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Analysis of modal frequencies of beams by a numerical and analytical approach

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Abstract. While static loads are more predictable and commonly applied in a project, dynamic ones are more complex due to its irregularity and its dependency on time. But the importance of knowing how the structures are affected and behave when dynamic loads are applied is essential to a building. Into the dynamic analysis, the natural frequency is one of the modal parameters vastly used to describe the comportment and the problems that the dynamic loads cause, such as resonance that induces to higher deflections. Thus, this paper will perform modal analysis of a steel beam, undamaged and damaged, through two different calculus approaches: analytical and numerical. The analytical approach data used in the modal analysis was calculated based on three generical equations; for the numerical study, ABAQUS software based in the finite element method was used. The numerical model was created in 1D, 2D, and 3D for the undamaged beam to compare the results with the parametric ones, and in 2D for the damaged beam. The natural frequencies obtained by all method was satisfactory, and this outcome might be useful for more complex research as composite structures composed by a steel beam and a concrete slab.

Keywords: dynamic behavior, numerical modeling, natural frequency.

1 Introduction

The loads that can affect the structure must be well-known, considering that any impact can interfere with the integrity and security of a building. The dynamic loads are more complex because, differently from the static loads, they depend on time. The behavior of the structure depending on time can be comprehended by the modal parameters, which are tools responsible for determinate the dynamic characteristic of the structure, known as natural frequencies, mode shapes, and damping.

The natural frequencies describe the vibration of the structure when there is no force applied. According to Mello [1], nowadays, the new architecture and the evolution of the construction are working with lighter materials together with bigger spans. Because of this modernity in the structure, the natural frequencies are lower and closer to human activities. When this frequency of the structure is the same that the dynamic load applies, causes resonance, which provokes higher deflection or the structure collapse.

Many standards around the world mentioned the consequences that can cause in the structure. According to ISO 2631/1 (1997) [2], the vibration in the structure can not only affect the structure but causes nausea and loss of comfort and productivity for the people. NBR 15575[3] defined that the structure cannot have the consequences of vibration that affects its lifetime or the comfort of the people.

To avoid the consequences of vibration, some standards define rules for the natural frequency of the structure. The Brazilian standard NBR 6118 (2014) [4] does not specify how to calculate the natural frequency but defines that any structure susceptible to dynamic loads must have that frequency 1.2 times higher than the critic frequency established by the standard for each type of construction. The AISC [5] determines the acceleration limit and the range of frequency for each type of construction and gives a simplified equation, in the absence of experimental or numerical approach, to calculate the natural frequency.

The calculation of the natural frequency can be made analytically, numerically, and experimentally. Although the results must be similar, each type of calculation has its peculiarity, and to prove the efficacy of the method, its necessary to calculate in more than one form. A couple of research were made comparing the natural frequency results obtained by experimental and analytical results. Fammy and Sidky [6] worked with a composite floor and achieved a difference, of the fundamental frequency, of 7,29% - 10,045% by the calculus AISC [5] standards and experimental results. Ahmed and Badaruzzaman[7] tested a panel, and the fundamental frequency captured by the experiment was 3,29% lower than the analytical calculus made using the Design Guide on Vibration of Floors [8].

Although small cracks and the deterioration of the structure are common to see over time, the equations by AISC [5] and Design Guide on Vibration of Floors [8] are used for the undamaged structure. Then to calculate analytically, the natural frequency for a damaged beam is used the equations based on Gillich and Praisach [9].

The objective of this comparison is to evaluate the numeric model on different analyses, and the efficacy of the simplifies equation in a simple model, Figure 1, to apply on complex structures lately.

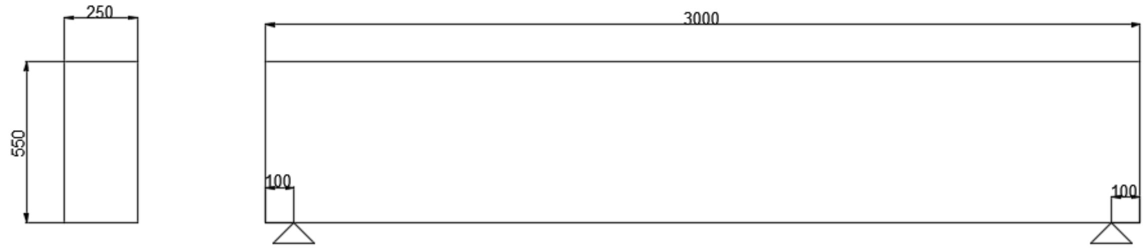

Figure 1. Transversal and longitudinal sections in mm

2 Numeric Modelling

The numeric analyses were carried out by using a finite element software ABAQUS/CAE to determine the natural frequency of the beam undamaged and damaged. For both outcomes were used the material characterized in, Table 1, where E_s is the modulus of elasticity of steel, I_t is the moment of inertia, w is the uniformly distributed weight per unit length, L is the length, ρ is the density of the steel, ν is the poisson's ration.

Table 1 Parameters of the steel beam

Parameters	Value
E_s (MPa)	200000
I_t (m^4)	0.003
ρ (g/cm^3)	7.8
ν	0.3
w (kN/m)	10.518
L (m)	3

2.1 Undamaged Beam

The beam was modeled in three different forms: as 3D solid and for meshing purpose, a C3D8R was used, as 2D shell and mesh of CPS4R and as a 2D wire and mesh of B21. The goal was to study the same structure in one (2D wire), two (2D shell), and three dimensions (3D solid). The natural frequencies obtained, displayed in, were from the first three mode vibrations for the three distinct models, Figure 2.

a)

b)

c)

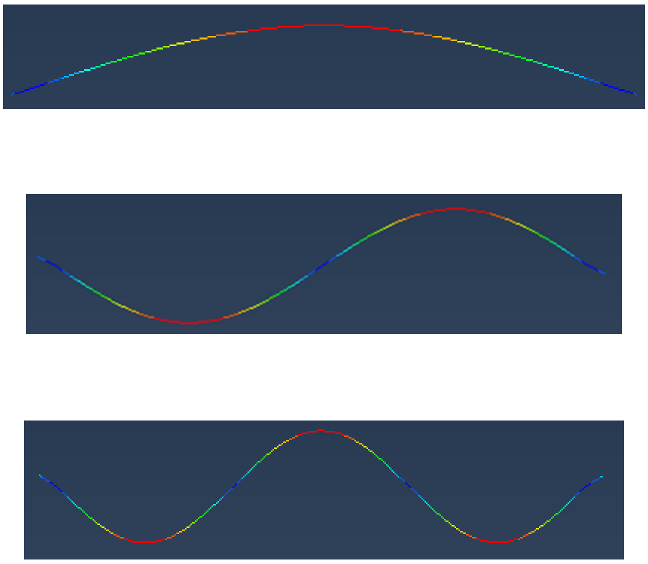

Figure 2. a) First b) Second and c) Third mode of vibration

Table 2. Numeric Values for the Natural Frequency of the Undamaged Beam

	Natural Frequency for each Mode of Vibration (Hz)		
Model	First	Second	Third
1D	150.95	524.85	1005.4
2D	178.94	462.31	1350.5
3D	186.08	494.97	1377.2

2.2 Damaged Beam

The damaged beam was modeled in 2D shell with a crack of 2cm and 4cm in the middle bottom of the beam, as illustrate Figure 3. The natural frequencies obtained for both beams are in Table 3.

Figure 3. Beam with a 4 cm crack

Table 3. Numeric values for the natural frequency of the damage beam

	Natural Frequency for each Mode of Vibration (Hz)		
Model by size of crack	First	Second	Third
2cm crack	174.24	463.65	1337.5
4cm crack	173.67	463.70	1334.5

3 Analytical Calculus

3.1 Undamaged Beam

This calculus will use three general solutions to calculate the natural frequency of the steel beam, shown in Table 4: the equation of AISC [5], the Design guide on the vibration of floors [8], and an equation-based in Euler–Bernoulli beam theory [9]. This method has the advantage of being fast and no need to experiment or make a numeric model. But as any other simplified calculus, it gives a generic result and is more untruthful than an experimental or numerical result.

The AISC uses the equation (1) to determine the natural frequency of a simply supported and uniformed loaded beam. Where f_1 is the fundamental frequency (Hz), those values of the incognitos needed for the equation (1), (2) and (3) below are listed in the Table 1.

$$f_1 = 0{,}18\sqrt{\left(\frac{g(384E_sI_t)}{5w_jL^4}\right)} = 161.72\ Hz\ . \tag{1}$$

The design guide on the vibration of floors utilize another calculus, equation (2). Where Cb is 1,57 for simply supported beams and m is the mass per unit length.

$$f_1 = Cb\sqrt{\frac{E_sI_t}{mL^4}} = 160.99\ HZ\ . \tag{2}$$

The equation based in Euler–Bernoulli beam theory [9] is calculated by equation (3). Where λ_i for i vibration mode.

$$f_i = \frac{\lambda i^2}{2\pi}\sqrt{\frac{E_sI_t}{\rho AL^4}} = 161.081\ Hz\ . \tag{3}$$

Applying the same concept of equation 3 to find the natural frequency for the second and third mode of vibration, the results are displayed in Table 4.

Table 4. Analytical values for the natural frequency of the undamaged beam

	Natural Frequency for each Mode of Vibration (Hz)		
Number of equations	First	Second	Third
Equation 1	161.762	647.048	1456
Equation 2	160.99	643.996	1449
Equation 3	161.081	644.323	1450

3.2 Damaged Beam

For the damaged beam, the equations used to find the natural frequency for the damaged beam (f_{D}) are based in the equation (3) and work by Gillich and Praisach[9] and equation (4), which is the maximum deflection for this type of beam of load and support.

$$V_{máx} = \frac{wL^2}{384EI}(5Ls^2 - 24a^2). \tag{4}$$

Where $V_{máx}$ is the maximum deflection of the beam, Ls is the space between the supports, and a is the distance overhanging. Joining the equation (3) and (4):

$$\frac{EI}{\rho A} = \frac{gL^2}{384V_{máx}}(5Ls^2 - 24a^2),$$

$$f_i = \frac{\lambda i^2}{2\pi}\sqrt{\frac{E_s I_t}{\rho A L^4}} = \frac{\lambda i^2}{2\pi}\sqrt{\frac{g(5Ls^2 - 24a^2)}{384L^2 V_{máx}}}.$$

V_{max} was calculated numerically by the model 2D shell displayed in chapter 2, presented in Table 5.

Table 5. V_{Dmax} and natural frequencies for each damaged beam

		Natural Frequency for each Mode of Vibration (Hz)		
Model by size of crack	V_{Dmax}(mm)	First	Second	Third
2cm	0.010233	174.93	699.72	1574
4cm	0.010360	173.85	695.402	1565

4 Discussion

The results obtained by analytical and numeric calculus for the undamaged beam is presented in Table 6.

Table 6. Numeric values for the natural frequency of the undamaged beam

		Natural Frequency for each Mode of Vibration (Hz)		
	Type	First	Second	Third
Numerical Results	1D	150.95	524.85	1005.4
	2D	178.94	462.31	1350.5
	3D	186.08	494.97	1377.2
Analytical Results	Equation 1	161.762	647.048	1456
	Equation 2	160.99	643.996	1449
	Equation 3	161.081	644.323	1450
	Arithmetic average	161.277	645.122	1451.667

The difference between the numeric results and the arithmetic average of the analytical results, displayed I in Figure 4, has shown that for the first mode of vibration, all models stayed almost in the same rate of percentage of 10% of the difference, but the 1D is most similar to the analytical result. For the second mode, the discrepancy of the frequencies was a minimum of 20%, and that made the second mode not useful for this study. The third mode had the closest value to analytic results by 3D model.

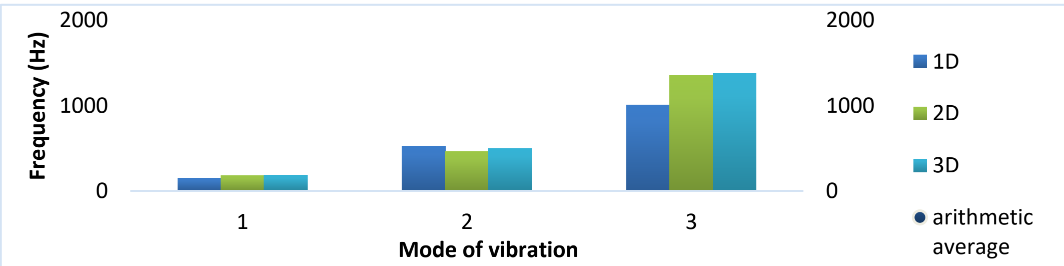

Figure 4. Difference between numeric and analytic result for undamaged beam

Table 7 and Figure 5 presented that the damaged beam had a great result for the first natural frequency, of only 1% and 1,03% of the difference between the numerical and analytical results for the crack of 2cm and 4cm, respectively. But as the outcomes of the second mode for the undamaged beam, the results were not satisfactory also for the cracked beam. The third mode had an average difference of 15%.

Table 7. Numeric Values for the Natural Frequency of the Damaged Beam

		Natural Frequency for each Mode of Vibration (Hz)		
	Type	First	Second	Third
Numerical Results	2cm	174.24	463.65	1337.5
	4cm	173.67	463.70	1334.5
Analytical Results	2cm	174.93	699.72	1574
	4cm	173.85	695.402	1565

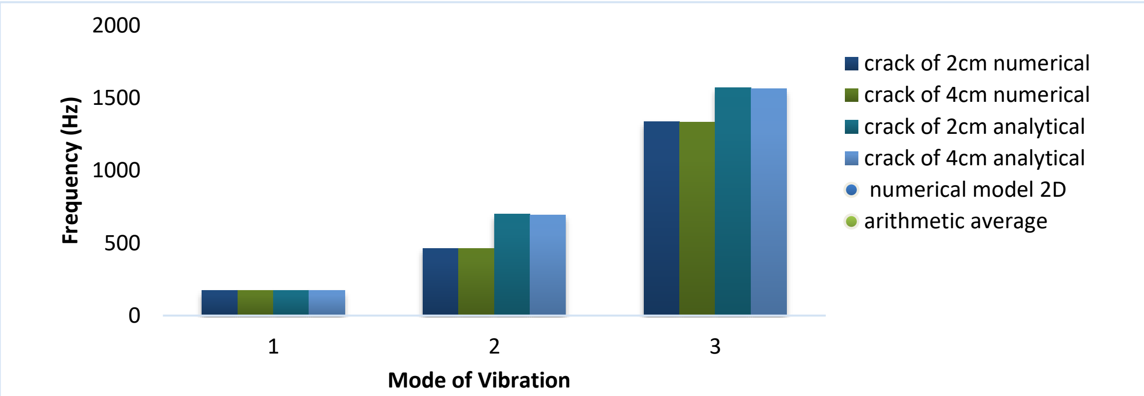

Figure 5. Difference between numeric and analytic result for damaged beam

Comparing the outcomes of the calculus, Figure 4 and Figure 5, with the studies by Fammy and Sidky [7] and Ahmed and Badaruzzaman [8] that had 4-10% of difference between the experimental and analytical frequencies, it is possible to state that the in this discrepancy of average 10%, the model 1D and 2D was promising for first mode of vibration and the 2D and 3D for the third mode for the undamaged beam, the 2D model, overall, was the most close to the analytical result. And for the damage structure the first mode was the only one that got result compatible with the 10%.

As predicted, the frequencies for the cracked beam were lower than the undamaged results, because the structure lost its integrity and rigidity, according to Figure 5 and Table 7. Numeric Values for the Natural Frequency of the Damaged BeamTable 7. The relation between the frequency and rigidity for one degree of freedom is presented in the equation (5). Where k is the rigidity (EI) and m mass.

$$fi = \sqrt{\frac{k}{m}} \tag{5}$$

The new rigidity of the beam for the crack of 2cm and 4cm was calculated by equation (4) and correlated with the loss of fundamental frequency by the 2D model. Knowing that the inertia of the section does not change with damage, the modification occurs in the modulus of elasticity of steel. The Figure 6 show the equation corelating the frequency and the percentage of the modulus of elasticity of the undamaged beam.Table 6

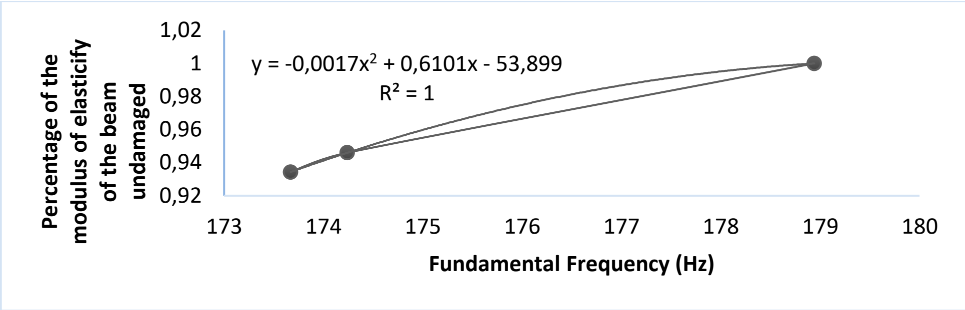

Figure 6. Relation between the modulus of elasticity and natural frequency

5 Conclusion

The dynamic loads constantly appear in the building, and many standards around the world made rules to avoid its consequences, as higher deflection and loss of comfort for people. The regulations are based on the natural frequency of the structure, and this parameter can be determined by experimental, analytical, or numeric calculus. But over time, the structure can lose rigidity, and this causes changes in the natural frequency, so to avoid the damages, the frequencies are calculated for an undamaged and damaged beam.

The natural frequencies were obtained by two different calculus: analytical, based in three general equations for the undamaged beam and in maximum deflection and one equation for the damaged beam, and numerical, made in the software ABAQUS.

The results were satisfactory overall, the numeric models 1D and 2D are promising for the first mode of vibration and the 2D and 3D for the third mode for the undamaged beam. For the damaged structure the natural frequencies decay, as expected, and the first mode was more precise than the others. The result of the damaged beam can be useful to measure the fundamental frequency of an existing structure that is suffering any loss of rigidity because it is possible to measure the deflection on the spot.

For further studies in a complex structure, is it necessary to improve the numeric model.

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Authorship statement.

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