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ERROR INTERPOLATION FOR REFERENCE VALUE CHARACTERIZATION FOR COMPLEX COLUMN BUCKLING PROBLEMS

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Abstract. For convergence verification of the column buckling problem it is necessary to obtain a good reference value to compare the values obtained during the numerical process, in most cases the reference value used is given by an experimental data or the result obtained by an analytical solution. In this way, the purpose of the present study is to analyze a critical load buckling problem in a continuous beam that have analytical solution and compared with curve fitting reference value obtained by Finite Element Method. The methodology applied seen to be reliable and applicable to problems that does not have analytical solution.

Keywords: convergence order, finite element, curve fitting, discretization error.

1 Introduction

The commonly use of numerical approaches to solve engineering problems brings with it the problem of having a reliable reference value to verify that the analysis are being performed in the right way. Buckling problems are very important in engineering projects, and are not trivial to solve, having many ways to solve it numerically. Soltani [1] said in his paper work that, the accuracy on critical buckling load could aid engineers to develop structures with satisfying stability. For a numerical analysis with a reliable result it is necessary an error analysis, Marchi and Silva [2] presents some procedures for estimating the error in numerical procedures for multi and one dimensional problems. Richardson extrapolation is widely used combined with finite element method, as says Mestrovic [3], the idea to implement Richardson extrapolation is to get better solutions with less computational cost. This type of method can be applied for many engineering research fields, Giacomini *et al.* [4] used for reduce the discretization error in CFD fields.

As several engineering problems have no analytical solution and experimental procedures may be unviable, some reliable reference value should be used to ensure a good convergence analysis. In the present work the analysis of the buckling problem of a continuous beam was made by the finite element method and compared to the available analytical solution in Timoshenko and Gere [5]. An estimation of analytical solution was performed by curve interpolation through finite element method results. Analysis was performed for continuous and stepped columns, considering the different moment of inertia between the section transitions. Results were stored and curve fitting was performed, the result obtained by curve fitting was used as the best approximation of the analytical solution.

2 ANSYS FE validation

The Finite Element Model (FEM) was executed in ANSYS APDL language, the modelling was performed using BEAM3 element that is a uniaxial element capable of withstanding traction, compression and moment. Each node has three possible degrees-of-freedom: displacement in x and y, and rotation around the z axis, as it stated in ANSYS Element Reference [6]. The BEAM3 element is based on the Euler-Bernoulli beam column theory. This theory assumes that a normal straight to the neutral line will be kept normal after deformation, consequently, shear deformations are neglected affirms Timoshenko and Gere [5].

As the stepped columns, to avoid miscalculations regarding the geometric property of the elements at the intersection of the geometric discontinuities, the mesh was generated so that each section has an equal number of elements. The problem was divided into four sections, where the two ends have lower stiffness and the other two central sections have higher stiffness according to the ratio (R). The first mesh generated was a coarse mesh with only 8 elements, being two elements per section, after the first generated mesh was added one element per section for convergence analysis; this process was repeated until the occurrence of numerical instability.

The solution was executed through Block Lanczos method, this method is very used to solve eigenvalues problems in symmetric and sparse matrix in big scale, the Subspace method could be used, and however, it tends a slow convergence. Block Lanczos algorithm is a variation of the classical Lanczos algorithm, where recursion is performed using a block vector instead of a unit vector, as contained in ANSYS Performance Guide [7].

3 Numerical procedure

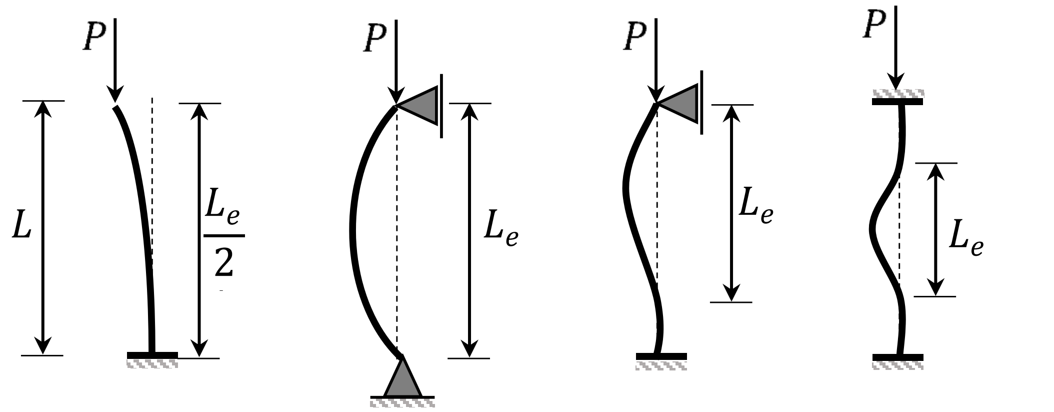
The convergence analysis of the Finite Element Method was performed through an adjustment curve of the critical load values obtained by the finite element method for a continuous column as a reference problem and stepped column with a different moment of inertia between the section transition with ratio (R) equal 1.5 and 2. Data were calculated for different meshes, with an increasing number of elements. Using the case of a continuous column simply supported (b) in which the analytical solution is known, the following procedures show the use of a fit curve in order to validate the results obtained.

To verify the reliability of the proposed alternative, the analysis was performed for a continuous column, with known analytical solution, calculated by:

$$P_{crit} = \frac{\pi^2 EI}{L_e^2}, \tag{1}$$

where, E =Young's Modulus; I =Section Moment of Inertia; L_e is the column equivalent length.

The equivalent length for critical load calculation depends on the boundary conditions at the column end. represents the equivalent length for each boundary condition, as shows Fig. 1.



(a) $\boldsymbol{L_e = 2L}$ (b) $L_e = L$ (c) $L_e = 0.699L$ (d) $L_e = 0.5L$

Figure 1. Boundary condition for Euler's critical load.

For a stepped pinned-pinned column (Fig. 2) , using the procedures in Timoshenko and Gere [5] is possible to find an analytical solution through

$$\tan\left(\sqrt{\frac{P}{EI_1}}\frac{(L-a)}{2}\right)\tan\left(\sqrt{\frac{P}{EI_2}}\frac{a}{2}\right) = \sqrt{\frac{I_2}{I_1}}. \tag{2}$$

The critical load (P) of the transcendental equation can be calculated by numerical method, like Newton-Raphson method.

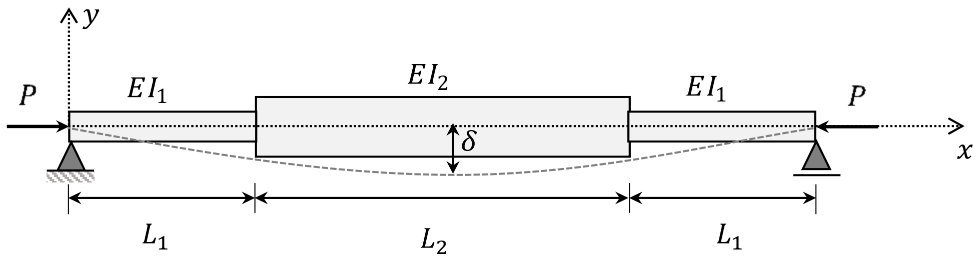


Figure 2. Continuous column buckling under axial load.

In order to obtain a reliable solution for problems that does not have available analytical solution, it was proposed a methodology, interpolate the reference value of finite element method analysis, using CFtool/MATLAB toolbox through adjustment

$$y = \phi_{Fit} + bn^c, \tag{3}$$

being ϕ_{Fit} the estimation of the analytical solution, b is constant, n is the number of elements of the discretized model and c is the apparent order of the error.

For convergence analysis, the true error of the numerical solution was determined by

$$\varepsilon(\phi_{ANSYS}) = |\phi_{Fit} - \phi_{ANSYS}|. \tag{4}$$

where, ϕ_{ANSYS} is the numerical solution by ANSYS for each mesh.

To verify the accuracy of the proposed method, the true error of the numerical solution was also calculated using

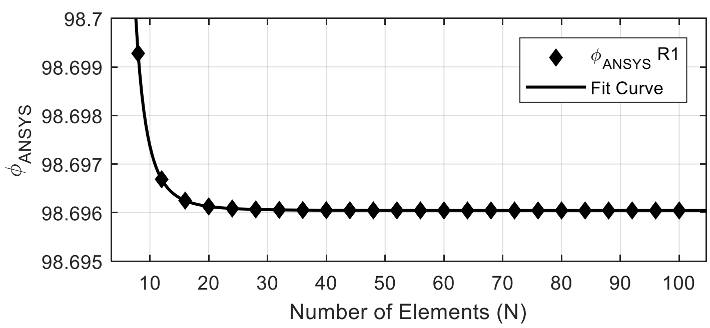
$$\varepsilon(\phi_{ANSYS}) = |\phi_{Analytical} - \phi_{ANSYS}|, \tag{5}$$

being $\phi_{Analytical}$ the analytical solution.

Data were calculated for different meshes, with a number of elements range from 8 to 100. This analysis was performed by ANSYS running in batch mode, the results were stored in arrays and interpolation was performed afterward. Then, through eq. 4 and 5 it is possible to identify the convergence order and the consistency of the model proposed.

4 Results

This section shows the results for the approached methodology, from Fig. 3 to Fig. 5 the results of the interpolation, being Fig. 3 for a continuous column, Fig. 4 for stepped column with ratio 1.5, and Fig. 5 for stepped column with ratio 2.



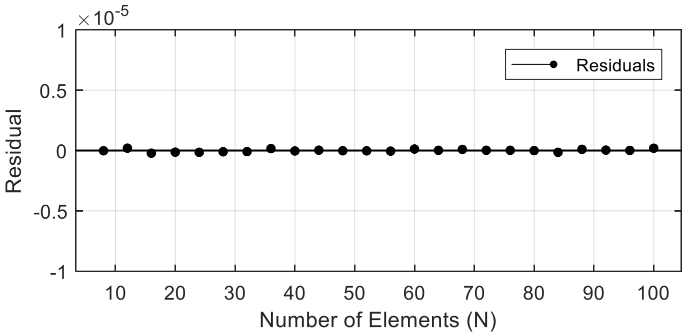


Figure 3. Interpolation of results obtained through finite element method for continuous column. (a) Curve fitting. (b) Interpolation residual analysis.

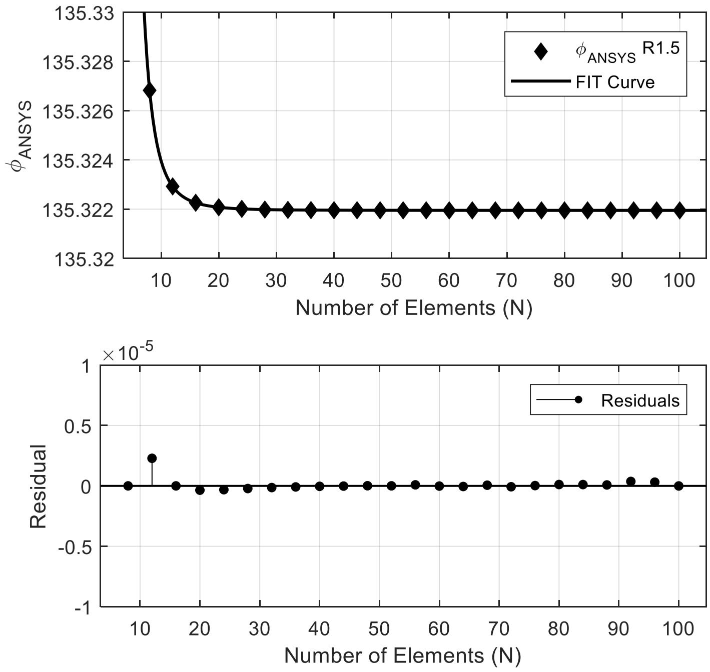
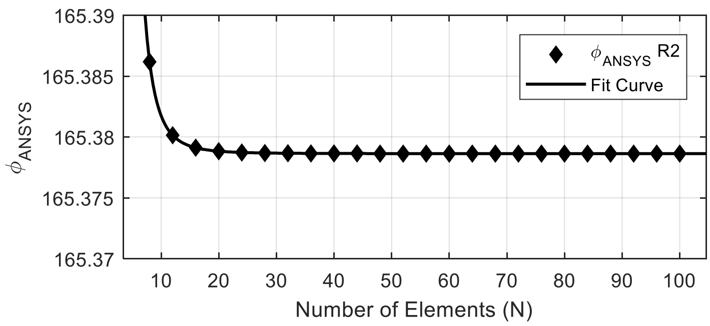


Figure 4. Interpolation of results obtained through finite element method for stepped column R1.5. (a) Curve fitting. (b) Interpolation residual analysis.



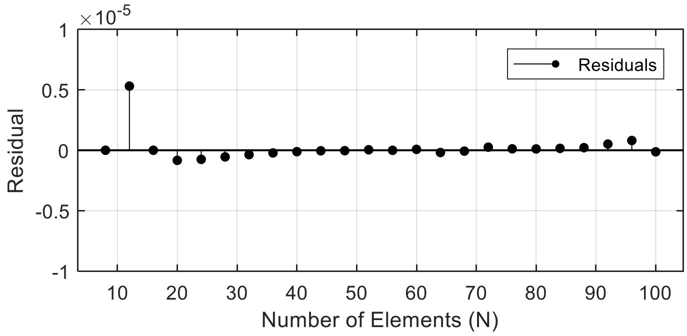


Figure 5. Interpolation of results obtained through finite element method for stepped column R2. (a) Curve fitting. (b) Interpolation residual analysis.

The interpolation analysis shows that the model has been good agreement with the equation proposed in methodology. For all cases the maximum residual was of the order of 10^{-5}, being the continuous column with the lower residual, this can be explained because of the continuity of the beam, with no difference geometry as stepped R1.5 and stepped R2 column, has no accumulation of error in the intersection of the different geometries, mainly with coarse meshes when the error is greater than finer meshes.

Table 1 shows the obtained results for curve fitting, the square residual has been 1 for all interpolations, showing that the fitting has good agreement. A comparison between the analytical solution ($\phi_{Analytical}$) and estimated reference value (a), shows that this method return good final reference value.

Table 1. Coefficients and residuals resulting from a polynomial adjustment of continuous and stepped column by FEM.

Inertia Ratio	$\boldsymbol{\phi_{Analytical}}$	$\boldsymbol{\phi_{Fit}}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{R^2}$
1	98.696044	98.696043992378	12.970366201446454	−3.989879141617130	1
1.5	135.321950	135.32195021962	19.046251616735855	−3.977284121146561	1
2	165.378631	165.37863155089	28.785500224644412	−3.966562224562297	1

Figure 6 to Fig. 8 shows the comparison between the analytical result as reference value, and curve fitting estimation for reference value.

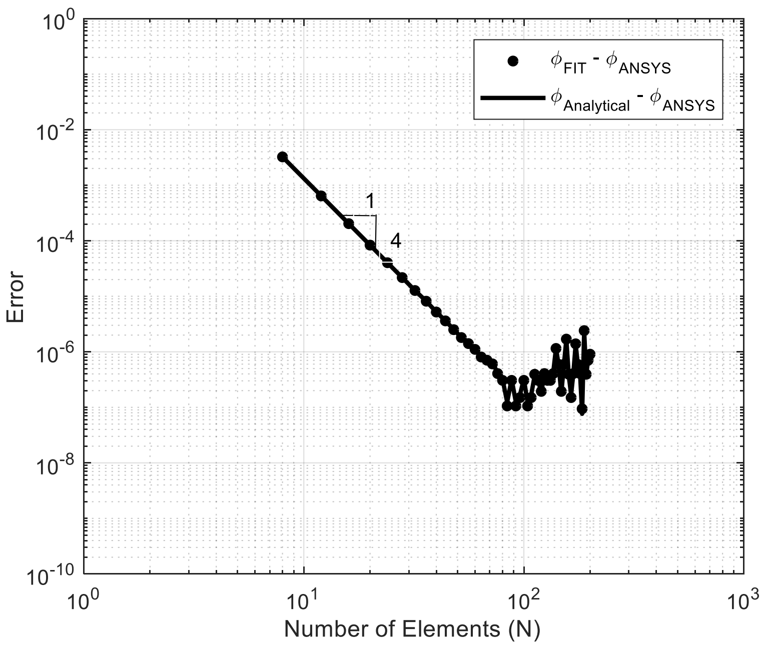


Figure 6. Convergence analysis of the result obtained through finite element method for continuous column using analytical and curve fitting result as reference value.

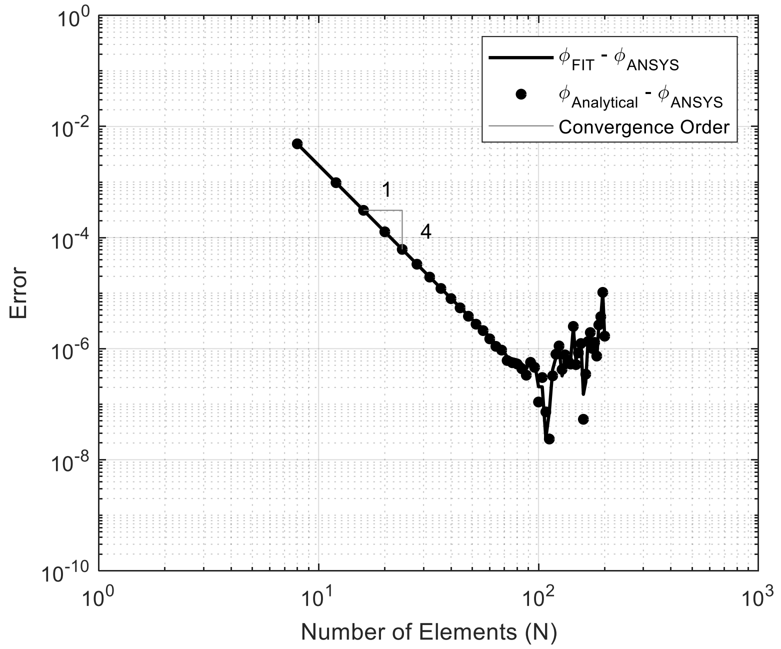


Figure 7. Convergence analysis of the result obtained through finite element method for stepped column R1.5 using analytical and curve fitting result as reference value.

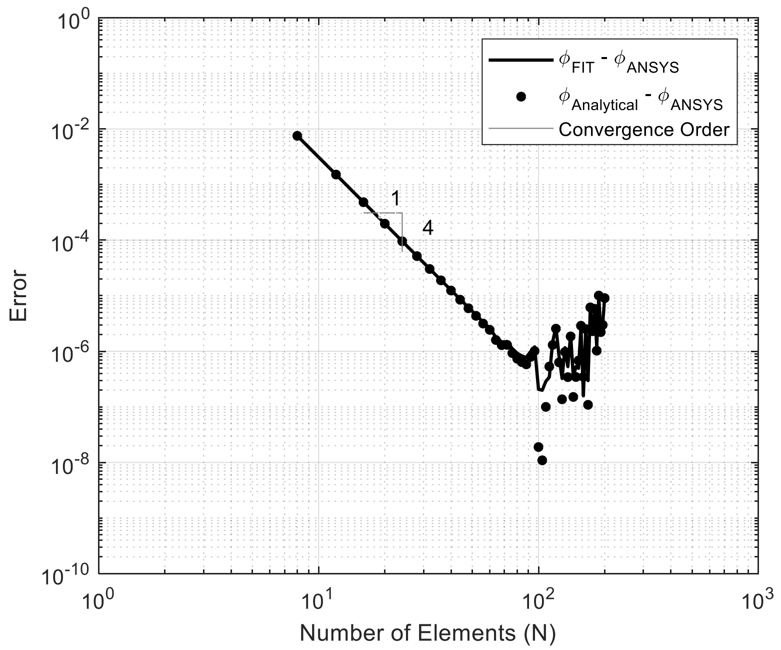


Figure 8. Convergence analysis of the result obtained through finite element method for stepped column R2 using analytical and curve fitting result as reference value.

For all cases the order of convergence was 4^{th} order asymptotic monotonic convergent, this can be explained because ANSYS use a 4^{th} order formulation to solve buckling problems. A numerical instability from 80 elements occurs for all cases, this demonstrate the importance of performing a convergence analysis .The results were consistent when reach the expected convergence order, when the problem does not reach the convergence order, an error accumulation can be happen due to mesh refinement or wrong considerations in the analysis principles.

5 Conclusions

The presented results showed to be convergent and consistent, the analysis reach the convergence order 4 as expected, and the behavior of continuous column and stepped column was similar, the difference between stepped and continuous column was due to the section difference, what generates an error accumulation with coarse meshes, when the mesh was refined the errors decreased and the model could be better adjustment.

Curve fitting reference value shows good accuracy with the analytical result, this methodology could be applied to others buckling problems that does not have available analytical solution as long as a convergence analysis be performed and the numerical instability verified. It is important to highlight that is determinant a previous analysis with simple problem to verify if the method and boundary conditions are correct, and then the more sophisticated analysis could be performed, using the step by step used in the simplest problem.

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