Application of strut-and-tie model in overhanging beams using CAST program

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Abstract. The strut-and-tie model has been useful to solve problems involving D-regions (holes, dapped end beam) in reinforced concrete. On this approach, overhanging beams supported by pile foundation has been found frequently in building on the limit of land although its study does not follow the same intensity. This work aims to compare the performance of some struts and tie topology for a beam under predefined load through CAST based on Brazil Codes. The tool allows users an iterative process verifying each truss element. Nevertheless, there isn't automatic optimum topology generation thus requiring an experience by the designer to choose the best model. It was proposed six varied layouts for a beam with constant geometric and materials properties, applying equal failure load to all models. Reduced the load in order to obtain in stress ratio bellow and near to 1.0 in each model. The results are presented comparing their reinforcement and limitations of layout. Some models reached equal maximum load working different members. From the results, it was possible to define the beam with reasonable performance and their applicability in practical design. It was discussed the potentials of the program like teaching the strut-and-tie model and also automation in node verification.

Keywords: strut-and-tie, topology, CAST, stress ratio, overhanging beams.

1 Introduction

The classical analogy of truss proposed by Rutter and Morsh derived the strut-and-tie models (STM) whose application broadens to a great number of structural elements. This widening became remarkable since papers by Professor Schlaich and Marti settled the analysis of behavior of members in bound states, whether elastic, whether plastic. Silva [1] defines the model as representation of stress field (compression or tensile) internal to structural element due to external forces and supports defined. The model consists to reduce the real structure to a truss where compressed elements are strut and those tensile, ties [2]. The joints are referred as nodes. The concrete around a node is called nodal zones. The mainly applications are in designing of discontinuity regions (D-regions) like overhanging beams, deep beams, joints, pile caps where Bernoulli hypotheses are not valid. Schafer and Schlaich [3] proposed a section of structural elements in Region B (application of Bernoulli hypotheses) and Region D (Discontinuity) which beam theory is not valid. Region-D can be geometric; abrupt section changes, holes in beams or even statical (near to concentrated load and reactions) and its extension is based in St. Venant´s principle [4]. The design of D-regions through empirical methods can lead to inappropriate solution. The STM is based on the lower-bound theorem admits the following: having load acting, the stress field of the system must attend to equilibrium conditions in internal and bound (force field) and also tensile of materials satisfied (material field) in order to attain an lower bound to the capacity of elastic and plastic materials [5]. In this way, the acting load is lower or equal to ultimate load of the structure and the stress field is said stable. However, it should be guaranteed crushing of strut and nodal zones do not occur before the yielding of the tension ties.

1.1 Load path method

The definition of de model is function of acting load and geometric. Silva [1] shows some aspects:

- Type of acting loads;
- Angle between struts and ties;
- Local application of loads and reactions;
- Number of layers of reinforcement;
- The surface of the concrete to the reinforcement;

Once settled the model, it looks for determining the load path inside the elements through the load path method. It consists in understanding the path in which the load applied scrolls the elements and find an reaction or force that equilibrates. Elastic Analysis are also possible via finite-element analysis. The literature highlights this approach conducts to values more effective because the failure load of structure is achieved, hence, the model will attend both service conditions (crack limitation) and ultimate bound state. Some requirements must be followed:

- The load path may be aligned and cannot intercept;
- The centroid of struts and ties and force action lines need to match in each node;
- Contrary loads should follow smallest path;
- Turns in load path shows stress concentration;

Thus, same geometric but with different loads it will reach others modellings. Pantoja [6] suggests that STM requires a touchiness from the designer to perform the most reasonable the structure working.

1.2 Struts

According to Schlaich [3] the concrete force in compression fields or even on nodes depends of a wide extension in multiaxial stress state. This field assumes the setting based on stress distribution;

- Idealized prismatic strut; It has parallel distribution without disturbance, do not generate transverse tensile stress;
- Fan-shaped strut: the stress field is radial and turns, insignificant. Transverse compression is favorable specially if it acts in both transverse directions e.g. in confined region [3].
- Bottle-shaped strut: The distribution of stress is shown by curved lines in Fig. 2 with section narrowing. Near to applied forces it appears biaxial and triaxial compression stresses. The transverse tension occurs now.

Souza [7] remembers bottle-shaped and fan-shaped are placed in D-Regions and those with prismatic shaped found in B-regions. The same author studied deeply many recommendations of resistance in Codes and authors. The NBR 6118:2014 chapter 22 [8] presents the verification to struts and nodal zones:

- f_{cd1}: Effective compressive strength in concrete without tensile strains transverse, just compressive stress in that direction (Fan-shaped or prismatic strut) and node CCC eq. (1).

$$f_{cd1} = 0{,}85 \cdot \alpha_{v2} \cdot f_{cd}. \tag{1}$$

- f_{cd2}: Effective compressive strength in concrete in regions with tensile strains transverse (bottle-shaped strut) and nodes CTT and TTT (finding two or more ties) eq. (2);

$$f_{cd2} = 0{,}60 \cdot \alpha_{v2} \cdot f_{cd} \text{ (ties.} \tag{2}$$

- f_{cd3}: Effective compressive strength in concrete and nodes CCT joining in one tie according to eq. (3);

$$f_{cd3} = 0{,}72 \cdot \alpha_{v2} \cdot f_{cd}, \tag{3}$$

$$\alpha_{v2}\text{: (1 - } f_{ck}\text{/250), } f_{ck} \text{in MPa.} \tag{4}$$

1.3 Nodes

According to Silva [1] node is a portion of concrete idealized in strut-and-tie meet and also acting forces or support conditions. In the structure this nodal zone owns length and width while in an idealized model is just pinned joints which forces meet and occurs quick changes in their directions. Pantoja [6] highlights a need to take in consideration their stresses, anchorage of ties in this region and concrete strength to the right design of nodal zone. Bellow, the classification of nodal zones;

- C-C-C: Node which three compressive forces meet;
- C-C-T; One force is tensile in node;
- C-T-T; two or more members are ties
- T-T-T; All are ties;

1.4 Ties

The design of tie is based on eq (5);

$$A_s = \frac{F_{sd}}{f_{yd}} \tag{5}$$

Where F_{sd} is the acting strength on the tie. Souza [7] says about the importance of anchorage of ties in D-region in strut-and-tie. The reinforcement needs to develop an acting strength in the support in order to not occur loss of anchorage. According to the author, the anchorage is reached through a portion of concrete surround the tie steel. For the purpose of guarantee an effective anchorage and does not occur crushing in nodal zone, should be considered a tensile zone of ties by several layers of reinforcement.

2 CAST Tool

Developed by Daniel Kuchma from University of Illinois at Urbana-Champaign, the software CAST (Computer Aided Strut-and-Tie) make easy the verification and designing the D-Region based on STM. It's recognized by its quick analysis during the graphical process of the model, showing the boundary, nodes, struts, supports, loads and holes. This tool doesn't own an automatic optimization to identify the best model, although does not restrict the user to only one solution, allowing to build a wide variety of trusses [9]. The iterative process beings after drawing the geometric model where truss stability is verified. Considerations used by CAST like uniform distribution of stress through the struts and ties, resulting forces in nodes allows a matrix analysis according to conventional trusses. Even it´s possible to choose elastic or non-linear analysis [2]. The strut design is based in ACI Code Committee 318 or defined by users. CAST offers a minimum automatic width thought a dimensionless parameter called stress ratio consists in a ratio between acting stress and resistant stress. The valuers up to 1,0 means failure whether in strut or/and ties. A stress ratio near to one indicates a desirable design. Another criteria is efficiency factor "v" having a value between 0 and 1. It affects compressive strength tending to let concrete more brittle. Nodal zones are joins from effective width strut and ties. There are two kinds of analysis: simplified (which calculates the level of stress in lay out of tie/strut and node) and detailed (subdivide the nodal region in "n" constant stress triangles) [2]. Detailed analysis results are obtained with subdivision of nodal zone in triangles with more than three sides. This distribution of discontinuity is provided since the stress state in triangles be constant and attended equilibrium on any triangle face. It applied the biaxial failure criteria to validate acting and resistant stress in each triangle. In C-C-C nodes is developed Mohr-Coulomb modified criteria and those C-C-T, C-T-T and T-T-T it's used a linear version of Mohr-Coulomb criteria to calculate the strength of nodes.

3 Materials

It was assumed concrete fck 30 MPa, coverage of 3 cm and steel yielding in 500 MPa. The beam width b_w is 50 cm and simple support simulating piles foundation was settled. The force applied indicates a pile acting in the overhanging beam. In order to compare it was stablished bars ∅ = 12.5 mm. The Figure 1 displays the size beam.

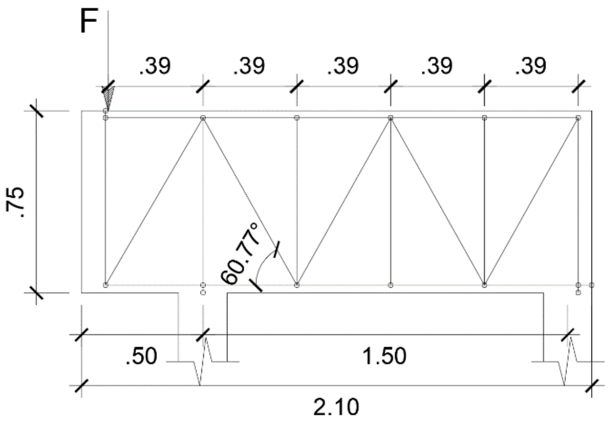

Figure 1. Overhanging beam anlysed

4 Results

After modeling in CAST, all members were verified. The acceptability relies on resistant strength in each element. It was studied six topologies starting with load equal to 800 kN and then decreasing it.

The Topology 1 (Figure 2) shows ties as solid lines and struts as dashed lines. Positive values indicate tensile and negative, compression. In first truss, failure happens on nodes and ties highlighted. Second truss, hotter colors show higher stress ratio (near to 1,0) as well as colder colors low values. In this model, it arrived maximum load of 600 kN and force of 479 kN in strut E7, about 70,6% from the capacity. Tie E2 was more required in truss (89,1% of resistant strength). Through simple analysis, node N09 obtained stress ratio 0.975 (97,5%) but in detailed analysis this node achieved 128,2%. Both Table 1 and Table 2 shows estimated area to each horizontal and vertical tie according to acting force. It's desirable to set the maximum bar to all elements. CAST gives like that on below here it was placed only to one model.

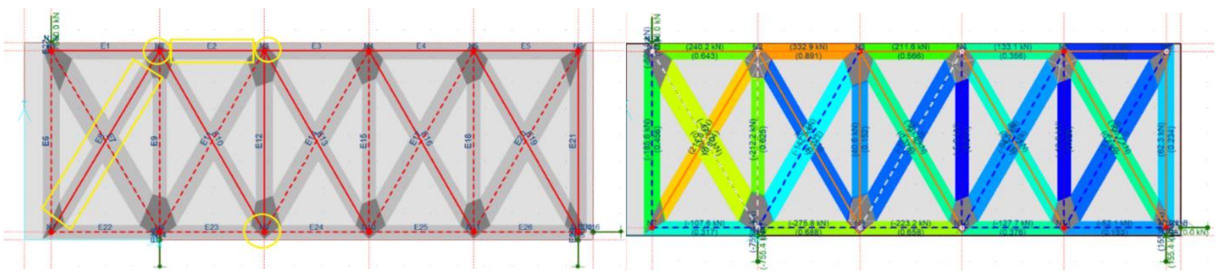

Figure 2. Strut and ties, Requirement in members (model 1)

Table 1. Horizontal ties (model 1)

Member	F_{st}(kN)	f_{yd}(MPa)	$A_{s,nec}$ (cm^2)	Layer	$A_{s,ef}$ (cm^2)	Stress Ratio
E2	332.9	435	7.65	5 (1c) + 2(2c)	7 ∅12.5 mm	0.891
E1	240.2	435	5.52	5 (1c) + 2(2c)	7∅12.5 mm	0.643
E3	211.6	435	4.86	5 (1c) + 2(2c)	7∅12.5 mm	0.566
E4	133.1	435	3.05	5 (1c) + 2(2c)	7∅12.5 mm	0.356
E5	34.9	435	0.80	5 (1c) + 2(2c)	7∅12.5 mm	0.093

Table 2. Vertical ties (model 1)

Member	F_{st}(kN)	f_{yd} (Mpa)	$A_{s,nec}$ (cm^2)	layer	$A_{s,ef}$ (cm^2)	Stress Ratio
E8	214.6	43.5	4.93	3 (1c) + 2(2c)	5∅12.5 mm	0.804
E10	30,5	43.5	0,70	3 (1c) + 2(2c)	5∅12.5 mm	0,114
E12	40,6	43.5	0,93	3 (1c) + 2(2c)	5∅12.5 mm	0,152
E13	101,0	43.5	2,32	3 (1c) + 2(2c)	5∅12.5 mm	0,378
E16	83,5	43.5	1,92	3 (1c) + 2(2c)	5∅12.5 mm	0,313
E19	106,7	43.5	2,45	3 (1c) + 2(2c)	5∅12.5 mm	0,400
E21	62,3	43.5	1,43	3 (1c) + 2(2c)	5∅12.5 mm	0,234

Reaching a maximum load of 250 kN, the failure in model 2 (Figure 3) occurred in strut E7 with 99% capacity and afterward horizontal ties E1 and E2 with stress ratio of 0.905. Vertical ties supported 60,5% of capacity. The more compressed strut had maximum width of 4,3 cm once upper values would surpass beam coverage. In nodes, simple analysis lead to node N1 near to strut E7 as the highest member required (stress ratio 0,99). Refined analysis was possible only in node N5 and N7 with capacity of 81,6% against 25,6 in simple verification. Here it needed 3∅12.5 mm in each horizontal tie. To vertical tie only to bars of 12.5 mm in each element.

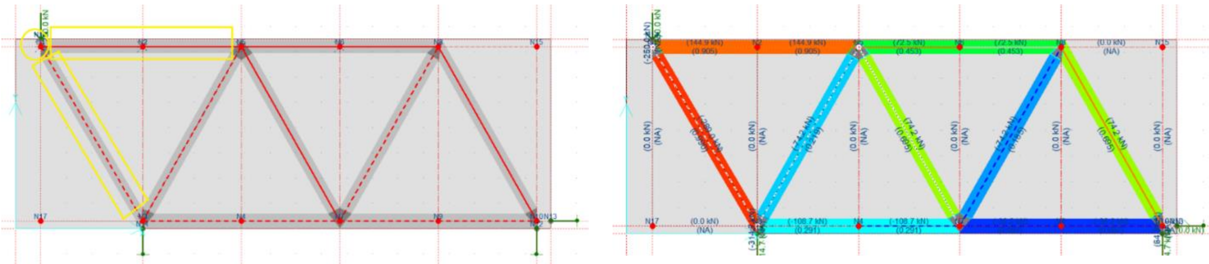

Figure 3. Strut and ties, Requirement in members (model 2)

The third model (Figure 4) applying maximum load of 600 kN has the most required tie (693,5 kN) and stress ratio 1,0 with unstable equilibrium. In struts E17 and E18 were noticed 99,5% of capacity. Horizontal ties E2 and E3 follows in failure with stress ratio of 97,8%. About nodes, N17 through simple verification answered with stress ratio 99,5% near to E17. Under detailed analysis N17 failed beforehand E7 with 127% of capacity.

Triangulation occurred in nodes N2 and N3 where values were close between nodal analyses with stress ratio 0.954 (simplified) and 0.939 in detailed. Node N3 displayed same behavior in both verifications (99,5% and 86,9%). It was required five bars of 12.5 mm to horizontal tie and 13 bars of 12.5 mm being nine on the first layer and 4 on the second layer.

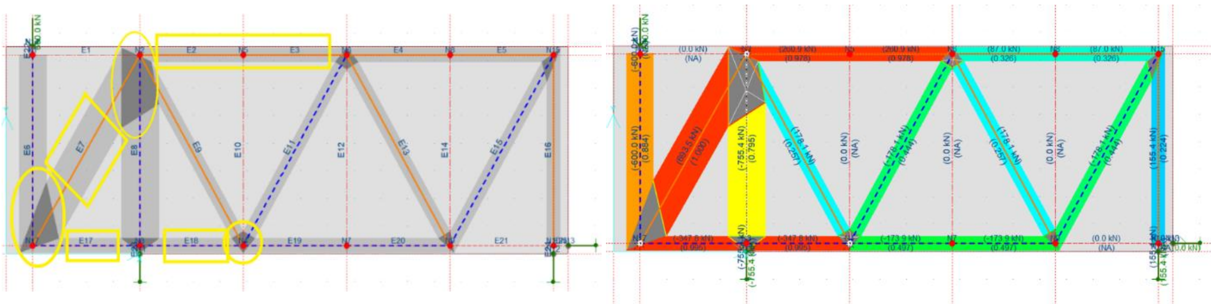

Figure 4. Strut and ties, Requirement in members (model 3)

The fourth topology (Figure 5) with maximum load of 250 kN required 99,0% of strut E7 and tie E1 with 90,5% (144,9 kN) according fig. 10 highlighting the mentioned members. Other elements didn't suffer strong requests like some struts with 11,9% of capacity (E21). It was noticed variations between kinds of analyses in nodes N4, N7, N9. In simple analysis, N4 close to member E9 appears with stress ratio 0,219 while in refined analysis showed 84,5% of capacity. Node N1 was more requested with 99% of capacity. In this model only three bars of 12.5 mm were needed in horizontal ties and two in vertical ties.

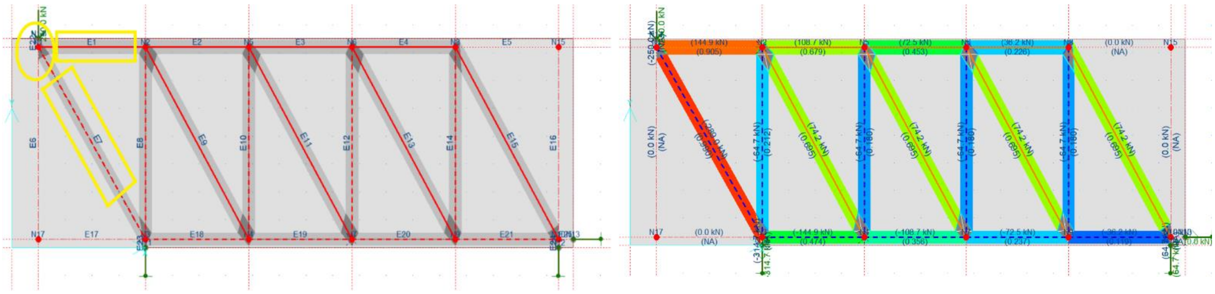

Figure 5. Strut and ties, Requirement in members (model 4)

The fifth model changes smoothly from fourth thus achieved same maximum load of 250 kN. As shown in (Figure 6), strut E7 with ratio of 0.99 and tie E1 with 0.905. Once both models are similar, there weren't big changes in results and horizontal and vertical ties kept the same acting forces. As the strains are similar, number of bars in horizontal tie and vertical tie does not change.

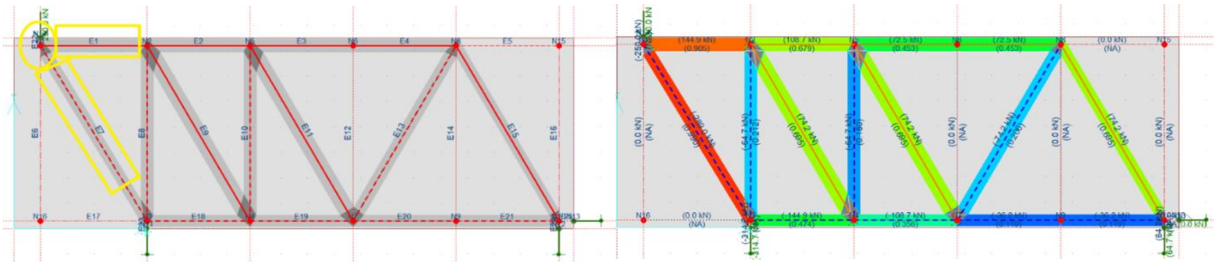

Figure 6. Strut and ties, Requirement in members (model 5)

The last topology (6) also failed with maximum load of 250 kN. In Figure 7 views the most required member, E7 with stress ratio of 0,99 and tie E1 near to 0,905. About nodes, N1 and N3 through detailed analyses answered with stress ratio of 1.121 and 2.154. In simple verification there were discrepancy which N1 was 368,26% and N3 83,9%. Here there isn't change in number of bars being three in horizontal ties and two in vertical ties of 12.5 mm.

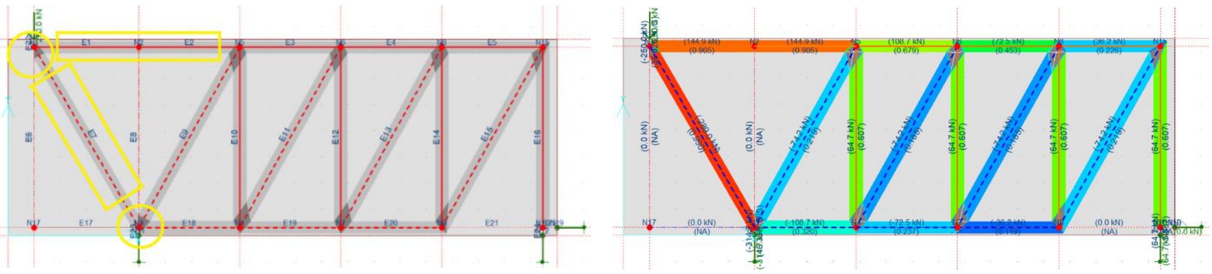

Figure 7. Strut and ties, Requirement in members (model 6)

In Table 3keeps all highest values of stress ratio for each model being in compression (-) and tensile (+). The reinforcement ratio (area of steel/area of concrete) in each beam model it is considered only vertical and horizontal ties not involving anchorage, vertical stirrups and skin reinforcement.

Table 3. Comparison stress ratio and Reinforcement ratio

Topology	Stress Ratio Max (-)	Stress Ratio Max (+)	Reinforcement ratio
1	.706	.891	1,378%
2	.990	.905	0,229%
3	.995	1	1,96%
4	.990	.905	0,361%
5	.990	.905	0,295%
6	.990	.905	0,361%

5 Conclusions

The factored load in models I, IV, V and VI was close once the topology was similar. Others beams showed different factored load and also required higher area of steel. However, theses beams (I and III) reached the tie yielding before crushing concrete strut. Knowing the need of designing building near the limit of land and clearer analysis, the strut-and-tie method has been effective in designing D-regions. The CAST tool speeds the verification and can be applied in other beams like pre fabricated beam. It was demonstrated users need to own some experience to consider the right path of load as well as economic and practical aspects because the tool does not generate automatic analysis. Using sloped reinforcement may reduce productivity and should be considered. Some mistakes can occur like crossing the load path and may lead to unrealistic solutions. As a limitation in this study, non-linear analysis wasn't developed and should be studied by others researchers or even compare results via finite elements analysis. Furthermore, CAST may be a strong tool in teaching STM once it's easy to handle beyond allows detailed analysis in nodal zones quickly.

6 Authorship statement

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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