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Physical Non-Linear Analysis Using the Finite Element Method One-Dimensional by Iterative Potra-Pták Method

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Abstract. In recent years, structural analysis has has gained a notoriety due to the technological advance. The structures that require a non-linear analysis, require the execution of a large number of calculations, with large and sparse matrices, requiring the use of iterative methods to solve the problems. This context, there is a need to search for more efficient solution methods or those that are adapted to the needs of advances in Civil Engineering analysis. Within the process of structural analysis, different analyzes have to be done in order to achieve structural security. Physical non-linear analysis considers a non-linear constitutive relationship, which when added to the analysis of the structure increases the use of the resistant capacity of the materials, in addition to making it more realistic. Currently, frameworks are used in many practical engineering applications from simple to more complex structures. Increasing the resistant capacity of these new materials can produce more economical structures. Structures that have plastic behavior only after the yelding was modeled using a parameter for hardening module. The current work aim is compare the solution methods Newton-Raphson, Modified Newton-Raphson and Potra-Pták. To analyses the performance of the methods, frameworks problems with physical non-linearity are analyzed by algorithms using Finite Element Methods developed in Fortran90. The both Newton-Raphson methods are largely used in non-linear analysis and have quadratics convergence and the Potra-Pták is a new method that has cubic convergence. According with the result, the Potra-Pták method become an advantageous comparing to others iterative methods.

Keywords: Nonlinear Physical Analysis, Iterative Methods, Steel Trusses, hardening module.

1 Introduction

Many structures have a linear behavior in the initial state. However, this only can be observed when the material has a linear elastic response with small displacements. When the load imposed on the structure exceeds the value of the load of yielding the deformation becomes an elastic and a plastic part as long as the structure does not have brittle rupture. Latter the deformation become permanent in the material defining the plastic regime. The sources of non-linearity are due to the non-linear behavior of the material, to the geometric non-linearity or to a combined effect of these (Pintea [1]). The accurate analysis of plane trusses requires accurate constitutive relationships, which account for several failure modes of member such as buckling, yielding, inelastic postbuckling, unloading, and reloading (Thai and Kim[2]).

Santos [3] analyzed plane trusses, which had also been analyzed in the work of Rodrigues[4], both observed that the results obtained by linear and non-linear geometric behavior showed negligible differences. On the other hand, when physical non-linearity was considered, the displacements had a significant increase in relation to linear analysis, demonstrating the importance of considering such a source of non-linearity.

It is necessary to use iterative methods to solve the nonlinear systems that occur during the structural analysis, due to the non-linear behavior of the material. In the case of analysis of these structures, the iterative method of Newton-Raphson is widely used (Yang and Kuo [5]). In recent years, with the development of efficient and fast computers, the investigation of non-linear problems and numerical methods for their resolution has increased dramatically. According to Souza et al. [6], before the 1980s order of convergence higher than

Newton-Raphson method the iterative method require the computation of higher order derivatives. It is known that greater the computational cost lower its applicability. There are methods that have a cubic convergence rate, which are better than Newton-Raphson method in this aspect, such as methods belonging to the Chebyshev-Halley class (Candela and Marquina [7]) and the Potra-Pták method (Potra and Pták [8]).

In this paper, we present algorithm for the incremental and iterative procedures based on Newton-Raphson and Potra-Ptak methods. The analyzes were performed in trusses problems with physical non linearity. The Finite Element formulation is used. The comparison between the results obtained by the proposed program and the results of the literature is done to show the ability of the proposed program to capture the inelastic nonlinear responses of plane trusses structures under static loads.

2 Theoretical Basis for the Physical Non-Linear Analysis

2.1 Constituve Modeling

For the physical non-linearity analysis of steel trusses structures subject to static actions it is necessary the development of mathematical equations that simulate the structural behavior of the steel and the creation of a suitable computational algorithm that stores all the previous history of the relation tension *versus* deformation of the structural elements. The basic concepts of elastoplasticity is essential to understand the behavior of post critical material rigidity reserves.

We introduce the notion of elastic-plastic behavior to a element of truss. Consider a simple experiment in which a bar is subjected to a force in the direction of its length. The force per unit cross-section of the one-dimensional element corresponds to the tension, σ , while the deformation, ε , corresponds to the change in length per unit length of the bar. A Figure 1(a) show the relationship between tension and deformation linear for an elastic bar. This is Hooke's law, $\sigma = E\varepsilon$, where E is the elastic modulus.

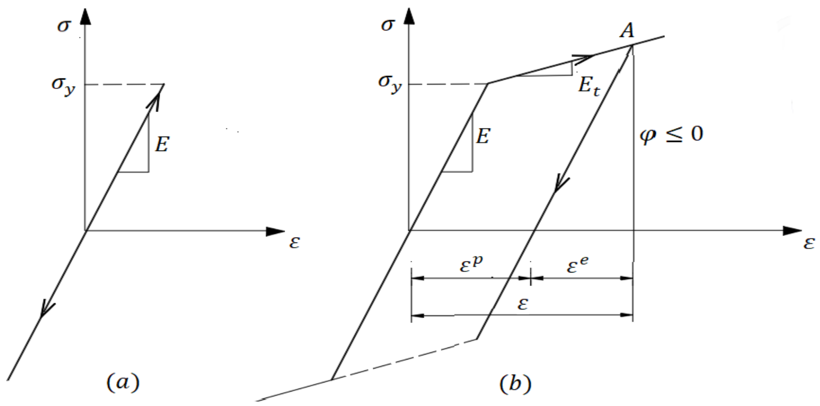

Figure 1. (a) relationship between tension and deformation linear (b) relationship between tension and deformation nonlinear

2.2 Incremental Elastic-plastic Analysis

When analyzing the Figure 2 within an incremental process, we have that in some after the initial yielding, the increase in tension dσ is accompanied by aincrease of deformation $d\varepsilon$.

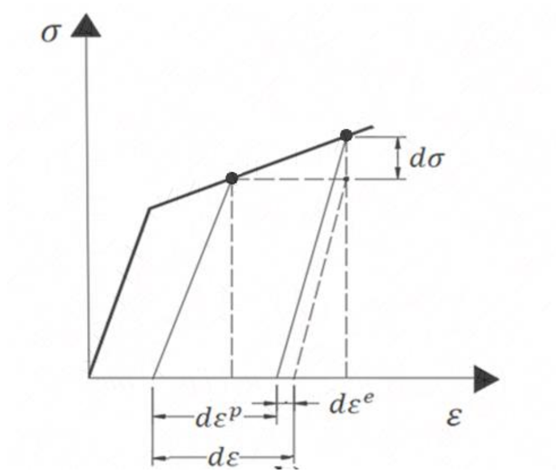

Figure 2. Incremental elastoplastic model

Adapting to equation (1) for a physical non linear analysis, it is:

$$d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p} \tag{1}$$

If it $\varphi \leq 0$ the element is in elastic regime and the Hooke law's is obeyed, and the addition of stress is given by:

$$d\varepsilon^{e} = \frac{1}{E} d\sigma \tag{2}$$

In this case, the elastic deformation is obtained by equation (2) and plastic deformation $d\varepsilon^{p} = 0$.

If the system is loaded with a load value above the yield load, it will deform according to the tangent modulus E_t. In this case, the increase of tension is accompanied with an increase of the elastic deformation and of the plastic deformation, given by the equation (3).

$$d\varepsilon^{p} = d\varepsilon - \frac{1}{E} d\sigma \tag{3}$$

The hardening parameter k is defined by:

$$k = \frac{d\sigma}{d\varepsilon^{p}} \tag{4}$$

and equation (5) can be rewritten as follows:

$$d\varepsilon = \left(\frac{1}{E} + \frac{1}{k}\right) d\sigma = \left(\frac{K+E}{Ek}\right) d\sigma \tag{5}$$

The element is in plastic regime and the addition of soils is given by:

$$d\sigma = E_t d\varepsilon = \left(\frac{Ek}{E+k}\right) d\varepsilon \tag{6}$$

For ideally plastic materials $E_t = 0$ and $k = 0$.

3 Incremental-Iterative of the Potra-Pták Method

3.1 Geometric interpretation of the Potra-Pták method

According to Soleymani et al. [9], the Potra-Pták iterative method requires two steps to solve a non-linear problem, keeping the stiffness matrix constant within each iteration. For non-linear structural analysis the method requires two function evaluations.

$$y_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{7}$$

$$x_{n+1} = x_n - \frac{f(x_n) + f(y_{n+1})}{f'(x_n)} \quad (8)$$

The geometric interpretation of the Potra-Pták method can be made based on the geometric interpretation of the standard Newton-Raphson method, x^{NR}, and Newton-Raphson modified, x^{NRM}. In the first step is equivalent to say that is the result obtained using Newton's standard Raphson method.

$$y_n = x_{n+1}^{NR} = x_n - \frac{f(x_{n-1})}{f'(x_{n-1})} \quad (9)$$

As in the Potra-Pták process the Jacobian matrix, is constant, it is equivalent to say that the second step is equal to an iteration using the modified Newton-Raphson. Soon,

$$x_n = x_{n+2}^{NRM} = x_{n+1}^{NR} - \frac{f(x_{n+1}^{NR})}{f'(x_n)} = \left(x_n - \frac{f(x_n)}{f'(x_n)}\right) - \frac{f(x_{n+1}^{NR})}{f'(x_n)} \quad (10)$$

Simplifying the equation (10), we have

$$x_{n+2}^{NRM} = x_{n+1}^{NR} - \frac{f(x_n) + f(x_{n+1}^{NR})}{f'(x_n)} \quad (11)$$

The solution of equation (11) equals the result of an iteration of the Potra-Pták method.

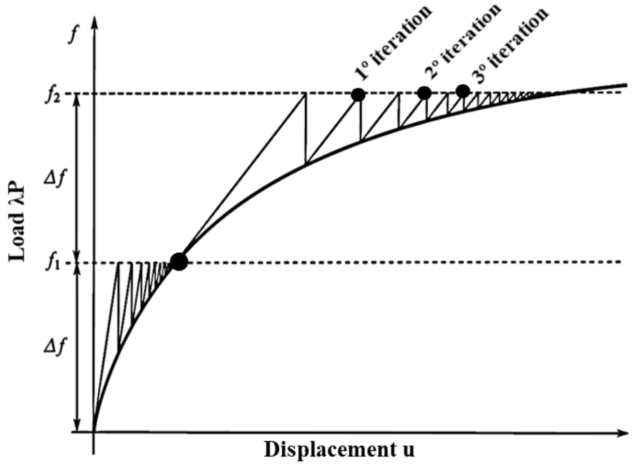

Figure 3. Geometric representation of Potra-Pták iterative method

The Figure 3 shows that a iteration of Potra-Pták method resembles two iterations of the modified Newton-Raphson method, with an update of $f'(x_n)$ (or updating the stiffness matrix in the structural system) every two iterations.

3.1 Physical non-linear structural static problem solution

The nonlinear static solver consists of obtaining the equilibrium between internal and external forces for each load increment, as follows (Bathe [10]):

$$g = \lambda F_r - F_i = 0 \quad (12)$$

in which F_i and F_r, are the global internal and external force vectors, respectively, λ is the load factor and g is the unbalanced load vector.

Calculate the correction vector of the displacements of the first step, Δy_k:

$$\Delta y_k = K_{k-1}^{-1} g\left(u_{k-1} + \Delta_{k-1}\right) \quad (13)$$

From this result, a new vector of unbalanced loads is calculated

$$g_k = g(u_{k-1} + \Delta y_k) \quad (14)$$

and a new displacement correction vector, Δu_k, maintaining the constant stiffness matrix

$$\Delta u_k = K_{k-1}^{-1} g\left(u_{k-1} + \Delta y_k\right) \tag{15}$$

determining the displacement vector of each iteration

$$u_k = u_{k-1} + \Delta u_k \tag{16}$$

The incremental-iterative process determines the equilibrium configuration of the structural system. At the end of each iteration, the displacements must be within a certain tolerance of the actual displacement solution. Thus, a convergence criterion is based on the convergence of the displacements.

$$\left\| \Delta u_k^i \right\| \leq \xi \left\| {}^{t+\Delta t}u_{k-1}^i \right\| \tag{17}$$

4 Numerical Analysis

4.1 Elastoplastic Analysis of Hyperstatic Plane Truss

Consider the truss shown in Figure 4, where the bars $\overline{14}$, $\overline{24}$ and $\overline{34}$ have the same elastic module $E = 20500\, kN/cm^2$, the same yelding strain $\sigma_y = 34.5\, kN/cm^2$ and the same cross-sectional area $A = 12.51 cm^2$, $L = 200 cm$, $P = 1050 kN$.

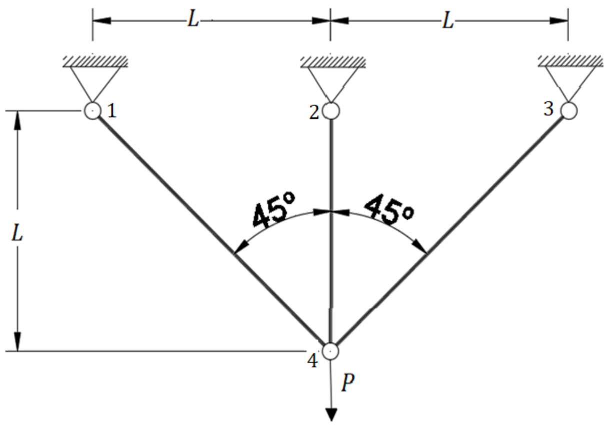

Figure 4. Truss in elastoplastic regime with

From the Mechanics of Solids, applying, for example, the method of displacement, follows:

$$\delta_4 = \frac{L}{EA} P \tag{18}$$

$$P_2 = \frac{P_e}{\left(2 - \sqrt{2}\right)} \tag{19}$$

$$P_r = \left(1 + \sqrt{2}\right) P_e \tag{20}$$

in wich δ_4 is the displacement of point 4; P_2 is the load that cause the yielding of the truss and P_r is the collapse load of the system.

From equation (19) and (20), the theoretical values for P_2 and P_r can be obtained: $P_2 = 736.78 kN$ and $\mathrm{P_r} = 1041.96\mathrm{kN}$.

The Figures 5, 6 and 7 show the curve load *versus* displacement of the physical non-linear analysis with the implemented program, based on the Isotropic Hardening Parameter, by Newton-Raphson Standard method, Newton Raphson modified method and Potra-Pták method, respectively. For the simulations with the incremental iterative technique, we adopted the increase of force equal $\Delta P = -10.5kN$ or $\Delta P = -105kN$ and tolerance equal $\xi = 10^{-5}$.

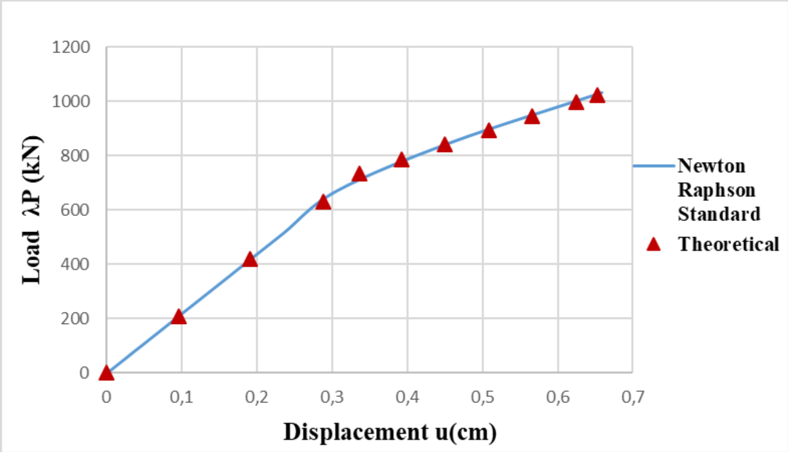

Figure 5. Physical non-linear analysis by Newton-Raphson's standard iterative method

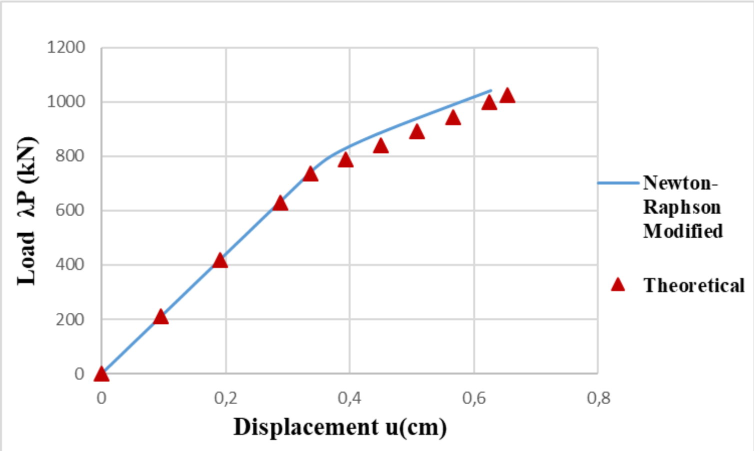

Figure 6. Physical non-linear analysis by Newton-Raphson's standard iterative method

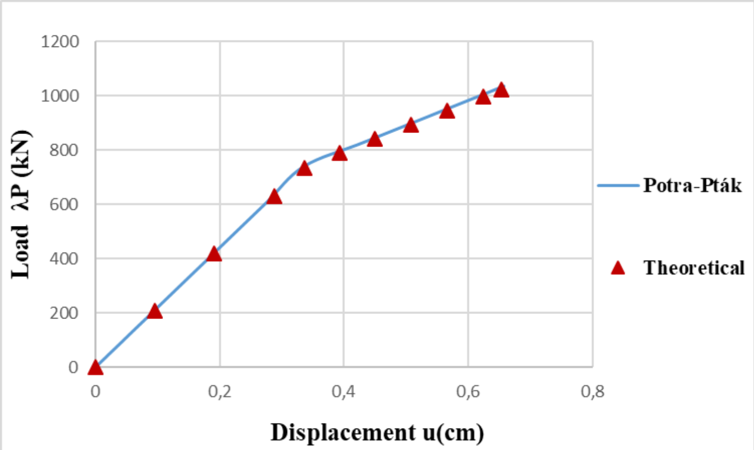

Figure 7. Physical non-linear analysis by Potra-Pták iterative method

It can be seen that the physical non-linear analysis using the three iterative methods reached the yielding load and the burst load very close to the theoretical values. Validating the implementation of the Potra-Pták method for non-linear structures analysis.

The table 1 presents the number of increment and number of iterations of each iterative method.

Table 1. Comparison between iterative methods for nonlinear analysis

Iterative Method	$\Delta P = -10.5kN$		$\Delta P = -105kN$	
	Number of increments	Accumulated iterations	Number of increments	Accumulated iterations
Newton-Raphson Standart	100	198	34	66
Newton-Raphson Modified	100	226	34	74
Potra-Pták	100	101	34	35

Two parameters of increment of load, one small and one greater, were used in order to verify the effectiveness of the method of Potra-Pták before the method of Newton-Raphson. As expected, the modified method presented a number of iterations higher than the standard method. The small number of iterations required for the convergence of the Potra-Ptah method, due to its cubic convergence, is observed. For this example, it was found that the Potra-Ptah method was quite efficient.

5 Conclusion

The numerical results show that a smaller number of iterations required by the convergence to a given tolerance compensates the greater computational effort of the iterative method of Potra-Pták than the Newton-Raphson method, due to two evaluations of the function. The efficiency of the implemented method is verified by the trend of having half iterations than the standard Newton Raphson method or modified Newton Raphson method.

The Isotropic Hardening Parameter model implemented was able to obtain good results, reaching the theoretical and numerical values of yelding load and collapse load. Therefore, for simple models it is possible to avoid models that have a high computational cost, such as the plastic label method and the plastic zone method.

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