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A PARAMETRIC STUDY OF FOOTBRIDGE VERTICAL BENDING RESPONSE CONSIDERING HUMAN-STRUCTURE INTERACTION

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Abstract. The effect of human walking in slender footbridges have been experimentally observed by many authors such as changes in the modal properties of the coupled human-structure system, especially damping ratio, which is responsible for the attenuation of the structural vibration oscillations. To simulate the pedestrian - structure dynamic interaction biodynamic models have been used to represent human walking effects (in contrast to the live load traditional model). The structural response is then governed by the modal parameters of the biodynamic models in addition to those of the structure itself. This paper presents a parametric study of footbridge vertical bending response considering the modal properties of a single degree of freedom spring-mass damper biodynamic model coupled to the structure within a mathematical model of the human-structure interaction. The results highlight the important role played by the frequency of the biodynamic model in addition to that of the walking frequency in relation to the frequency of the structure.

Keywords: Structural dynamic, Human-structure interaction, Footbridges, Parametric study.

1 Introduction

Design codes and practical guidelines, such as ISO 10137 [1] and Sètra [2], considers the effect of human walking in footbridge as a moving load model (ML). However, this procedure presents a great limitation, specially, in slender footbridges, because it does not consider the human structure interaction (HSI) effect. According to Shahabpoor *et al*. [3], the HSI is defined as a continuous and mutual dynamic effect that the pedestrian acts on the structure and consequently the structure acts on the person.

There is experimental evidence indicating that HSI is relevant both for the response amplitudes of the structure and the dynamic properties of the footbridge-pedestrian coupled system, especially the damping ratio. Brownjohn [4] carried out tests on a simply supported beam structure considering a person in different positions and observed important increases in the damping of the system. Nimmen *et al*. [5] noted similar results when a varying number of persons walks in group on a footbridge in the University of Warwick. Wang *et al*. [6] reported an increase of up to 4-fold of the damping ratio of the Olga footbridge in Germany, when it was occupied in comparison to the empty structure. According to Jones *et al*. [7], the increase of damping ratio of the pedestrian-footbridge coupled system occurs due to the capacity of the human body to absorb the oscillations of the structure.

To evaluate the HSI effect in footbridges, biodynamic models represented by a single degree of freedom spring-mass-damper model (SMDM) have been used. However, a great number of different biodynamic models are been developed and the variety of their parameters can represent a problem to simulate the real effect of human walking on structures. In this way, the influence of the modal properties of the SMDM and of a hypothetical structure in the HSI effect were assessed in this paper.

2 Pedestrian Force Model

Due to the accelerating and decelerating of the human body mass, a pedestrian produces, during walking, forces in the lateral, longitudinal and vertical directions. Nevertheless, the vertical component is the most investigated [8].

Experimental tests were conducted to describe the vertical force produced by human walking [9-11]. Measurements of continuous walking time histories indicated a nearly periodic behavior of the force. This hypothesis was used by many authors, codes and guides [1, 2, 8-12]. In this way, the vertical reaction force due to a pedestrian walking is commonly written as a Fourier series, as follow in Eq. (1):

$$F(t) = G + \sum_{i=1}^{nh} G\alpha_i \sin(2\pi i f_p t - \varphi_i) \tag{1}$$

where $F(t)$ is the vertical force, G is the pedestrian weight, α_i is the dynamic load factor (DLF); f_p is the pedestrian walking frequency, t is the time, φ_i is the phase angle, *i* is the order of the harmonic and *nh* is the total number of contributing harmonics.

The DLFs are generally obtained from experimental campaigns and it depends on the physical characteristics and gait of the pedestrian. Therefore, it is common to observe a large deviation of these values in different references. Costa [10] and Varela *et al.* [11] carried out an experimental campaign in which 53 persons walked individually on a rigid mixed steel-concrete footbridge. The vertical ground reaction force of the floor and the acceleration of the human body were measured and the expressions of the DLF were adjusted to the measured data as indicated in Table 1:

Table 1: DLFs and phase angle of the Fourier series [10-11]

DLFs	Phase Angle
$\alpha_1 = 0.156 f_p^3 - 0.182 f_p^2 + 0.036$	$\varphi_1 = 0.0^{\circ}$
$\alpha_2 = \begin{cases} 0.065 f_p, \text{ if } f_p \le 2Hz \\ 0.1958 f_p - 0.3266, \text{ if } f_p > 2Hz \end{cases}$	$\varphi_2 = 85.6^{\circ}$
$\alpha_3 = 0.06$	$\varphi_3 = 89.4^{\circ}$

According to Ross [8], usually the first three harmonics are taken into account and the others higher harmonics do not have much influence. However, Varela [13] pointed out a considerable amplification of the response of the fourth harmonic of the human load in a mixed steel-concrete slab when persons walked on it.

3 Spring-mass-damper model (SMDM)

Many authors have reported in their work the effect of Human Structure Interaction (HSI) on the modal properties of the structure [3-7, 10, 12]. Zivanovic *et al.* [14] pointed out two aspects in relation to HSI background: (i) firstly, in the structure, it is responsible for changes in the dynamic properties such as the natural frequency and damping ratio, (ii) the second aspect involves the pedestrian motion which alters its gait (walking frequency, step length and speed) under structure's vibration perception.

In spite of the HSI has been reported by several researchers [3-7, 10, 12, 14], it is known very little about how the modal properties of the structure changes. In this way, biodynamic models have been developed in order to simulate the HSI.

The modal properties of the first biodynamic models were based on the publications from the biomechanical literature that considers the person stand and stationary [12]. Moreover, other biodynamic models were calibrated to consider the person walking, although the modal parameters of some SMDM do not depend of the physical characteristics [18] and gait parameters of the person walking [5], [15], [16,17], such as its mass (M) and walking frequency (f_p). Table 2 indicates the modal properties of some biodynamic models.

Table 2: Modal properties of the biodynamic models

Reference	Modal mass of the model (m_p) $[kg]$	Frequency of the model (f_{mod}) $[Hz]$	Damping ratio of the model (ξ_p) [%]
Alonso and Saéz [15]	$0.84M$	2.75	47.0
Nimmen *et al.* [5]	$0.95M$	3.34	26.0
Zhang *et al.* [16]	$1.00M$	1.85	30.0
Lou *et al.* [17]	$1.00M$	$1.25-1.60$	$37.5-50.0$
Shahabpoor *et al.* [18]	70.00	$2.75-3.00$	$25.5-30.0$
Zhang *et al.* [19]	$1.00M$	$1.1043f_p$	30.0
Wang *et al.* [20]	$1.00M$	$0.3049f_p+1.3670$	$-0.2116f_p+0.8737$

Recently, Silva [21], Toso [22], Costa [10] and Varela *et al.* [11], carried out experimental campaigns to determine the modal properties of the SMDM using regression equations taking into account the physical and gait parameters of a pedestrian walking. They identified the modal parameters of the biodynamic model of a person walking analysing the correlation between the walking force and the acceleration of the human body.

However, the modal parameters of the SMDM of Silva [21] and Toso [22] differ somewhat from the results obtained by the references mentioned in Table 2 and the values related from the biomechanical literature, especially the frequency of the model and the damping ratio. On the other hand, the modal parameters of the biodynamic model of Costa [10] and Varela *et al.* [11], indicated by the regression equations in Eqs. (2) to (4), showed better correlations with the results found in the mentioned references.

$$m_p = 12.940 + 0.874M - 9.142f_p \tag{2}$$

$$k_p = 360.30m_p - 1282.5 \tag{3}$$

$$\xi_p = -20.818f_{ma} + 87.513 \tag{4}$$

where M is the mass of the person, f_p is the walking frequency, m_p is the modal mass, k_p is the modal stiffness, ξ_p is the damping ratio and f_{ma} is the damped frequency of the SMDM.

4 Mathematical Model

Two mathematical models are developed to consider the action of a single pedestrian walking in a lively footbridge. In the first model (4.1), the person is considered as a moving load. The second model (4.2), is based on the work of Costa [10] and the pedestrian is modelled as a coupled single-degree-of-freedom system with damping (c_p), stiffness (k_p), and mass (m_p).

4.1 Moving force model

Considering the footbridge as a beam-likely structure, the equation of motion (5) can be transformed into a set of uncoupled equations. Each *j*-th mode is described by an equivalent single degree of freedom equation:

$$m_{sj}\ddot{q}_j(t) + c_{sj}\dot{q}_j(t) + k_{sj}q_j(t) = Q_j(t) \tag{5}$$

where m_s is the modal mass of the structure, c_s is the modal damping of the structure, k_s is modal stiffness of the structure and q is the generalized coordinate of the structure.

Assuming the structural vibration energy is mostly concentrated on the first vertical bending mode $(j=1)$, the modal force, $Q(t)$, can be obtained multiplying the vertical reaction force Eq. (1) by the mode shape function,

$\phi(x,t)$, as follows in Eq. (6):

$$Q(t)=\frac{1}{m_{sj}}F(t)\phi(x,t) \tag{6}$$

The displacements of the footbridge can be written as in Eq. (7):

$$y(x,t)=\phi(x)q(t) \tag{7}$$

4.2 Coupled Spring Mass Damper Model

To take into account the human-structure interaction, a new force $f(x,t)$ is considered in this model. This force is related to the dynamic effects that the pedestrian acts on the structure and consequently, the structure acts on the pedestrian, while the persons walks on it. As shown in Figure 1 the interaction force can be obtained by the sum of the elastic and dampen portion of the biodynamic model:

$$f(x,t)=c_p[\dot{u}(t)-\dot{y}(t)]+k_p[u(t)-y(t)] \tag{8}$$

where y is the displacement of the footbridge in the contact point of the pedestrian and u is the generalized coordinate that represents the displacements of human body modelled as a SMD.

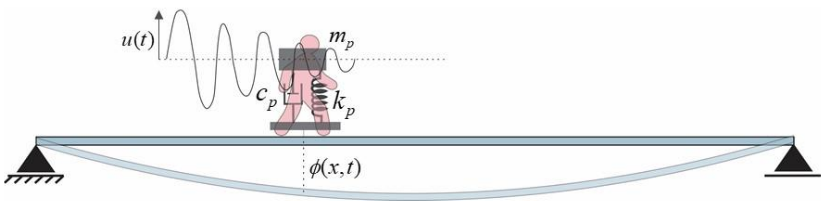


Figure 1. Pedestrian (SMDM) walking in a slender footbridge

The equation of motion of the pedestrian walking on the footbridge is given by the equilibrium of the forces at pedestrian's center of mass (CM) indicated in Figure 1, as follows:

$$m_p\ddot{u}(t)+c_p[\dot{u}(t)-\dot{y}(t)]+k_p[u(t)-y(t)]=0 \tag{9}$$

The equilibrium of forces at the contact point of the structure and the pedestrian's foot provides the Eq. (10):

$$m_s\ddot{q}(t)=-c_s\dot{q}(t)-k_s q(t)+\phi(x,t)[f(x,t)+F(t)] \tag{10}$$

Substituting Eq. (7) and (8) in Eq. (10) and rearranging the terms, the equation of motion of the structure is then described by Eq. (11):

$$m_s\ddot{q}(t)+[c_s+\phi^2(x,t)c_p]\dot{q}(t)+[k_s+\phi^2(x,t)k_p]q(t)-\phi(x,t)c_p\dot{u}(t)-\phi(x,t)k_p u(t)=Q(t) \tag{11}$$

Rewriting Eq. (9) and Eq. (10) in matrix form, the motion of coupled structure-pedestrian can be written as in Eq. (12):

$$\begin{gathered}\begin{bmatrix} m_s & 0 \\ 0 & m_p \end{bmatrix}\begin{Bmatrix} \ddot{q}(t) \\ \ddot{u}(t) \end{Bmatrix}+\begin{bmatrix} c_s+c_p\phi^2(x,t) & -c_p\phi(x,t) \\ -c_p\phi(x,t) & c_p \end{bmatrix}\begin{Bmatrix} \dot{q}(t) \\ \dot{u}(t) \end{Bmatrix}+ \\ \begin{bmatrix} k_s+k_p\phi^2(x,t) & -k_p\phi(x,t) \\ -k_p\phi(x,t) & k_p \end{bmatrix}\begin{Bmatrix} q(t) \\ u(t) \end{Bmatrix}=\begin{Bmatrix} Q(x,t) \\ 0 \end{Bmatrix}\end{gathered} \tag{12}$$

5 Parametric study

In order to evaluate the effect of the HSI in the attenuation of the response of footbridges, a parametric study is carried out in this section. Firstly, the influence of the parameters of the SMDM, such as modal mass, walking frequency and damping ratio are variables while the others parameters are kept constant. Later, the modal properties of the structure are variables and the parameters of the SMDM are kept fixed. The response of the footbridge is obtained solving numerically the mathematical models presented in section 4 using the Runge-Kutta method.

5.1 The influence of the parameters of the SMDM

It was evaluated the response of a structure in relation to the modal properties of a single pedestrian represented by a SMDM and its gait properties, such as walking speed (v_p), walking frequency (f_p), modal mass (m_p), damping ratio (ξ_p) and frequency of the model (f_{mod}). These input variables are normalized with respect to the base values obtained in accordance to [1-3], [5], [10-12], [15-16], [18-20], as follows: $v_{p,base} = 1.4\ m/s$, $f_{p,base} = 2\ Hz$, $m_{p,base} = 75\ kg$, $\xi_{p,base} = 30\ \%$, and $f_{mod,base} = 2.85\ \text{Hz}$. The DLFs and phase angles proposed by Costa [10] and Varela *et al*. [11] are used in the vertical reaction force.

The footbridge analyzed is a simply supported beam structure. The footbridge length is 35 m and the first vertical bending vibration mode is resonant with the base value of walking frequency of the pedestrian. The ratio between the base mass of the pedestrian and the modal mass of the structure is equal 3% [10]. The base value of damping ratio of the footbridge is $\xi_{s,base} = 0.5\ \%$. Figure 2 presents the variation of the ratio between the maximum mid-span RMS value normalized the corresponding response of the structure due to the passage of the SMDM with the base parameters.

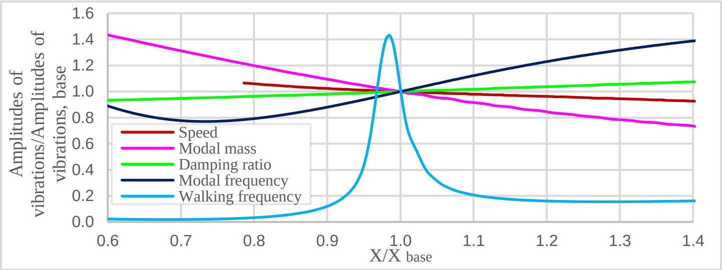


Figure 2. Sensitivity of the parameters of the SMDM in the dynamic responses of the structure

From the chart, it can be noticed that the walking speed and the damping ratio of the model are the less sensitive parameters that influences the response of the structure. Despite that, it can be seen a slight decrease on the vibration amplitudes of the structure with an increase on the walking speed. This result is in disagreement with the results observed by Nimmen *et al*. [5] and Wang *et al*. [6] that concluded that the speed of a pedestrian is inversely proportional to the interaction effect between the structure and person walking. In relation to the damping ratio of the model, the response of the structure increase with the increase of the biodynamic damping. This occurs, probably, due to the restriction of the motion of the spring-mass of the biodynamic model. High values of damping ratio can lock the SMDM and disable its effectiveness in reduction of the oscillations of the footbridge from the point of view of human-structure interaction.

Considering the modal mass of the SMDM, the role effect of this variable in the reduction of the amplitudes of the footbridge's motion is verified in a quasi linear relation with the increase of the model mass. Costa [10] reported similar results when two pedestrians with different masses crossed individually a lively footbridge. This conclusion discloses to the potential effect of groups and crowds that crosses the structure and their favorable contribution in terms of the modal mass.

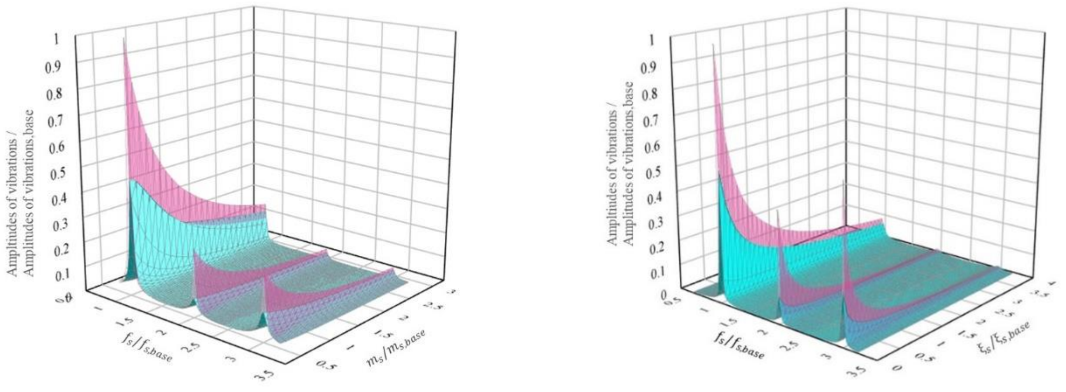
With respect to the frequency of the SMDM (f_{mod}), a considerable reduction in the RMS value was noted for the ratio $(f_{mod} / f_{mod,base})$ between 0.7 - 0.8. In this range of values, the frequency of the SMDM is in quasi resonance with the frequency of the structure and the frequency of the person walking, then a double resonance occurs. However, a decrease in the footbridge response was noted, possibly, due to the amplification of the oscillations of the SMDM and consequently the reduction of the vibration of the structure. This result highlights the overestimation of the amplitudes of vibrations of the footbridge when using the simplified methodology adopted by the most codes and guidelines that does not consider the interaction effect especially in the resonance.

The walking frequency is the most sensitivity parameter in the response of the structure and its responsible by high amplitudes of vibrations in the footbridge in the resonance. Moreover, an interesting aspect noted in Figure 2 is the peak of the RMS value that has a slight deviate to the left in relation to the resonance between the excitation frequency and the frequency of the structure $(f_p / f_{p,base} = 0.985)$. This result occurs due to a slight decrease of the natural frequency of the footbridge in the presence of the pedestrian. This conclusion was also pointed out by Shahabpoor *et al*. [3], Brownjohn [4] and Zivanovic *et al*. [14] that reported changes in the modal properties of the coupled pedestrian-structure system, mainly, the modal damping.

5.2 The influence of the parameters of the structure

To assess the favorable contribution of the biodynamic model in the reduction of the amplitudes of vibrations of footbridges in comparison to those obtained with the moving force model, the modal properties of the structure, such as natural frequency (f_s), modal mass (m_s) and damping ratio (ξ_s), are variables while the parameters of the SMDM are fixed as the base values mentioned in 5.1.

The results of the analyses are shown in Figure 3 for the ML model (pink) and SMDM (cyan) as RMS acceleration at midspan normalized with respect to the corresponding value obtained for the baseline parameters in the moving force model. The natural frequency, modal mass and damping ratio of the structure are also normalized with respect to the respective values: 2.0 Hz, 2500.0 kg, 0.5%.



(a) Influence of the variation of the natural frequency and the modal mass

(b) Influence of the variation of the natural frequency and the damping ratio

Figure 3. Response of the footbridge considering the ML model (pink) and SMDM (cyan)

As expected, the greatest amplitudes of vibrations are observed near the resonance for the lowest values of modal mass and for the lowest values of damping ratio of the footbridge. Moreover, when the natural frequency of the structure increases and becomes equal to one of the multiple harmonics of the human walking the response is amplified. Despite being lower than that of the first harmonic this amplification should not be neglected.

In relation to the interaction effect, a great contribution of the SMDM is verified when the structure is resonant with the walking frequency and its multiple harmonics. However, this reduction decreases with an increase of the modal mass of the footbridge. This result indicates a proportionality relationship between the ratio of the pedestrian mass and the footbridge mass in HSI effect.

With respect to the structure's damping ratio, the HSI favorable effect increases with the decrease of this parameter. This occurs because the amplitude of vibration of the footbridge becomes larger and, consequently, the motion of the base of the pedestrian that induces the SMDM increases, making it more active. This result draws attention to the importance of the use of the interaction model in low-damping footbridges, such as those made of steel or composite materials.

6 Conclusions

In this paper, two parametric studies were performed to consider the effect of the HSI in the attenuation of the response of footbridge considering the influence of the modal properties of the SMDM and the influence of the modal properties of the structure. The results obtained showed damping ratio and walking speed of the SMDM as the less sensitive parameters that influences the amplitudes of vibration of the structure. On the other hand, the frequency of the biodynamic model is an important parameter in the HSI effect, however, a large deviation of values for this variable is found in literature. In relation to the modal mass of the pedestrian, a beneficial contribution of this variable was observed indicating the potential effect of crowds to attenuate the response of the structure. In relation to the structure, it was noted a reduction in the HSI effect with the increase of the modal mass and damping ratio of the structure. Moreover, it is near resonance that the effect of the SMDM is more evident.

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