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Design of a conveyor belt idler roller using a hybrid topology/parametric optimization approach

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Abstract. A significant part of iron ore cost is due to transportation. Around 30% of this cost consists in replacing idlers in conveyor belts. Thus, a proper design of the idler is of great economic importance. Another crucial aspect is the ergonomics related to idlers replacement due to its weight and logistics. Having this in mind, the usage of a robust optimization methodology for the conveyor belt idlers becomes essential to reduce the cost of the mining process and improve idler replacement conditions. Polymeric materials have a great potential to reduce idlers weight due to their low density. However, polymers have lower Youngs's modulus when compared to metals, making a hybrid design (metal + polymer) a very suitable option for the idler. Polymers can be easily molded by additive manufacturing, thus, a great range of different shapes can be obtained. In this case, topology optimization methods are suitable to obtain an optimum manufacturable design. However, the steel part (shaft) has less flexibility in the design. Its topology and shape being fixed, levaves only a few parameters to define the part design. In this case, a parametric optimization is applicable. This paper presents a systematic procedure that combines both parametric and topology optimization to obtain an optimum design for a conveyor belt idler. The topology optimization scheme is used within the parametric optimization iterations with different combinations of the geometric parameters of the shaft to build a surrogate model. Then, the surrogate model is optimized using an improved sequential least-squares quadratic programming (SLSQP) method.

Keywords: Topology optimization, Conveyor belt idler roller, Surrogate modelling, Finite element analysis

1 Introduction

The design of conveyor belt components involve a set of different technologies. To guarantee the safety and quality of the each component of the system, several standards must be followed. The most famous one is given by the Conveyor Equipment Manufacturers Association (CEMA [1]), which presents guidelines to design the whole system. Since it is a standard for the whole system, in the idler chapter, there are guidelines to select idlers instead of designing them. A more specific standard, the ABNT [2], presents requirements that are specific for conveyor belt idlers and the conveyor belt idler rollers. Figure 1 shows a typical conveyor belt and an idler assembly.

ABNT [2] presents clear directives for selection and design of conveyor belt idlers and and idler rollers. Using this standard it is possible to design a roller using simple analytical equations and tables. The roller is usually composed of an external envelope, an internal shaft, and two rolling bearings and labyrinth seals. A closer look in ABNT [2] unveils that the stiffness criteria are based solely on the the structural behaviour of the shaft. Cousseau and Borges [3] showed that in structural terms, the stiffness is the critical point in structural design. This means that, in general, the allowable stress values are not exceeded on metal rollers. Although these conclusions cannot be extrapolated to polymeric rollers, it can be deduced, at least, that in addition to strength, special attention should be paid to rigidity.

The literature about optimization of idler rollers is scarce. Pol and Jadhav [4] showed a methodology to optimize conveyor pulleys. They are different components but the construction is very similar. They performed a parametric optimization of the structure of the pulley, using a finite element model to represent the mechanical behaviour of the component.

This paper aims to present, develop and test a robust method to optimize a conveyor belt idler roller. Since the envelope is made of a polymeric material, it is possible to build it with additive manufacturing, making possible different types of topology configurations. Thus, in this case, an efficient optimizer must also include a topology optimization strategy.

Figure 1. Example of a conveyor belt (left) and a conveyor belt idler with three rollers (right) [1]

2 Methodology

Cousseau and Borges [3] present a method to perform the structural analysis of idler rollers in which the contact pressure due to the belt in the roller is considered to be constant. Using a global analysis in the conveyor belt system performed by Fedorko and Ivančo [5] and the Saint-Venant principle, the constant pressure approach is valid as long as the maximum stresses does not occur on the contact area. This previous methodology to analyze the rollers was improved by introducing a COMBI214 element [6]. This element allows to replace the mesh of the bearing for the appropriate radial stiffness. The bearing stiffness is computed using Palmgren method, described by Gargiulo [7]. Another advantage is that, the model described here is linear and, therefore, is straightforward to apply topology optimization.

The analyzed roller is mass manufactured by a high-density polyethylene (HDPE) to build the envelope and mounting in the machined shaft via two 6310-2Z rolling bearings [8]. The stiffness and strength of HDPE can vary significantly depending on composition of constituents, additives e processing. To find such quantities, a tensile test was performed. The value of Poisson's ratio was not measured, and it was used 0.46, taken from Polymer Properties Database [9]. The material constants for the steel can be found in ABNT [2]. The properties of the materials of the roller are summarized in Tab. 1.

Table 1. Material data of the conveyor belt roller

Material	Young's modulus E [GPa]	Poisson's ratio $\nu[-]$	Allowable stress [MPa]	Density [kg/m^3]
Steel	210	0.3	200	7850
HDPE	1.2119	0.46	6.21	940

ABNT [2] states the main mode of failure of rollers is fatigue of bearings. In order to operate in optimum conditions, the bearings must not exceed 9' of misalignment due to shaft deformation [2]. Thus, an optimization method must include a constraint of maximum misalignment due to load. Figure 2 shows the bearing misalignment angle.

The design region shown in Fig. 3 must follow some constrains. First of all, since the roller is 3D printed in the axial direction, there must be no overhang angles over 45^o. Another constraint is that the volume is equal to a fraction of the initial value. It has a fixed 197.2 mm of diameter and 800 mm of length. Thus, according to ABNT [2], it must withstand a load of 15917 N.

Figure 3 shows the roller used as a baseline and the topology optimization approach. The optimized roller must have the same external dimensions so it can be interchangeable with the original. Additionally, a hollow shaft is proposed to improve the structural performance of the roller.

The solid isotropic material with penalisation (SIMP) method was applied to the model. The problem can be expressed as:

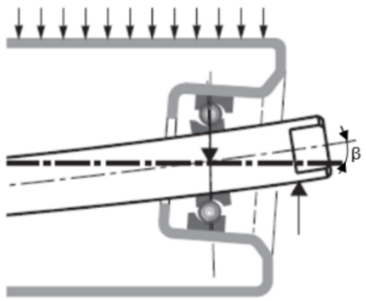

Figure 2. Bearing angle of misalignment [2]

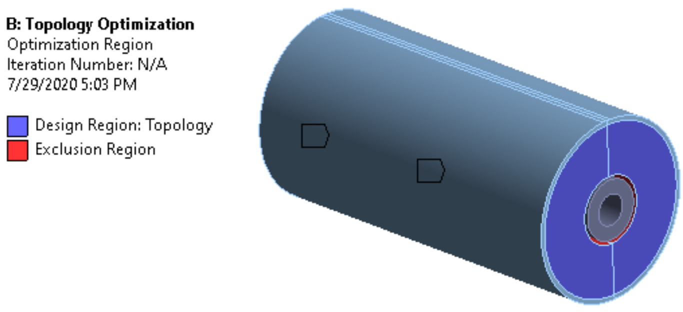

Figure 3. Design region for the topology optimization

$$
\begin{aligned}
\text{minimize:} \quad & \mathbf{f}^T\mathbf{u} \\
\text{subject to:} \quad & \left(\sum_{e=1}^{N} \rho_e^p \mathbf{K}_e\right)\mathbf{u} = \mathbf{f} \\
& \sum_{e=1}^{N} v_e\rho_e \leq V \text{ , } 0 < \rho_{min} \leq \rho_e \leq 1 \text{ , } e = 1, ..., N \\
& \text{Overhang angle} \leq 45^{\text{o}} \\
& \text{Cyclic symmetry pattern repetitions} = 4
\end{aligned} \tag{1}
$$

where:
$\mathbf{f}$ is the force vector
$\mathbf{u}$ is the deformation vector
$\mathbf{K}_e$ if the stiffness matrix of the e_{th} element
ρ_e is a pseudo-density used as design variable.
v_e is the volume the the e_{th} element
V is the fraction of the initial volume used as a constraint

Bendsøe [10] also suggests that, for 3D models, the penalization factor p is chosen such that :

$$
p \geq \max\left\{15\frac{1-\nu^0}{7-5\nu^0}, \frac{3}{2}\frac{1-\nu^0}{1-2\nu^0}\right\} \tag{2}
$$

For the HDPE, with Poisson's ration equals to 0.46, the p the value found with equation 2 is 10.125. The problem shown in equation 2 minimizes a quantity known as *compliance*, that is inversely proportional to the *global* stiffness. The number of cyclic repetitions was arbitrarily chosen to simplify the optimization process. Since it can only take integer values, a discrete optimization algorithm would be required to incorporate this variable in the optimization process.

Even with the problem posted, some configuration remain open:

- Volume fraction to be actually used
- Diameter of the shaft between bearings

- Diameter of the hole the shaft

A Pareto front could be built [11] , but here we present a different approach. Given that there is one topology optimization problem for each choice of parameters, it is possible to put the topology optimization withing a larger parametric optimization, such as:

$$\begin{aligned} \text{minimize:} \quad & \text{mass} = f(D_S, D_H, V_f) \\ \text{subject to:} \quad & \beta \leq 9' \end{aligned} \tag{3}$$

where:
D_S is the diameter of the shaft
D_H is the diameter of the shaft hole
V_f is the volume fraction of the topology optimization
β is the bearing misalignment due to belt load

The mass is the one found using the topology optimization method using the input parameters. The misalignment angle is also obtained using an static structural analysis of the roller. One may argue that this approach could lead to a non-conservative design, since the structure of the roller is not guaranteed to be axisymmetric. The structure has some degree of symmetry since it repeats itself every $\pi/2$ radians. Thus, a second load case was considered, displaced $\pi/8$ radians of the original case, which correspond to the region between the reinforcements in the structure in the topology optimization. The first load scenario generates a misalignment angle β_1 and the second load scenario generates a β_2 angle.

Since each topology optimization process takes a couple of hours to run, a surrogate model [12] is built to perform the parametric optimization and thus alleviate the computational burden. According to Forrester et al. [12], a surrogate model is an analytical, simplified model that represents the high-fidelity model and can estimate with a sufficient accuracy the response to be used as objective and/or constraint in a optimization algorithm.To built the surrogate, the first part is to define the points in which the high-fidelity model is going to be evaluated. This part is called *design of experiments*. Jin et al. [13] describe a so-called Latin hypercube sampling (LHS), that is, a way to sample in which no point share a common design variable value. The intention here is to use as few points as possible. In order to improve the effectiveness of the sampling, Jin et al. [13] also suggest a way to optimize the sampling by maximize the minimum distance between points. This is usually refereed as max-min approach of the LHS.

2.1 Design of experiments

The design of experiments was performed using 20 points. The process of running the design points was not automatic. It was possible to run automatically the topology optimization algorithm, but the post-processing of the *.stl* geometry was made individually for each geometry. Thus, it was difficult to automatically include refinement points in the model. However, the model was validated using the cross-validation technique.

The implementation used was made by Bouhlel et al. [14] in Python, and it includes the design of experiments, the surrogate model and its derivatives. Using the derivatives, the flexibility in the optimization algorithms is increased [15]. The DOE sampling points are shown in Fig. 4.

2.2 Surrogate model

After defining a proper design of experiments, the surrogate model must be built and its effectiveness evaluated. Here, Kriging was chosen as the surrogate method. This approach has two main advantages: first, the surrogate has no error in the training points (making a $R^2 = 1$ model in every situation) and it is defined in terms of the variance expected in the estimate. A Kriging function is a special case of a radial basis function, which can be expressed in the following terms:

$$\hat{f}(\mathbf{x}) = \mathbf{w}^T \boldsymbol{\psi} = \sum_{i=1}^{n_c} w_i \psi \left(\left\| \mathbf{x} - \mathbf{c}^{(i)} \right\| \right) \tag{4}$$

where:
$\mathbf{w}$ is a vector containing the coefficients of the training function and
$\boldsymbol{\psi}$ is the radial base function.

In the Kriging model, the basis function is defined as:

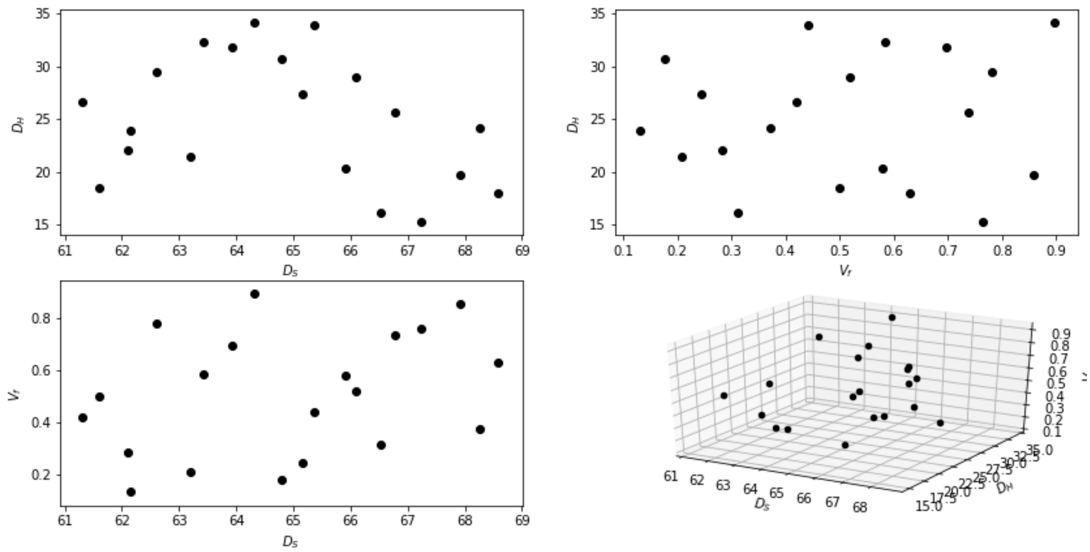

Figure 4. Design of experiments for the parametric optimization

$$\psi = \exp\left(-\sum_{j=1}^{k}\theta_j\left|x_j^{(i)} - x_j\right|^{p_j}\right) \tag{5}$$

where:
θ_j are coefficients to be adjusted and
p_j are coefficients chosen prior to the training of the surrogate (usually p_j is a constant value, independent of j).

Using the design points shown in Fig. 4, it was possible to determine the coefficients of the surrogate model. To calculate the value of the angles, finite element analysis was performed and the mass of the bearings was added to calculate the total roller mass. Since the computation of the design points was not carried out in an automatic way, it became unfeasible to implement an automatic refinement of the model using an algorithm such as the maximum likelihood estimation (MLE) [12]. Instead, only a verification of the model is performed. This is made using a cross validation scheme. This scheme consists in, for each design point, build a surrogate without it, and evaluate the difference the expected value (in the high-fidelity model) and the estimated with the reduced surrogate model. This was performed for the 3 surrogate models and for all 20 design points. The results are summarized in Fig. 5.

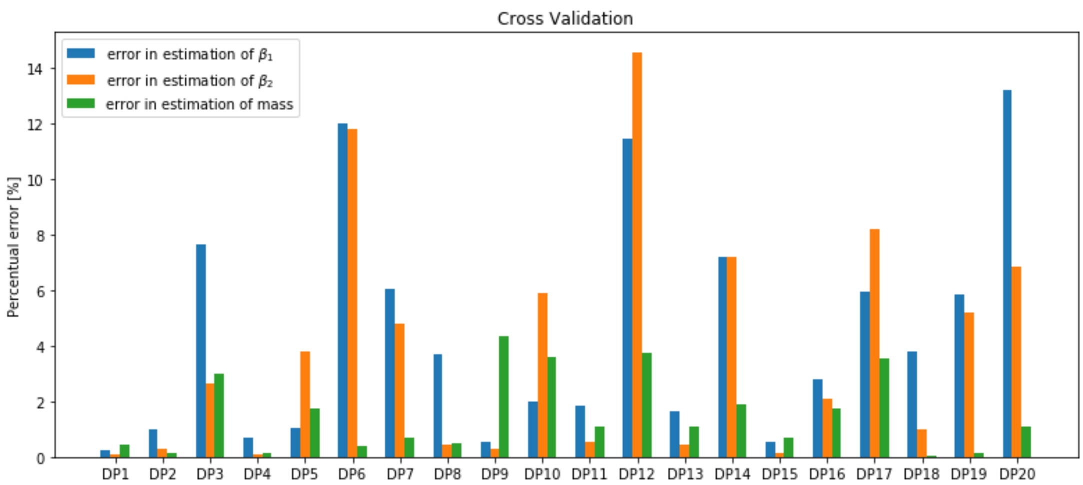

Figure 5. Cross validation results for the surrogate models

As shown in Fig. 5, significant errors were found in the surrogate models. It would be the case of refining the model, but, as mentioned earlier, this is not an option. This is a point of further improvement, but it is expected that even though the absolute value of the constants present some error, the location of the optimum does not vary significantly. Besides, after finding the optimum point, some adjustments in the dimensions of the shaft can still be made.

2.3 Parametric optimization

The sequential least-squares programming (SLSQP) algorithm, taken from the open source repository SciPy [16], was employed as the optimizer in the parametric optimization approach. SLSQP is a gradient-based method which runs fast but does not not guarantee global optimality. Hence, a *walkaround* strategy was performed, which consists in running a LHS DOE with 1000 points and perform 1000 optimizations with different initial guesses. Among the final points of each optimization, the one that provides the best design is considered the global optimum.

From the parametric optimization results, it was not possible to find a feasible design (i.e, a design with $\beta \leq 9'$). Hence, it was considered as the optimal point of each optimization the design of the last iteration, and it was adjusted in a final analysis to comply to the constraint.

2.4 Final design verification

The parametric optimization strategy leaded to $D_S = 62.74$ mm, $D_H = 35$ mm and $V_f = 0.7985$. These values were inputted in an ANSYS topology optimization framework. Summarized results (surrogate vs. high-fidelity model) are shown in Tab. 2.

Table 2. Summarized results.

	$\beta_1 [']$	$\beta_2 [']$	Mass [kg]
Surrogate-optimized model	10.25	9.63	31.37
Topology-optimized model	9.86	9.85	34.09

Table 2 shows that the maximum error between the surrogate and high-fidelity model were less than 3.5%, showing no need to improve the surrogate and redo the optimization. The design of the roller was manually adjusted as shown in Fig. 6. The allowable used to calculate the safety factors are shown in Tab. 1.

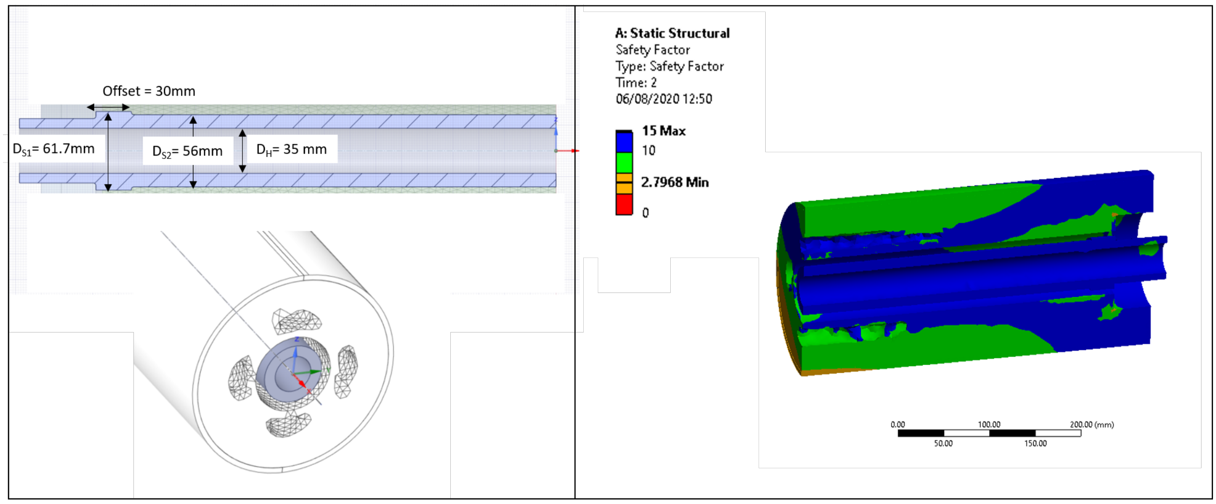

Figure 6. Final design of the roller (left) and the safety factor distribution along the component for this design.

This design ended up with $\beta_1 = 8.40'$, $\beta_2 = 8.40'$ and mass of 30.76 kg.

3 Conclusions

The results of the topology optimization showed that different designs are possible. It was showed that a great amount of mass in the polymeric roller is necessary, but the shaft can be as light as possible.

Even thought this work already found a light and strong roller design, certainly it has room to improve the algorithm, as the following ideas:

- Only four repetitions in the cyclic symmetry was considered. The number of cyclic symmetries could be a design variable (i.e., a integer value which may vary during the optimization loop);
- The shaft could be manufactured from aluminum, since this study showed that the shaft from steel could be hollow (i.e., not solid and therefore less stiff);
- Still in the materials area, the material itself could be a design variable, both for the shaft (steel or aluminum) and the different materials for the roller (polyethylene, polyamide, HDPE and others);
- Other possibility is to take into account the viscosity properties of the polymers. Perhaps, due to its viscoelasticity, its behaviour might change with the rotating speed.

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