XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Computational routine for the generation of artificial earthquakes through the Kanai-Tajimi spectrum and dynamic analysis using the Newmark method

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Abstract. The dynamic analysis of structures becomes each day more important, due to the constant cases of earthquakes around the world, which compromises the stability of buildings. In this context, the present study aims to develop and validate a tool that allows analyzing the behavior of structures facing different earthquakes. This tool was developed in the form of a computational routine in the MATLAB® software, through the implementation of a code based on the generation of an artificial earthquake to obtain an artificial seismic accelerogram, by applying the Kanai-Tajimi filter [6] and [7], and a routine with the Newmark method developed to solve the dynamic motion equation. Then, the code was validated through results available in the literature, being them a ten-story shear building by Mohebbi et al. [9] and a six-story shear building by Miguel et al. [10]. The results obtained with the program allow dynamic analysis of any building, important for the dimensioning of energy-dissipation devices. From the literature results, it was possible to validate and prove the employability of the computational routine developed, since the results obtained in the present code converged to the literature reference values, with some variations due to the randomness of the earthquake.

Keywords: computational routine, dynamic analysis, kanai-tajimi spectrum, newmark method.

1 Introduction

The dynamic control of the structures has become more expressive due to the increase of world seismic records, which compromises the stability of buildings if they are not foreseen and duly considered in the design. Most of these earthquakes have a low magnitude, often not even being felt by humans, or they occur with their epicenter far from more populous locations with simple houses. Then, from that point on, many researchers turned their eyes to the development of systems to stabilize buildings and absorb the energy generated by earthquakes.

The beginning of these studies is dated from the 90's, when Frahm [1] proposed a device for damping vibrations of bodies. This study started with the aim of developing an energy absorption system for bodies subjected to periodic vibratory forces generated by the propelling system of ships. Reports show that many tremors do not have a very high energy potential, but when their epicenter is in the center of a big city, or a city located in mountainous terrain, it can cause a tragedy, because of the constructions with simplified design, like houses.

So, that it is almost impossible to accurately predict the time and energy of an earthquake, research continues to advance in the analysis of dynamic systems and ways to develop structural designs resistant to shocks and vibrations. Cardoso et al. [2] demonstrated in their researches the first masonry structures with anti-seismic provision, using a braced timber structure called "gaiola pombalina" above the first story, that was developed after the 1755's Lisbon Earthquake, being called the "Pombalino" buildings after reconstruction. The larger number of these buildings is located in Lisbon Downtown, with the anti-seismic and anti-fire provisions.

An earthquake produces energy to accelerate the ground so that stimulates the base of a structure in forced displacement, this causes deformation in the structures, which must be considered in the design, as large deformations can cause rupture and small deformations can break certain materials. Therefore, since the 90's the development of earthquake-resistant and energy absorption system has advanced a lot, from that, Brazilian and international standards were developed for the design of structures with provision for earthquakes. These standards use simplification for the calculations of the energy of the earthquake and the dynamic analysis, some of them are used only for minimum requirements for simple buildings. For example, the Brazilian standard NBR 15421/2006 [3] is only used for residential and commercial buildings, with usual materials and conventional construction methods. The Eurocode 8 [4] presents regulations for both usual structures and special designs, such as silos, bridges, reservoirs. Finally, the Uniform Building Code [5] does not initially present structure restrictions, but it does set restrictions on the types of structures under analysis for better results. In short, some of the most important standards consider the earthquake as an equivalent static model, even though sometimes recommended a dynamic analysis, which is a model a little distant from reality, even for cases where there is no risk of earthquakes.

Therefore, it is necessary for a project of earthquake-resistant structures, the use of probabilistic methods that consider the randomness of the earthquake in space and time, being them models where the excitation from the earthquake and the dynamic response of the structure are defined as stochastic processes. In this concept, many mathematical models and methods for generating earthquakes have been developed, including one of the most used in the scientific community, also used in the references of this work, which is the method of generating a seismic signal in a frequency domain from the Kanai-Tajimi, [6] and [7], power spectral density function.

Within this context, this article presents the results of the validation of this computational routine developed in MATLAB® software for the generation of a seismic signal through Kanai-Tajimi, [6] and [7], and the analysis of the dynamic response of two buildings subject to this excitation, being possible to consider any degrees of freedom in the analysis. The earthquake is generated using the method mentioned above, and then this signal is transformed into a ground acceleration from Shinozuka and Jan [8]. Then, this excitation is applied to the base of a ten-story shear building and a six-story shear building, to analyze the dynamic response of them using the Newmark method. The validation of this routine is done by comparing the research results with the studies of Mohebbi et al. [9] and Miguel et al. [10]. This comparison makes possible to conclude the research and confirm that it is in accordance with the methods used. Also, it can be a basis for further studies of more advanced probabilistic methods, and even, as desired by the authors, the implementation of vibration dampers and their optimization.

2 Methodology

Most of the researches developed in this area use some geographical location as a reference, using its seismic accelerations, soil type and its characteristics. This study does not use a specific region of the world for the analysis, using only the corresponding peak ground acceleration (PGA) values of the articles used for validation, but the computational routine can be used for any earthquake for the analysis, regardless of the region where it is located.

The study is based on the denomination of the earthquake as an experiment, which occurs in function of time and space. This experiment will then be repeated over and over in an identical manner, obtaining different results, called sample functions by Roberts and Spanos [11]. Agglomerating all these results, different if compared to each other, but similar in an overview way, an infinite set of sample functions appears, called by Clough and Penzien [12] a stochastic process, or random process by Bendat and Piersol [13].

Since the earthquake is represented through a stochastic process, its properties and results can be analyzed at any time, from the evaluation of the average of the results of the sample functions. In the same way that the results can be obtained through the sample functions, it is possible to calculate the correlation between them, or the autocorrelation function. According to Bendat and Piersol [13], if the sample functions and the autocorrelation function do not vary over time, the average is constant, then the stochastic process is called a stationary stochastic process. This autocorrelation function is used to solve the problem of using classical analysis of Fourier theory to solve the stationary stochastic process. According to Newland [14], this difficulty arises because the sample functions are aperiodic and can be solved because the autocorrelation function can provide information about the stochastic process indirectly, as the function is maximum when the reference sample functions are in phase, and minimum when they are in antiphase. With that, it is possible to use the Fourier transform from the autocorrelation function to generate the power spectral density called the Wiener-Khintchine theorem [15] and [16].

To solve the dynamic equation, a numerical approach must be taken, since the solution of this equation is outside the analytical domain. One of the most well-known numerical integration methods is the Newmark [17] method, an implicit method of integration that uses the results of acceleration in two instants of time. For that, it is necessary to satisfy the equation for discrete time intervals defined by *"Δt"*, performing iterations in the subsequent intervals, admitting variations for displacement, velocity and acceleration within each interval, using two constants parameters defined by Rao [18] as the proportion of acceleration participation in the velocity and displacement equations in the time interval. Starting from the dynamic equation, the whole computational routine uses dynamic analysis as an experimental procedure to be repeated numerous times over time, this is part of the stochastic process approach. The code composition process starts with the use of the methods described below.

2.1 Dynamic system equation

In general, when requested by dynamic loading, a structure responds in displacement. However, it is necessary to identify that this study uses non-deterministic methods, as the structure is subject to a random dynamic loading. The differential equation that governs a system was proposed by Birkhoff [19] and enhanced by Clough and Penzien [12] assuming a generic form that can be used for any degree of freedom, given by eq. (1).

$$\mathbf{M}\ddot{Z}(t) + \mathbf{C}\dot{Z}(t) + \mathbf{K}Z(t) = p(t). \tag{1}$$

Where ***"M"***, ***"C"*** and ***"K"*** are, respectively, the mass, damping and stiffness matrices of the structure. *"Z(t)"* is the displacement vector with respect to the analysis time and the dots over them are the differentiation from the displacement over the time corresponding to the acceleration and speed of the structure, as well as *"p(t)"* is the vector function of the load, the energy that the structure is subjected, as a function of time. The energy vector, in this case, showed by Clough and Penzien [12] and also Miguel et al. [10], can be defined by eq. (2).

$$p(t) = -\,\mathbf{MB}\,\ddot{Y}(t). \tag{2}$$

Since ***"M"***, again, is the mass matrix of the structure, ***"B"*** represents a matrix "*r x s*", where *"r"* is composed of the cosine directors of the angles formed between the base of the structure and the direction of the degrees of freedom and *"s"* the number of movements considered. Finally, *"Ÿ(t)"* is the second differentiation of the displacement over time considered at the base, the ground acceleration, representing the seismic excitation. Only for simplification methods, the structure damping matrix can be given by a relationship with the mass and stiffness matrix, called Rayleigh Damping, proposed by Strutt [20], aka Lord Rayleigh, and given by eq. (3).

$$\mathbf{C} = \mu\mathbf{M} + \lambda\mathbf{K}. \tag{3}$$

Where *"μ"* and *"λ"* are the proportionality coefficients of the mass and damping matrices.

2.2 Kanai-Tajimi filter and seismic excitation

As a first step in the computational routine to solve the dynamic equation, it is necessary to define the acceleration of the soil as a stationary, one-dimensional stochastic process, being filtered from Kanai-Tajimi [6] and [7] by a process of Gaussian white noise, defined by the eq. (4).

$$S(\omega) = S_0\left[\frac{\omega_s^4 + 4\omega_s^2\xi_s^2\omega^2}{(\omega^2 - \omega_s^2)^2 + 4\omega_s^2\xi_s^2\omega^2}\right]. \tag{4}$$

Where *"ω"* and *"ξ"* are the ground frequency and ground damping, respectively, who can be picked from any location previously defined, not being restricted to the place of development of this code and the references. The *"S(ω)"* is the constant portion of the spectral density, given by the eq. (5).

$$S_0 = \frac{0,03\xi_s}{\pi\omega_s(4\xi_s^2 + 1)}. \tag{5}$$

Using the concept defined by Shinozuka and Jan [8] it is possible to solve eq. (4) to generate a signal of acceleration of the ground in the frequency domain defined by the routine developed. Thus, this acceleration is defined in the time domain, obtaining an artificial seismic accelerogram, through the eq. (6).

$$\ddot{Y}(t) = \sqrt{2}\sum_{j=1}^{N_\omega}\sqrt{S(\omega_j)\Delta\omega}\cos(\omega_j t + \phi_j). \tag{6}$$

Since *"N_ω"* is the frequency interval defined, *"$\Delta\omega$"* is the frequency increment and *"ϕ"* is the phase angle, defined at random, with its distribution between 0 and 2π. With the result of eq. (6), the value must be normalized so that it is in accordance with the peak ground acceleration characteristic of the region under study.

3 Results

To start evaluating the results, an artificial earthquake was first developed through the computational routine for generating the ground acceleration in the vertical direction from Kanai-Tajimi, [6] and [7], with the conversion of the signal to the time domain by Shinozuka and Jan [8]. From the artificially generated accelerogram, shown in Fig. 1 as an example of the 20 artificial excitations generated for the simulations, a computational routine was also applied to obtain the natural frequencies of buildings with different numbers of stories.

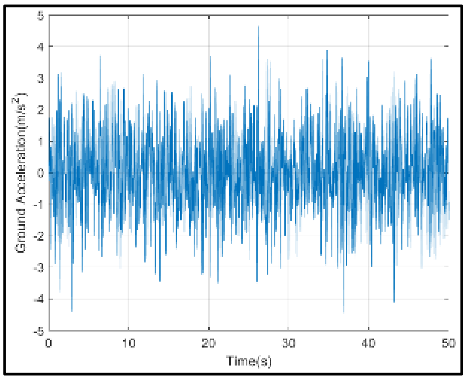

Figure 1. Ground acceleration graphic of an artificial earthquake generated by Kanai-Tajimi [6] and [7]

Finally, a routine with the Newmark method was developed to solve the dynamic motion equation and obtain the results in displacement and acceleration of each floor and the inter-story drift. After completion, the code was validated through results available in the literature, being them a ten-story shear building by Mohebbi et al. [9] having on each floor a concentrated mass of 36.0×10^4 kg, equivalent stiffness of 650.0×10^6 N/m and equivalent damping of 6.2×10^6 Ns/m, and a six-story shear building by Miguel et al. [10] with each story having concentrated mass of 82.0×10^3 kg, equivalent stiffness of 290.0×10^6 N/m and damping ratios for the first and second modes of 0.5%, resulting in $\mu = 1.0699 \times 10^{-1}$ and $\lambda = 1.7695 \times 10^{-4}$ according to the Rayleigh [20] damping equation.

The two buildings used for validation have the same frequency domain for generating the artificial earthquake, with a minimum frequency of 0Hz, a maximum frequency of 25Hz and a frequency range of 0.01Hz. The time duration and the ground characteristics of the buildings are different, being 50s of excitation in a ground with 37.3 rad/s of frequency, 0.3 of ground damping and 0.475g of PGA, being *"g"* the gravitational acceleration, for the Mohebbi et al. [9] building. The time duration for the Miguel et al. [10] building is 20s, in a ground with 20 rad/s frequency, 0.5 of ground damping and 0.20g of PGA. The time step of the two analyze is the same, 0.01s.

3.1 Dynamic response

For the dynamic analysis, the first evaluation of results was taken using the characteristics of a shear building used by Mohebbi et al. [9]. Acceleration values were obtained to be able to visualize the energy of movement of each floor and its direction. Thus, the maximum value of each floor is taken to find out what is the highest acceleration of each floor in module, which is shown in Tab. 1, with the lines corresponding to the stories 1-10.

Table 1. Comparative of the maximum acceleration (m/s²) results with Mohebbi et al. [9]

Mohebbi	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
3.4535	3.4577	3.8670	4.1419	3.3921	3.8403	3.5599	3.7517	3.2889	3.2463	4.0145
5.7160	5.3489	5.7865	6.3454	4.9650	5.9745	5.4585	6.4913	5.1684	5.6245	6.1221
7.0865	6.4389	6.8429	7.5284	6.1262	7.4944	6.7698	8.1112	6.6039	6.6598	7.5640
7.0889	7.3456	7.1830	8.1836	6.7737	8.4001	7.4439	8.6415	7.6757	6.7418	7.7972
7.4988	7.9243	7.1758	7.9602	6.9018	8.7371	7.5752	8.4498	8.1253	6.5274	7.7609
7.1911	7.7507	7.0685	7.3183	6.9900	8.3953	7.4994	8.1548	8.0231	6.5508	8.2711
7.1377	7.3080	7.1083	8.1264	7.4311	7.7601	7.5930	8.8298	7.7940	6.8934	8.3808
7.1373	7.0425	7.3504	8.9002	7.5887	7.8241	7.8636	9.0956	7.7318	7.4589	9.6575
7.3084	7.5836	7.6977	9.5180	7.5341	7.9396	8.1445	10.1685	7.5016	8.0655	10.7899
8.0268	8.3053	7.9528	9.8663	7.9429	8.4301	8.3482	10.8001	7.5043	8.4804	11.4410

As the same as the acceleration results, the displacement shows how much the stories are moving. Highlighting that these values are the maximum of the story, in module. Thus, Tab. 2 shows these values, comparing with the reference, according to the stories 1-10.

Table 2. Comparative of the maximum displacement (m) results with Mohebbi et al. [9]

Mohebbi	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
0.0217	0.0186	0.0219	0.0229	0.0203	0.0241	0.0206	0.0261	0.0219	0.0191	0.0234
0.0430	0.0361	0.0428	0.0451	0.0398	0.0474	0.0402	0.0510	0.0421	0.0375	0.0458
0.0628	0.0521	0.0618	0.0661	0.0580	0.0690	0.0582	0.0738	0.0602	0.0550	0.0667
0.0801	0.0665	0.0788	0.0855	0.0747	0.0879	0.0760	0.0944	0.0773	0.0715	0.0862
0.0950	0.0798	0.0936	0.1029	0.0897	0.1041	0.0921	0.1127	0.0937	0.0867	0.1040
0.1075	0.0922	0.1061	0.1181	0.1031	0.1192	0.1058	0.1288	0.1081	0.1001	0.1201
0.1186	0.1035	0.1164	0.1310	0.1146	0.1325	0.1169	0.1428	0.1200	0.1113	0.1343
0.1288	0.1126	0.1245	0.1410	0.1238	0.1428	0.1252	0.1541	0.1293	0.1200	0.1457
0.1363	0.1191	0.1301	0.1481	0.1302	0.1499	0.1308	0.1622	0.1359	0.1260	0.1538
0.1403	0.1225	0.1330	0.1517	0.1336	0.1535	0.1336	0.1664	0.1392	0.1290	0.1580

The inter-story drift results of the analysis are given in Tab. 3. It demonstrates the variation between floors, which is the height of the pillars, this can be used to select which materials to use, and if these materials can support this deformation, where the results of 10 simulations corresponding to floors 1-10 are presented.

Table 3. Comparative of the maximum inter-story drift results (m) with Mohebbi et al. [9]

Mohebbi	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
0.0217	0.0186	0.0219	0.0229	0.0203	0.0241	0.0206	0.0261	0.0219	0.0191	0.0234
0.0213	0.0176	0.0210	0.0223	0.0196	0.0235	0.0198	0.0250	0.0202	0.0184	0.0226
0.0199	0.0162	0.0193	0.0211	0.0184	0.0218	0.0196	0.0234	0.0194	0.0176	0.0215
0.0176	0.0154	0.0173	0.0196	0.0170	0.0201	0.0185	0.0215	0.0183	0.0166	0.0202
0.0164	0.0145	0.0152	0.0180	0.0153	0.0184	0.0168	0.0195	0.0165	0.0152	0.0187
0.0157	0.0131	0.0134	0.0160	0.0135	0.0163	0.0145	0.0171	0.0144	0.0134	0.0169
0.0138	0.0113	0.0112	0.0137	0.0116	0.0135	0.0126	0.0143	0.0121	0.0112	0.0148
0.0112	0.0093	0.0087	0.0109	0.0093	0.0104	0.0101	0.0115	0.0097	0.0087	0.0120
0.0079	0.0067	0.0063	0.0076	0.0066	0.0071	0.0071	0.0082	0.0069	0.0061	0.0085
0.0041	0.0035	0.0033	0.0039	0.0034	0.0037	0.0037	0.0043	0.0036	0.0031	0.0045

To facilitate the visualization of these results, a graph is generated with the 10 simulations to compose a comparison in the form of dispersion around the Mohebbi et al. [9] results. Figure 2, with the support of Tab. 1, Tab. 2 and Tab. 3, demonstrates the trend of the values, which even with the randomness of the earthquake, follow a pattern similar to the reference. Acceleration results show a maximum variation of 20%, displacement also, and the inter-story drift shown a maximum variation of 15%, considering all of the stories, in the three dimensions.

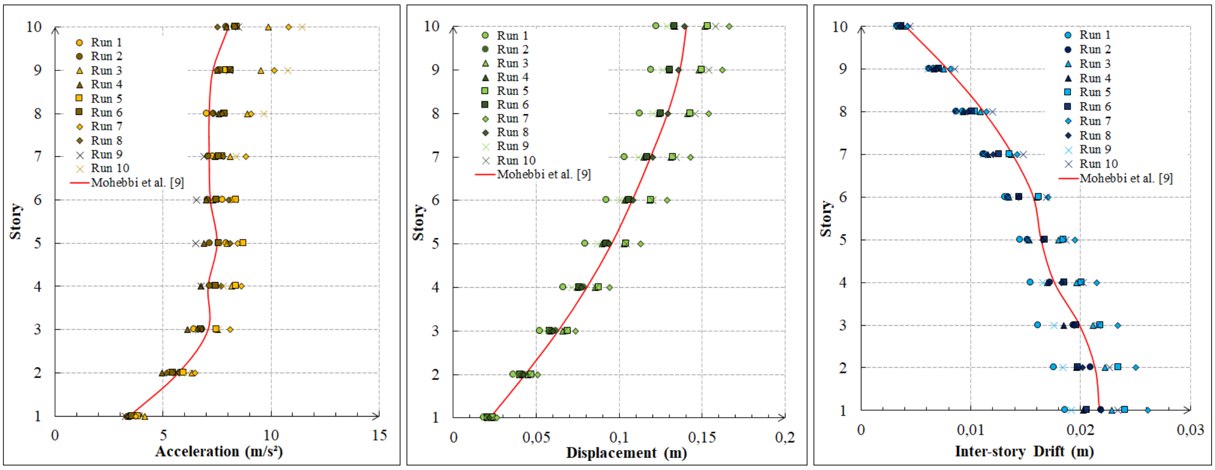

Figure 2. Acceleration, displacement and inter-story drift comparison with Mohebbi et al. [9]

Likewise, due to the importance of drift, another building was analyzed, with a different number of floors and other characteristics, like mass, stiffness and damping, to see the dynamic response and compare with the results of Miguel et al. [10]. These results can be seen in Tab. 4, from the stories 1-6, respectively.

Table 4. Comparative of the maximum inter-story drift (m) results with Miguel et al. [10]

Miguel	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
0.0128	0.0143	0.0150	0.0142	0.0137	0.0113	0.0124	0.0122	0.0136	0.0109	0.0134
0.0121	0.0130	0.0140	0.0133	0.0131	0.0105	0.0114	0.0113	0.0127	0.0105	0.0125
0.0109	0.0112	0.0124	0.0118	0.0118	0.0094	0.0097	0.0097	0.0112	0.0098	0.0111
0.0091	0.0094	0.0103	0.0098	0.0096	0.0079	0.0079	0.0083	0.0091	0.0086	0.0090
0.0066	0.0069	0.0075	0.0072	0.0069	0.0060	0.0057	0.0062	0.0065	0.0064	0.0067
0.0035	0.0037	0.0039	0.0039	0.0038	0.0033	0.0031	0.0033	0.0035	0.0035	0.0037

In the same form of the other results, Fig. 3 shows the graphical representation of Tab. 4, revealing the accuracy of the simulations in comparison with the reference chosen to validate the routine.

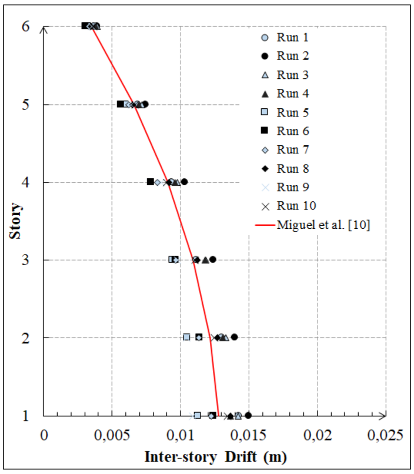

Figure 3. Inter-story drift graphic comparison from the results of the routine with Miguel et al. [10]

With this comparison between the results of the inter-story drift of the routine and Miguel et al. [10], the maximum variation of the results, considering all stories, was 14%. This reinforces previous comparisons and confirms the effectiveness of the computational routine created, making it possible to use it in new researches.

4 Conclusions

The results obtained with the code allow the dynamic analysis of any building, anywhere in the world, using the characteristics of the ground under analysis, providing displacement, velocity and acceleration data resulting on the stories facing a random earthquake with any time duration and frequency domain, in addition to the inter-story drift, which is very important for the dimensioning of energy-dissipation devices and the selection of structural materials. From the results observed in the literature, it was possible to validate and prove the employability of the computational routine developed, since the results of the obtained in the present code converged to the values of the literature chosen for the validation, presenting an average variation, considering all the simulations results of the two buildings, of around 10% on each dimension due to the randomness of the earthquake. Thus, it is concluded that the developed code is applicable to the problem of dynamic analysis of structures using the Newmark method, besides providing a way to generate artificial earthquakes for other analyzes with different methods. Also, it is possible the later use of the code in routines of damped systems and optimization problems of devices, using algorithms to establish the best positioning and optimum damping forces.

Acknowledgments. We thank the Postgraduate Program in Mechanical Engineering (PROMEC) and Federal University of Rio Grande do Sul (UFRGS) for providing all the institutional support and equipment needed for the research. We also thank CAPES and CNPq, for the financial support to both the institution and the student.

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