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Topology optimization tools applied to the design of anchor systems in offshore structures

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Abstract. The design of anchor systems in offshore structures consists in three general steps: conceptual design, sizing and analysis, and verification. In this work, the topology optimization is applied on the first step, as an effort to support the design of new anchor lines topologies. There are demands on the oil and gas industry that seek to minimize the anchor radius of a production unity, which is the horizontal projection of the line connecting the platform and the anchor point, and to maximize the anchor pattern, which is the angle between the last anchor line of a cluster and the first anchor line of the neighbor cluster, for example. Topology optimization formulations can incorporate these demands in order to achieve better (more economic) solutions to be used in the conceptual design of new production unities. In this work, we use a discrete topology optimization formulation with multiple load cases and box constraints with a cable constitutive model. Results show that our approach has the potential of producing innovative topologies unlocking the creativity of the designer and new conceptual designs.

Keywords: Topology optimization, cable structures, anchor systems, offshore structures, structural design

1 Introduction

The design of anchor systems in offshore structures consists of three general steps: conceptual design, sizing and analysis, and verification. This methodology is intrinsically related to what the industry is already familiar to produce and to the designer experience – who acquires it on a trial and error basis, during the designing process –, therefore, it is a subjective process.

In this work, topology optimization tools are introduced in the first step of the process in order to support the design of more efficient strucutres in a general point of view. These tools have the power to suggest stiffer, lighter and cheaper structures – depending on the design objectives – which can be used as a start point on the designing process, based on a mathematical formulation.

There are demands on the oil & gas industry that seek to improve the design of anchor systems of the offshore production unities. For example, the anchor radius of a production unity is the horizontal projection connecting the platform and its anchor point. Minimizing the anchor radius of a production unity means to bring the anchor points closer to the unity, making the area of influence of that anchor system smaller. Another example is the anchor pattern, which is the angle measured between the last anchor line of a cluster and the first anchor line of the neighbor cluster. Maximizing the anchor pattern means giving more room for equipments and other production systems, which are the reason why the production unity is built in the first place.

2 Mathematical formulation

The mathematical formulation used in this work is a ground structure[1] based topology optimization formulation for cable structures undergoing possibly finite displacements and deformations and subjected to possibly multiple load cases:

[1]For a better understanding of the ground structure (GS) concept, the reader is referred to the paper by Ramos Jr and Paulino [1].

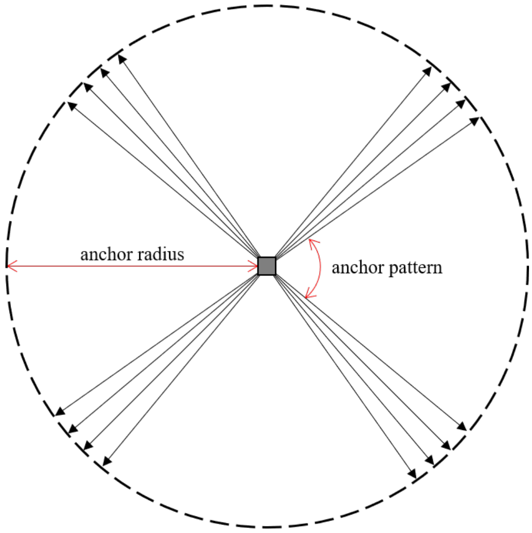


Figure 1. Anchor radius and anchor pattern of a production unity.

$$
\begin{aligned}
\min_{\boldsymbol{A}} \;\; & f(\boldsymbol{A}) = \min_{\boldsymbol{A}} \sum_{j=1}^{m} -w_j \Pi_j(\boldsymbol{A}, \boldsymbol{u}(\boldsymbol{A})) \\
s.t. \;\; & g(\boldsymbol{A}) = \boldsymbol{L}^T \boldsymbol{A} - V^{max} \leq 0 \\
& 0 \leq A_i \leq A_i^{max}, \text{ for } i = 1, ..., n \\
with \;\; & \boldsymbol{u}_j(\boldsymbol{A}) = \underset{\boldsymbol{u}}{\operatorname{argmin}} \left[\Pi_j(\boldsymbol{A}, \boldsymbol{u}(\boldsymbol{A})) + \frac{\alpha}{2} \boldsymbol{u}^T \boldsymbol{u} \right], \text{ for } j = 1, ..., m,
\end{aligned}
\tag{1}
$$

on which $\boldsymbol{A}$ is the vector of design variables representing the cross-sectional areas of the cable net, $\boldsymbol{u}$ is the vector of nodal displacements, $\boldsymbol{L}^T$ is the vector of lengths of the cable members, Π_j is the stationary total potential energy of a system under the j-th load case, $w_j > 0$ are the so-called weight factors associated with the j-th load case, n is the number of cable members and m is the number of load cases. The parameter α is part of a strategy called Tikhonov regularizaton used by Sanders et al. [2] which aims to avoid bad matrix conditioning during the process of optimization due to the zero lower bound and the max filter strategy (to be discussed later).

Minimizing the negative of the total potential energy has been shown to be equivalent to maximizing the stiffness through the end-compliance approach by Klarbring and Strömberg [3]. This means that the objective function of this problem seeks a cable network with maximum stiffness.

2.1 Sensitivity analysis

In an optimization problem the sensitivity of the objective function is responsible for indicating the path the algorithm should follow to find a stationary point. In this case, this means to determine which cable members will have their cross-sectional areas increased or reduced. This analysis is made not only for the objective function but also for the restrictions of the problem. In this work, the sensibilities of the problem are:

$$
\frac{\partial f(\boldsymbol{A})}{\partial A_i} = -L_i \Psi_i, \text{ and} \tag{2}
$$

$$
\frac{\partial g}{\partial A_i} = L_i. \tag{3}
$$

on which Ψ_i is the specific strain energy of the material.

2.2 Specific strain energy

This formulation considers a hyperelastic constitutive model of a tension-only material whose specific strain energy is defined by:

$$\Psi_i = \begin{cases} \frac{E_i}{2}(\lambda_i - 1)^2 & \text{if } \lambda_i \geq 1 \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

and therefore has its constitutive model $\sigma_i = \frac{\partial \Psi_i}{\partial \lambda_i}$ defined as:[2]

$$\sigma_i = \begin{cases} E_i(\lambda_i - 1) & \text{if } \lambda_i \geq 1 \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

on which $\lambda_i = \varepsilon_i + 1$ is the stretch of the cable member, defined according to its axial strain.

2.3 Optimum conditions

Despite defining the mathematical formulation, it is necessary to know when the optimal configuration is reached. For the problem in this work, the adopted metrics are the specific strain energy and the squared tension for each cable member. It is proved by Ramos Jr and Paulino [1] and Zhang et al. [5] using the so-called KKT conditions that these two metrics show constant values for all members of the final topology when the box constraints are not active. This situation correspondes to a full stress design in a linear case.

Beyond these two metrics, the value of the objective function at each iteration will also be studied, to ensure the problem is converging to a minimum. The value of the normalized member areas will be plotted to picture a better understanding of the design variables at the end of the simulation.

The final topology is the set of cables with non-zero cross-sectional area after a simple filtering scheme is performed, denoted by the following condition:

$$A_i = \text{filter}(\boldsymbol{A}, \alpha_f) = \begin{cases} 0 & \text{if } \frac{A_i}{\max(\boldsymbol{A})} \geq \alpha_f \\ A_i & \text{otherwise,} \end{cases} \tag{6}$$

on which α_f is a filter value which ensures that the final topology satisfies global equilibrium.

3 Results

Figure 2 shows the model used on the examples presented afterwards. The production unity is represented by a node with a first gender support (simulating water's buoyancy force). The candidate support areas are also expressed in the model.

3.1 Example 1 - 2D structure with simple GS

In this example, the GS with boundary conditions (BC) shown in Fig. 3 was simulated. Second gender supports were considered to simulate the interaction between the cables and the soil. A first gender support was considered at the load application point to simulate the buoyancy forces acting on the production unity. Only one load case was considered, which is a horizontal force.

First, in simulation a), only a volume constraint was considered, letting the structure accomodate material in the best way disregarding cross-sectional area limits. Afterwards, in simulation b), the box constraints were considered, generating two different final topologies.

[2]For a better understanding of this relationship, the reader is referred to the paper by Sanders et al. [4].

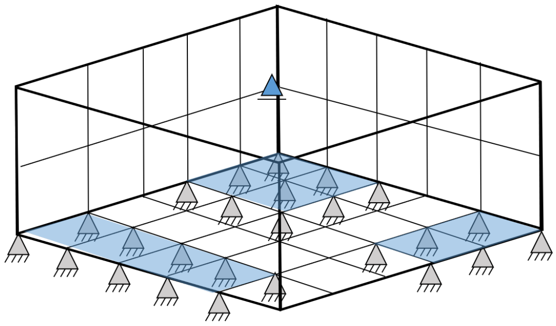


Figure 2. Model on which the simulations were based, with a generic supports configuration.

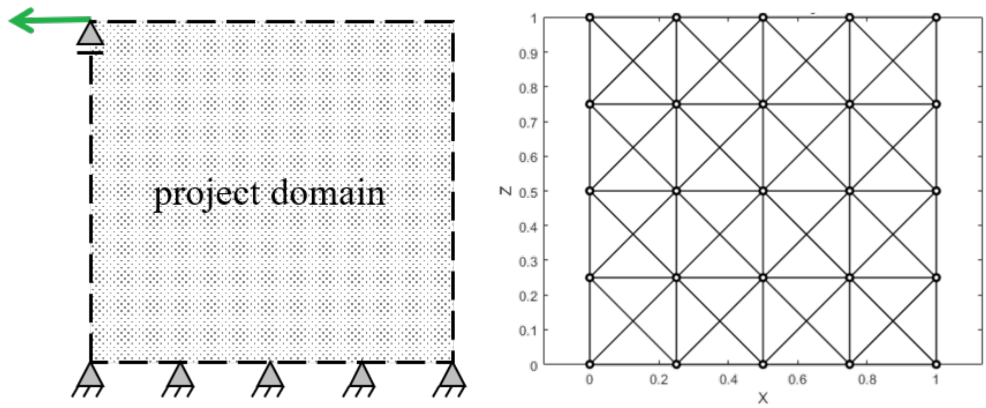
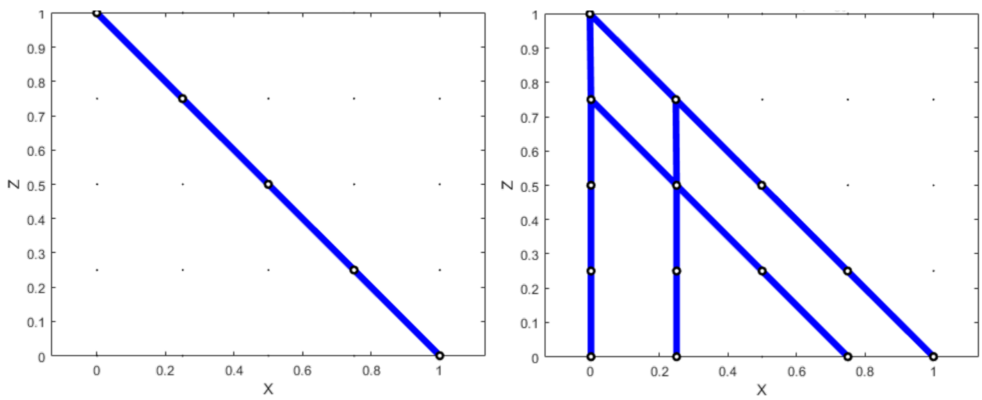


Figure 3. Project domain with boundary conditions and the initial GS used.



a) b)

Figure 4. Final topologies with: a) box constraints inactive; b) box constraints active.

The final topology in Fig. 4 a) is resumed to a cable anchored in the furthest node, which is very reasonable from a structural point of view, noted that this configuration has the greatest tension horizontal component to equilibrate the applied load, while in the situation b) the structure had to use more bars in order to equilibrate the applied load since there was a limit to the cross-sectional areas of the members.

The simulation metrics for a) are shown in Fig. 5. The optimum conditions were satisfied, because the specific strain energy and the squared tension for each cable member are constant. Therefore, this final topology is optimum for the problem.

The metrics for b) are shown in Fig. 6. The conditions of specific strain energy and squared tension for each cable member to be constant were not satisfied – as expected – since the box constraint was active. Also, the value of the objective function in the end of the simluation in this case was 400% higher than the previous one. This means that this structure is 400% less stiff than the previous one.

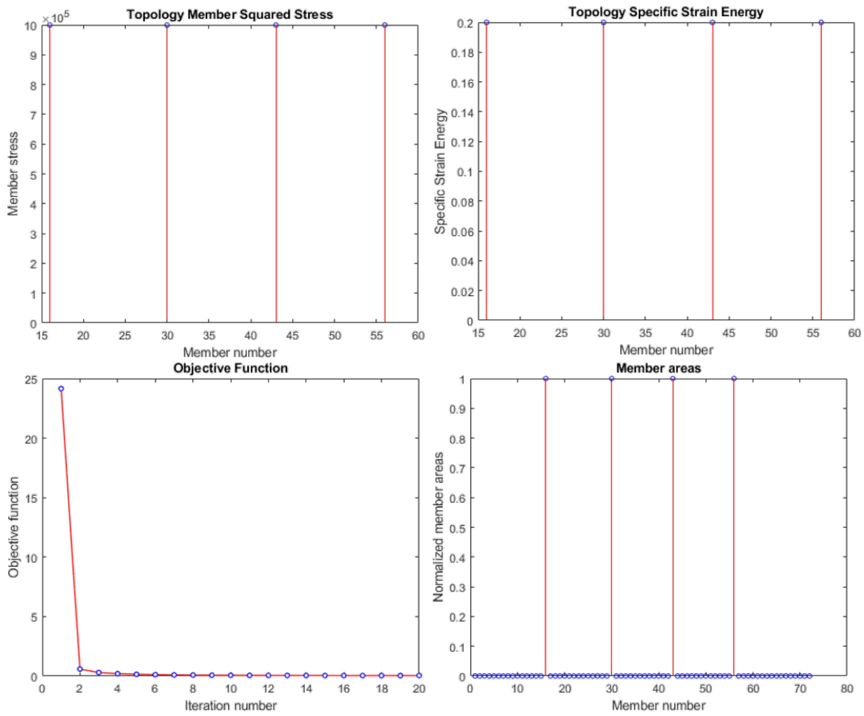


Figure 5. Metrics of simulation a).

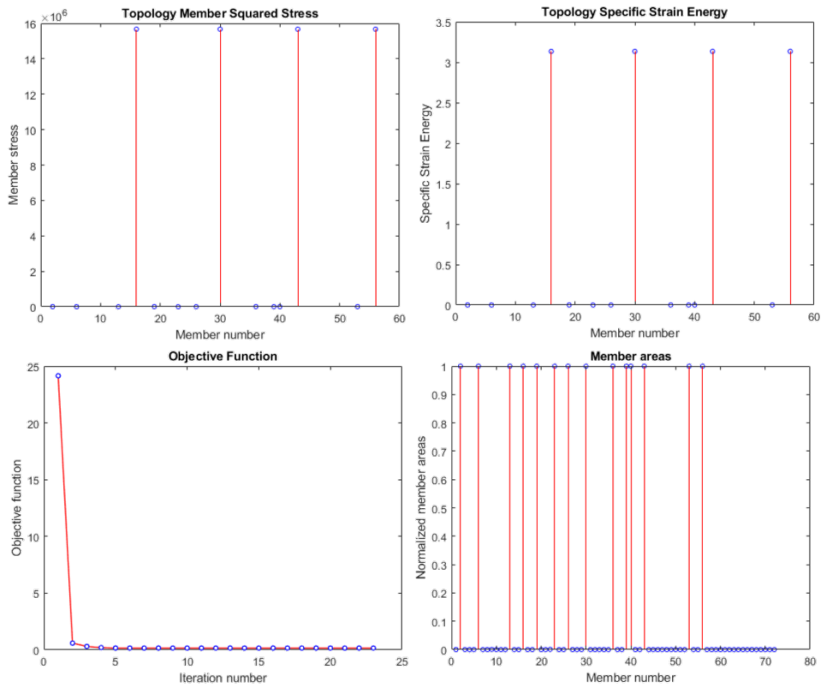
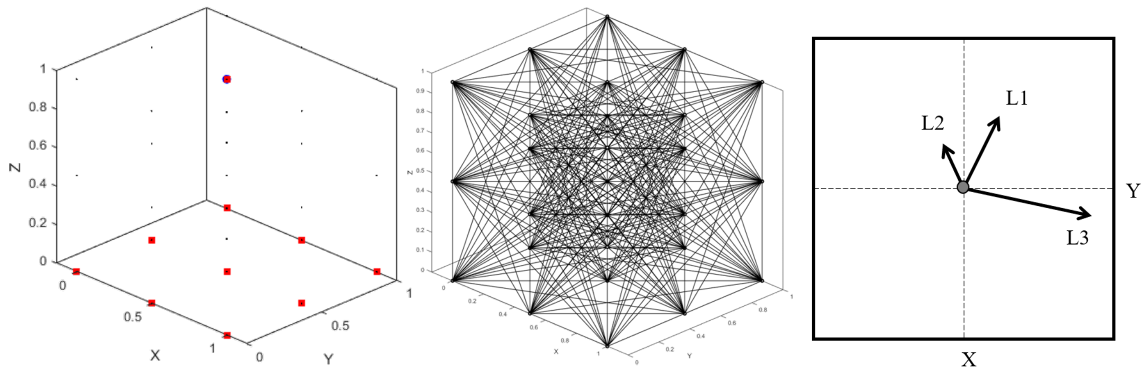


Figure 6. Metrics of simulation b).

3.2 Example 2 - 3D structure with three load cases

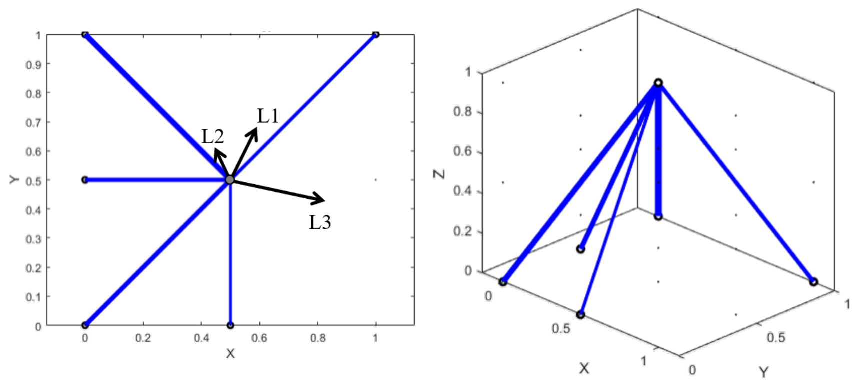
In this example, the GS shown in Fig. 7 was simulated. Second gender supports were considered to simulate the interaction between the cables and the soil. A first gender support was considered at the load application point to simulate the buoyancy forces acting on the production unity. Three load cases were considered, represented by $L1$, $L2$ and $L3$.



a) b) c)

Figure 7. a) $3 \times 3 \times 3$ 3D domain with BC of the simulation; b) GS of the simulation; c) top view with the three load cases acting on the production unity.

Only a volume constraint was considered. The final topology is shown in Fig. 8, on which it can be noted that the algorithm has distributed more material on the opposite direction of the greatest loads, as expected, since we are considering a tension-only cable net. Also, the final design is not an usual design from an offshore anchor system point of view, which shows the potential of this tool in suggesting more efficient topologies as a start point for the design of offshore anchor systems.



a) b)

Figure 8. a) top view of the final topology with the acting load cases; b) isometric view of the final topology.

The simulation metrics for the Example 2 are shown in Fig. 9. Despite considering three load cases and the cross-sectional areas of the cables being obviously different, the optimum conditions were satisfied, because the specific strain energy and the squared tension for each cable member are constant. Therefore, this final topology is optimum for the problem.

4 Conclusions

With the simple examples shown in this work, we can say that our approach has the potential of producing more efficient and innovative conceptual topologies for the oil & gas industry, unlocking the creativity of designers from the perspective of a mathematical, objective method.

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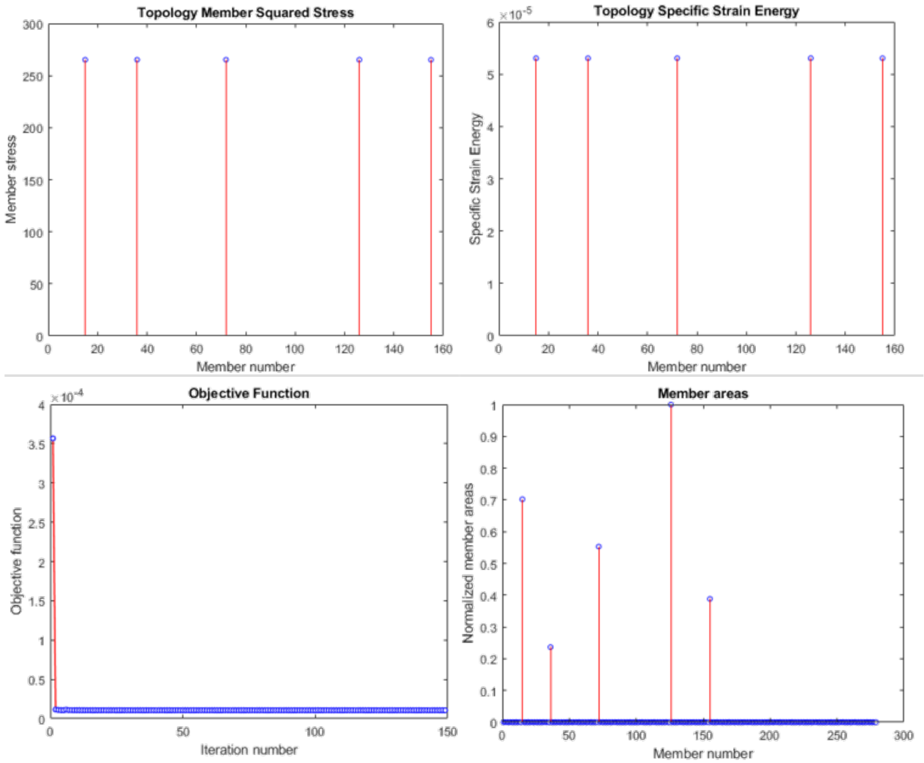

Figure 9. Metrics of Example 2.

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References

[1] Ramos Jr, A. S. & Paulino, G. H., 2015. Convex topology optimization for hyperelastic trusses based on the ground-structure approach. *Structural and Multidisciplinary Optimization*, vol. 51, pp. 287–304.
[2] Sanders, E. D., Ramos Jr, A. S., & Paulino, G. H., 2017. A maximum filter for the ground structure method: An optimization tool to harness multiple structural designs. *Engineering Structures*, vol. 151, pp. 235 – 252.
[3] Klarbring, A. & Strömberg, N., 2012. A note on the min-max formulation of stiffness optimization including non-zero prescribed displacements. *Structural and Multidisciplinary Optimization*, vol. 45, pp. 147 – 149.
[4] Sanders, E. D., Ramos Jr, A. S., & Paulino, G. H., 2020. Topology optimization of tension-only cable nets under finite deformations. *Structural and Multidisciplinary Optimization*, vol. 62, pp. 559 – 579.
[5] Zhang, X., Ramos Jr, A. S., & Paulino, G. H., 2017. Material nonlinear topology optimization using the ground structure method with a discrete filtering scheme. *Structural and Multidisciplinary Optimization*, vol. 55, pp. 2045 – 2072.