strength (f_t) and of 115 N/m to 137 N/m to the fracture energy. The smeared crack model of fixed direction with the Carreira and Chu [6, 7] stress-strain laws and plane stress state are considered in the analyzes. The material parameters adopted are: compressive strength $f_c = 31,0$ MPa; tensile strength $f_t = 2,70$ MPa; compressive strain $\varepsilon_c = 0,002$; tensile strain $\varepsilon_t = 0,0001925$; shear retention factor $\beta_r = 0,00$.

In the Fig. 4 the equilibrium path GL-P0 is far from experimental results and there is a disturbance in its descending branch. The same behavior is seen in the equilibrium path GL-P1, but enrichment P1 reduces the load peak in relation to GL-P0. The use of the polynomial function P2 led to the equilibrium path GL-P2 compatible with the experimental results of Petersson [11], both in the estimation of the maximum load and in the description of the post-critical regime.

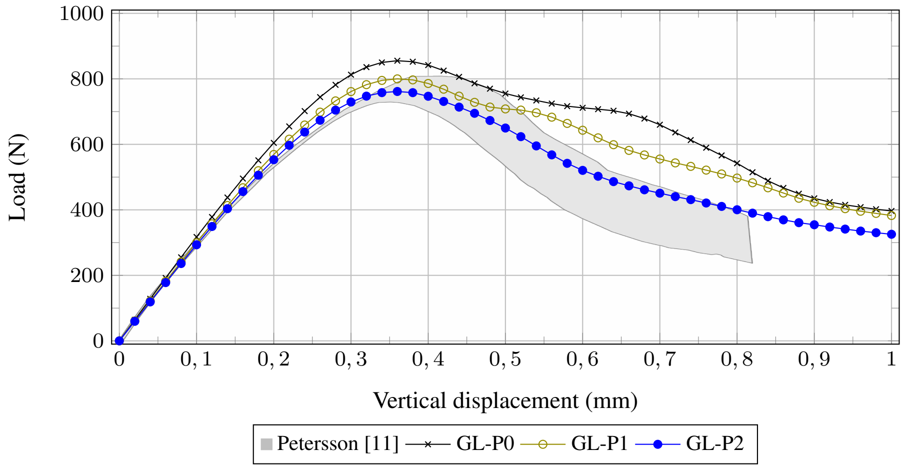

Figure 4. Equilibrium paths: global-local GFEM with P0, P1 e P2 enrichment functions.

Figure 5 shows the evolution of the damage in the local problem along the GL-P2 equilibrium path. It is observed that the damage is concentrated in the central region of the local domain corresponding to the region between the notch and the points of application of the load in the global problem.

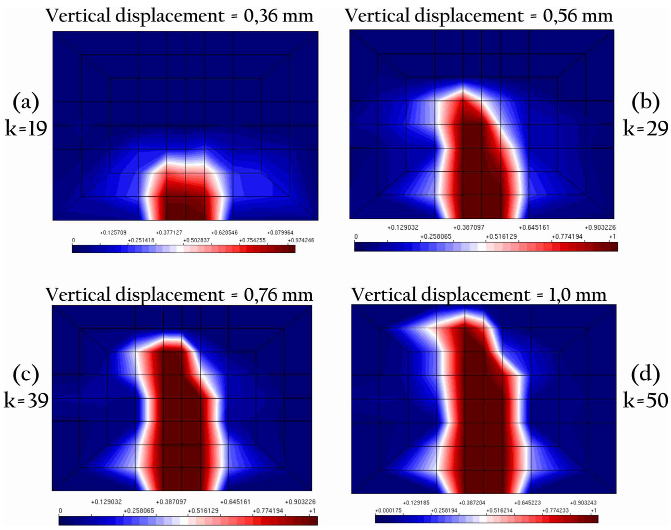

Figure 5. Evolution of damage.

3.2 Microplane Model

In this section, it is applied the microplane model of Leukart and Ramm [8], implemented in `INSANE` by Wolenski [14]. In this model is applied a kinematic constraint with components of volumetric and deviatoric strains, and a single damage variable that couples volumetric and deviatoric damage in the microplanes is adopted, controlling the degradation from damage evolution functions dependent on a single equivalent strain measure.

Table 1 presents the numerical parameters to the linear, bilinear, exponential and polynomial damage functions with the strain measure of de Vree *et al.* [13] (Wolenski [14]). The dimensionless parameters are defined as: κ_0 is the limit value of the damage initiation, κ_u determines the final damage value, κ_{cr} delimits an intermediate damage value, f_0 represents the material limit stress, f_{cr} defines the intermediate stress in the post-peak branch, α^{mic} is the maximum degradation of the material, β^{mic} is the the parameter that governs the shape of the post-peak curve, f^e is the equivalent stress relative to the material strength limit, and E^0 is the initial elastic modulus.

Figure 6 shows the equilibrium paths GL-P0, GL-P1 and GL-P2 obtained by the four damage functions. The general trend observed is that the application of polynomial enrichment in the nodes that surround the region where

Table 1. Parameters of the damage functions (Wolenski [14]).

Linear				
$\kappa_0 = 0,000190$	$\kappa_u = 0,00460$			
Bilinear				
$\kappa_0 = 0,000195$	$\kappa_u = 0,0055$	$\kappa_{cr} = 0,00155$	$f_0 = 4,0$	$f_{cr} = 2,25$
Exponential				
$\alpha^{mic} = 0,960$	$\beta^{mic} = 500$	$\kappa_0 = 0,0002$		
Polynomial				
$f^e = 5,95$ MPa	$E^0 = 30000,0$ MPa	$\kappa_0 = 0,000385$		

the enrichments with the global-local solution are applied brings the equilibrium paths closer to the Petersson [11] experimental results, more expressively with the aplication of the P2 enrichment. Linear and bilinear damage functions do not represent experimental behavior well. There is a good agreement between the GL-P2 results of the exponential and polynomial damage functions and the experimental results in the estimation of the maximum load, and the polynomial damage function describes the equilibrium path more adequately. Figure 7 shows the

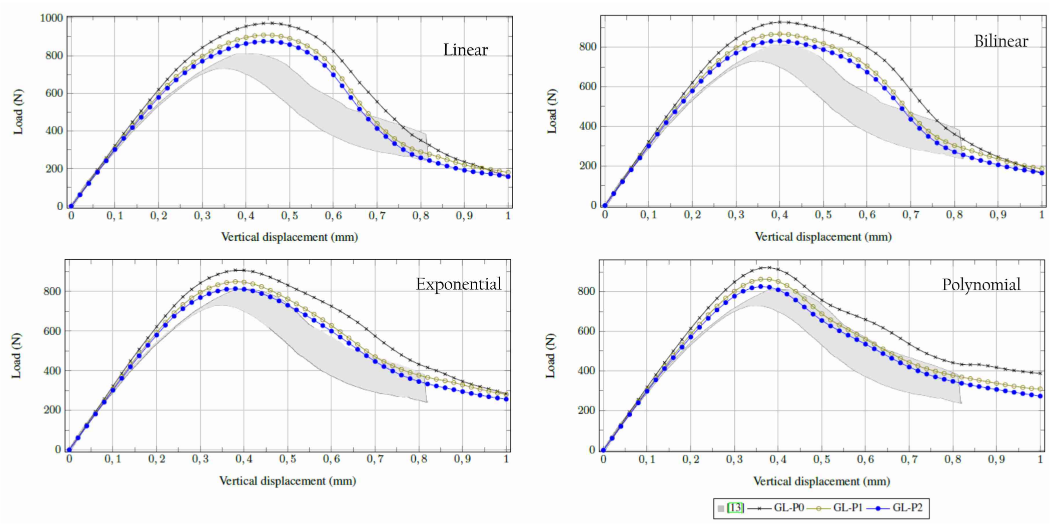

Figure 6. Equilibrium paths: global-local GFEM with P0, P1 e P2 enrichment functions.

evolution of the damage in the local problem along the GL-P2 equilibrium path. It is observed that the damage is concentrated in the central region of the local domain corresponding to the region between the notch and the points of application of the load in the global problem.

4 Conclusions

In the proposed GFEM global-local the evolution of the phenomena of material deterioration observed only in the local problem and its influence on the global behavior of the structures was captured from both smeared crack model of fixed direction with the Carreira and Chu [6, 7] stress-strain laws and microplane model of Leukart and Ramm [8]. The responses obtained were compatible with the experimental results available in the literature and it is concluded that the proposed global-local approach was able to improve the quality of the solution comparing with standard GFEM. New investigations must be performed aiming to verify other numerical examples and constitutive models.

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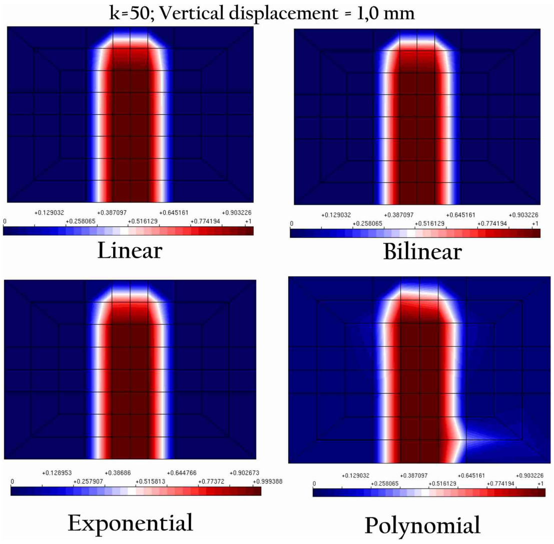

Figure 7. Evolution of damage to the four damage functions: vertical displacement of $1,0$ mm.

References

[1] Duarte, C. A. & Babuška, I., 2005. A global-local approach for the construction of enrichment functions for the Generalized FEM and its application to propagating three-dimensional cracks. In *Technical report, ECCOMAS Thematic Conference on Meshless Methods*, Technical Report 06.

[2] Kim, D. J., Duarte, C. A. & Proença, S. P., 2012. A Generalized Finite Element Method with global-local enrichment functions for confined plasticity problems. *Computer Methods in Applied Mechanics and Engineering*, 50, pp. 563–578.

[3] Freitas, A., Torres, D. A. F., Mendonça, P. T. R. & Barcellos, C. S., 2015. Comparative analysis of C^k- and C^0- GFEM applied to two-dimensional problems of confined plasticity. *Latin American Journal of Solids and Structures*, 12(5), pp. 861–882.

[4] Kim, J. & Duarte, C. A., 2015. A new Generalized Finite Element Method for two-scale simulations of propagating cohesive fractures in 3-d. *International Journal for Numerical Methods in Engineering*, 197, pp. 487–504.

[5] Evangelista Jr, F., Alves, G. S., Moreira, J. F. A & Paiva, G.O. F., 2020. A global–local strategy with the generalized finite element framework for continuum damage models. *Comput. Methods Appl. Mech. Engrg*, 50, pp.1–20.

[6] Carreira, D.J. & Chu, K., 1985. Stress-strain relationship for plain concrete in compression. *ACI Journal*, 82, pp. 797–804.

[7] Carreira, D.J. & Chu, K., 1986. Stress-strain relationship for reinforced concrete in tension. *ACI Journal*, 83, pp. 21–28.

[8] Leukart, M. & Ramm, E., 2006. Identification and interpretation of microplane material laws. *Journal of Engineering Mechanics*, 132(3),pp. 195–305.

[9] Monteiro, A. B., 2019. Análise não linear de meios parcialmente frágeis via abordagem global-local do Método dos Elementos Finitos. *Thesis, Federal University of Minas Gerais*.

[10] Gori, L., Penna, S. S. & Pitangueira, R.L.S., 2017. A computational framework for constitutive modelling. *Computers and Structures*, 187, pp. 1–23.

[11] Petersson, P. E., 1981. Crack growth and development of fracture zones in plain concrete and similar materials. *Technical Report TVBM-1006*, Lund Institute of Technology, Lund, Sweden.

[12] Batoz, J.-L. & Dhatt, G., 1979. Incremental displacement algorithms for nonlinear problems. *International Journal for Numerical Methods in Engineering*, 14(8), pp. 1262–1267.

[13] de Vree, J. H. P., Brekelmans, W. A. M. & van Gils, M. A. J., 1995. Comparison of nonlocal approaches in continuum damage mechanics. *Computer e Structures*, pp. 581–588.

[14] Wolenski, A. R. V., 2013. Ambiente Teórico-Computacional Unificado para Modelos Constitutivos: Inclusão de Modelo de Microplanos. *Dissertation, Federal University of Minas Gerais*.