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Applying a topology optimization projection scheme for structures under free and forced vibration

Lucas O. Zúñiga[1], Renata M. Soares[2], Sylvia R. M. de Almeida[3]

[1]*School of Civil and Environmental Engineering, Federal University of Goiás*
Av. Universitária, 1488, Qd 86, Bloco A, sala 13, St. Universitário,74605-220, Goiânia, GO, Brazil
lucasde@discente.ufg.br
[2]*School of Civil and Environmental Engineering, Federal University of Goiás*
Av. Universitária, 1488, Qd 86, Bloco A, sala 6, St. Universitário,74605-220, Goiânia, GO, Brazil
renatasoares@ufg.br
[3]*School of Civil and Environmental Engineering, Federal University of Goiás*
Av. Universitária, 1488, Qd 86, Bloco A, sala 9, St. Universitário,74605-220, Goiânia, GO, Brazil
sylvia@ufg.br

Abstract. This work addresses topology optimization applied to structures under free and forced vibrations using the density approach. A SIMP-like model is used to penalize the stiffness and the mass distribution. The density approach applied to structures under vibration is prone to intermediate densities in the final solution and to numerical instabilities on a larger scale than the static approaches. In this paper a projection scheme is applied to overcome the numerical instabilities characteristic of this type of approach using a Heaviside projection function is applied in both cases. The Heaviside function also reduces de intermediate densities in the final solution when compared both to the linear function and to the application of the classical sensitivity filter. Numerical examples are presented to illustrate the application of the projection-based technique applied to free and forced vibration problems.

Keywords: topology optimization, free vibration, forced vibration, Heaviside function, projection scheme.

1 Introduction

Methods to achieve optimal structural designs have been under investigation for decades and the traditional trial-and-error process is being systematically replaced by optimization techniques applied to several steps of the design process. With the improvement of the methods used to describe the structural behavior, the structures became more sensitive to dynamic actions and effects. In this context, considering such effects from the beginning of the structural layout determination phase can be essential for achieving structural efficiency. Thus, the study of topology optimization techniques taking into account dynamic effects is a natural path and contributes to the improvement of the design process.

Du and Olhoff [1] applied the density approach in topology optimization for free vibration problems and Olhoff and Du [2] for non-damped forced vibration. In both cases SIMP ([3], [4]) was applied. However, the density approach presents numerical instabilities and the final solutions contain a considerable amount of intermediate densities. Among the methods for controlling numerical instabilities, projection schemes stand out for, in addition to eliminating solutions with checkerboard patterns and mesh dependency, reduce intermediate densities in the final solution depending on the choice of the projection function.

Guest *et al*. [5] proposed a topology optimization approach for static problems using projection schemes, an alternative to overcome numerical instabilities providing control over the size of the resultant structural members. The results obtained using a Heaviside-like projection function presented low amounts of elements with intermediate densities. A similar technique was applied by Almeida *et al*. [6], who used inverse projection schemes to control the void distribution in the final solution. Latter Guest [7] used projection schemes with multi-phase projections and Li and Khandewal [8] studied the relation between continuity techniques and the preservation of volume of the structure.

The present work applies projection schemes to topology optimization problems of free vibration and non-damped forced vibration. The projection functions are studied in particular to obtain smaller amounts of elements with intermediate densities in the applications in dynamics problems.

2 The density approaches

The density method is based on the material distribution in an extended domain, with the presence and absence of material determining the structural layout. In the original approach, which is still the most widespread today, the design variables represent the proportion of material in each region of the extended domain, representing a pseudo density, simply called density. Although several methods can be used for structural analysis, in most cases a discretization of the extended domain in finite elements is used. The design variables represent the element densities, ρ_{el}, with the null value representing the void and the unit the solid material.

For structural analysis, it is necessary to associate density with the mechanical property representing the material strength, in this case the Young's modulus. The SIMP (Solid Isotropic Material with Penalization) model is the most widely material model associated to the density methods. Equation (1) presents the SIMP model, in which a density penalty leads to the void-solid final solution.

$$E_{el} = \rho_{el}{}^{p}\ E_s \tag{1}$$

Where: ρ_{el} is the density of the element *el*; E_{el} is the Young's modulus of element *el* with density ρ_{el}; E_s is the Young's modulus of the solid material; and p is the penalty factor.

2.1 Formulation for static analysis

The most studied optimization problem for static analysis is the so-called compliance problem, presented in eq. (2). The objective is to minimize the mean compliance of the structure while keeping the structural volume constant. A minimum value, x_{min}, is applied to the design variables in order to prevent singularity of the stiffness matrix associated to the degrees of freedom in the void region.

$$\begin{array}{ll} \text{Obtain :} & \mathbf{x} \\ \text{Minimizing :} & \mathrm{c}(\boldsymbol{\rho}(\mathbf{x})) = \mathbf{F}^T\ \mathbf{U}(\boldsymbol{\rho}(\mathbf{x})) \\ \text{Such that :} & \sum \rho_{el}\ V_{el} = f\ V_d \\ & 0 < x_{\min} \le x_j \le 1 \\ \text{With :} & E_{el} = E\left(\rho_{el}, E_s\right) \\ & \mathbf{K}(\boldsymbol{\rho}(\mathbf{x}))\ \mathbf{U}(\boldsymbol{\rho}(\mathbf{x})) = \mathbf{F} \end{array} \tag{2}$$

Where: **x** represents the design variables; **ρ** represents the element densities; *c* is structural mean compliance; **F** represents the nodal forces; **K** is the structure stiffness matrix; **U** represents the nodal displacements; V_{el} is the volume of element *el*; V_d is the volume of the extended domain; *f* is the prescribed volume fraction; x_{min} is the minimum value for the design variables.

The sensitivity of the objective function with respect to the element densities is given by eq. (3). If element-based design variables are applied with no projection scheme, the design variables **x** are equal to the element densities **ρ** and eq. (3) represents the sensitivity of the objective function with respect to the design variables as well.

$$\frac{\partial c(\boldsymbol{\rho})}{\partial \rho_{el}} = -\mathbf{U}^T(\boldsymbol{\rho})\ \frac{\partial \mathbf{K}(\boldsymbol{\rho})}{\partial \rho_{el}} \mathbf{U}(\boldsymbol{\rho}) \tag{3}$$

If nodal design variables are applied there must be a function relating the element densities **ρ** to the design variables **x** and the sensitivity of the objective function with respect to the design variables is evaluated by eq. (4).

$$\frac{\partial c(\boldsymbol{\rho})}{\partial x_j} = \frac{\partial c(\boldsymbol{\rho})}{\partial \rho_{el}} \frac{\partial \rho_{el}}{\partial x_j} \tag{4}$$

2.2 Formulation for free vibration

In the free vibration problems in topology optimization, the objective is to maximize the first natural frequency of the structure, associated to a compliant behavior. Bendsoe and Sigmund [9], point out that the solution thus obtained is very likely to be also suitable for static problems. The problem is presented in eq. (5).

$$\begin{aligned}
&\text{Obtain}: && \mathbf{x} \\
&\text{Maximizing}: && \omega_1^2(\boldsymbol{\rho}(\mathbf{x})) = \frac{\boldsymbol{\Phi}_i^T \mathbf{K}(\boldsymbol{\rho}(\mathbf{x}))\ \boldsymbol{\Phi}_i}{\boldsymbol{\Phi}_i^T \mathbf{M}(\boldsymbol{\rho}(\mathbf{x}))\ \boldsymbol{\Phi}_i} \\
&\text{Such that}: && \sum \rho_{el}\ V_{el} = f\ V_d \\
& && 0 < x_{\min} \le x_j \le 1 \\
&\text{With}: && E_{el} = E\left(\rho_{el}, E_s\right) \\
& && (-\omega_1^2\ \mathbf{M}(\boldsymbol{\rho}(\mathbf{x})) + \mathbf{K}(\boldsymbol{\rho}(\mathbf{x})))\ \boldsymbol{\Phi}_1 = \mathbf{0}
\end{aligned} \tag{5}$$

Where: ω_1 is the natural frequency of the structure; **K** is the structure stiffness matrix; **M** is the mass matrix; **Φ** is the eigenvector of the vibration modes.

The sensitivity of the objective function with respect to the element densities is given by eq. (6). If element-based design variables are applied with no projection scheme, the design variables are equal to the element densities and Eq. (6) represents the sensitivity of the objective function with respect to the design variables as well.

$$\frac{\partial \omega_i^2}{\partial \rho_{el}} = \boldsymbol{\Phi}_i^T \left(\frac{\partial \mathbf{K}}{\partial \rho_{el}} - \omega_i^2 \frac{\partial \mathbf{M}}{\partial \rho_{el}} \right) \boldsymbol{\Phi}_i \tag{6}$$

If nodal design variables are applied the sensitivity of the objective function with respect to the design variables is evaluated by eq. (4).

2.3 Formulation for non-damped forced vibration

For structures under non-damped forced vibrations problems, the objective of the topology optimization is to maximize the dynamic compliance. The formulation described by Olhoff and Du [2] is presented in eq. (7).

$$\begin{aligned}
&\text{Obtain}: && \mathbf{x} \\
&\text{Minimizing}: && C_d(\boldsymbol{\rho}(\mathbf{x})) = \left(\mathbf{P}^T\ \mathbf{U}(\boldsymbol{\rho}(\mathbf{x}))\right)^2 \\
&\text{Such that}: && \sum \rho_{el}\ V_{el} \le f\ V_d \\
& && 0 < x_{\min} \le x_j \le 1 \\
&\text{With}: && E(\rho) = \rho^p\ E_s \\
& && (-\Omega^2\ \mathbf{M}(\boldsymbol{\rho}(\mathbf{x})) + \mathbf{K}(\boldsymbol{\rho}(\mathbf{x})))\ \mathbf{U}(\boldsymbol{\rho}(\mathbf{x})) = \mathbf{P}
\end{aligned} \tag{7}$$

Where: C_d is the dynamic compliance; **P** is the vector of the amplitudes of the harmonic force; and Ω is the excitation frequency.

The sensitivity of the objective function with respect to the element densities is given by eq. (8). If element-based design variables are applied with no projection scheme, the design variables are equal to the element densities and Eq. (6) represents the sensitivity of the objective function with respect to the design variables as well.

$$\frac{\partial C_d}{\partial \rho_{el}} = \left(2\ \mathbf{P}^T\ \mathbf{U}\right)\left(-\mathbf{U}^T\ \left(\frac{\partial \mathbf{K}}{\partial \rho_{el}} - \omega^2\ \frac{\partial \mathbf{M}}{\partial \rho_{el}}\right)\ \mathbf{U}\right) \tag{8}$$

If nodal design variables are applied the sensitivity of the objective function with respect to the design variables is evaluated by eq. (4).

3 Projection schemes

Topology optimization problems solved using the density method are subject to numerical instabilities, such as checkerboard patterns and mesh dependency. A widely used technique used to overcome these instabilities is the so-called sensitivity filter. However, as much as the sensitivity filter bypasses such instabilities, the final topology still has a considerable number of elements with intermediate densities. In this work, projection schemes were used with the smoothed Heaviside projection function proposed by Guest *et al* [5]. By this approach, the design variables are associated with the nodes of the finite element mesh that discretizes the extended domain and related to the element density through the projection function represented in eq. (9). The density of the element is evaluated taking into account the influence of the nodes within a circle Ω_{el} of radius r_{min} through the distances of the nodes within the circle to the centroid of the element.

$$\rho_{el} = 1 - \mathrm{e}^{-\beta\, \mu_{el}} + \mu_{el}\ \mathrm{e}^{-\beta} \tag{9}$$

Where: β is the parameter that controls the curvature of the function; μ^{el} is the linear projection function, given by eq. (10),

$$\mu_{el} = \frac{\sum_{j\in S} x_j\, w(r, r_{\min})}{\sum_{j\in S} w(r, r_{\min})} \tag{10}$$

Where: μ_{el} is the linear projection function associated to element *el*; r_{min} is the radius of the region Ω_{el}, *S* is the set of nodes belonging to Ω_{el}; x_j is the design variable associated with the nodes belonging to *S*; *w* is the weight of node j in relation to the centroid of the element el erj distance from node j to the centroid of the element el.

$$w(r_j, r_{\min}) = \begin{cases} \dfrac{r_{\min} - r_j}{r_{\min}}, & \text{if } j \in \Omega_{el} \\ 0, & \text{if } j \notin \Omega_{el} \end{cases} \tag{11}$$

An additional instability present in topology optimization problems in which eigenvalues and eigenvectors are the objective or the constraint of the optimization is related to modes of vibration in void regions of the design domain. These vibration modes spuriously influence the final solution of the optimization process. In the classic SIMP model, the mass is not penalized as the stiffness is. Thus, the ratio between mass and stiffness is very high for small densities. The solution presented by Pedersen [10] is to change the penalty scheme for intermediate densities, changing the penalty for densities less than 0.1, as shown in eq. (12).

$$\text{stiffness} \begin{cases} \rho_{el}^3 \ \text{ for } 0{,}1 \le \rho_{el} \le 1 \\ \dfrac{\rho_{el}}{100} \ \text{ for } \rho_{\min} \le \rho_{el} \le 0{,}1 \end{cases} \tag{12}$$

Tcherniak [11] complemented the proposal presented by Pedersen [10] and changed not only the rigidity penalty, but the mass penalty, according to eq. (13). Tcherniak [11] adopted 0.15 for ρ_{thr}.

$$m_e = \begin{cases} m_s\ \rho_{el},\ \rho_{el} > \rho_{thr} \\ 0, \qquad \rho_{el} < \rho_{thr} \end{cases} \tag{13}$$

Where: m_s is the mass for solid material; ρ_{thr} is the density chosen as a parameter, and m_e is the new mass of the element *el*.

4 Examples and results

The first example in this section presents the results obtained for a beam under free vibration. Two support conditions are analyzed: studded at the left side and simply supported at the right side; and simply supported at both sides. The second example is a cantilever beam under forced vibration. In all examples in this section the Young's module is 10^5 and the Poison ratio 0.3.

4.1 Structure under free vibration

In the next two examples the length of the beam (L) is 6 times its height (H), discretized in a mesh with 240x40 elements. Continuity was applied as proposed in [12], for a fixed β equal to 15 and p varying from 1 to 10.

The first example is the simple supported beam presented in Fig. 1 with results presented in Fig. 2.

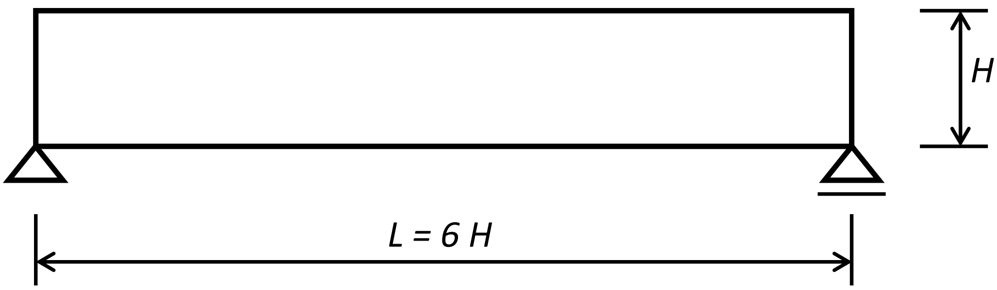

Figure 1. Structural domain for the simple supported beam

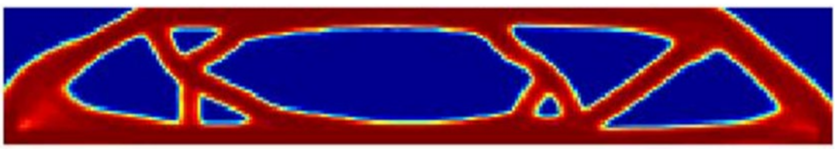

Figure 2. Final topology for the simple supported beam under free vibration (mesh with 240x40 elements and r_{min} = 4 elements)

The natural frequency for the topology presented in Figure 2 is 0,176 rad/s and the rate of elements presenting intermediate densities is 16.69%.

The second example is the cantilever – supported beam presented in Fig. 3 with results presented in Fig. 4.

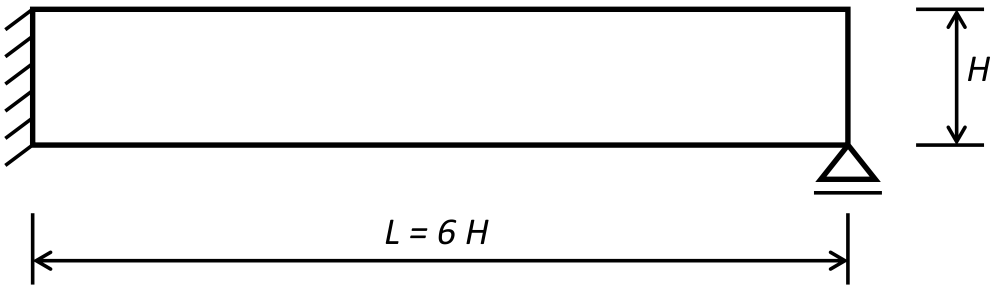

Figure 3. Structural domain for the cantilever – supported beam

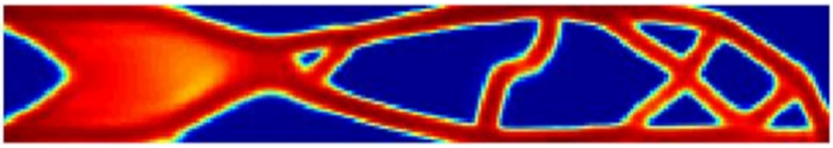

Figure 4. Final topology for the cantilever –supported beam under free vibration (mesh with 240x40 elements and r_{min} = 4 elements).

The natural frequency for the topology presented in Figure 4 is 0.237 rad/s and the ratio of elements presenting intermediate densities is 38.39%. The change in support conditions substantially affected the rate of elements with intermediate densities.

4.2 Structure under non-damped forced vibration

A cantilever beam with length (*L*) 1.5 times its height (*H*), discretized in a mesh with 60x40 elements. Continuity was applied as proposed in [7], for p varying from 1 to 3 and $\beta = 1$ in the first step and β varying at each iteration *k* as follows: $\beta = 1.1 + k$.

The cantilever beam presented in Fig. 5(a) was analyzed for three frequencies: 0.1 rad/s (Fig. 5(b)); 5 rad/s (Fig. 5(c)); 10 rad/s (Fig. 5(d)).

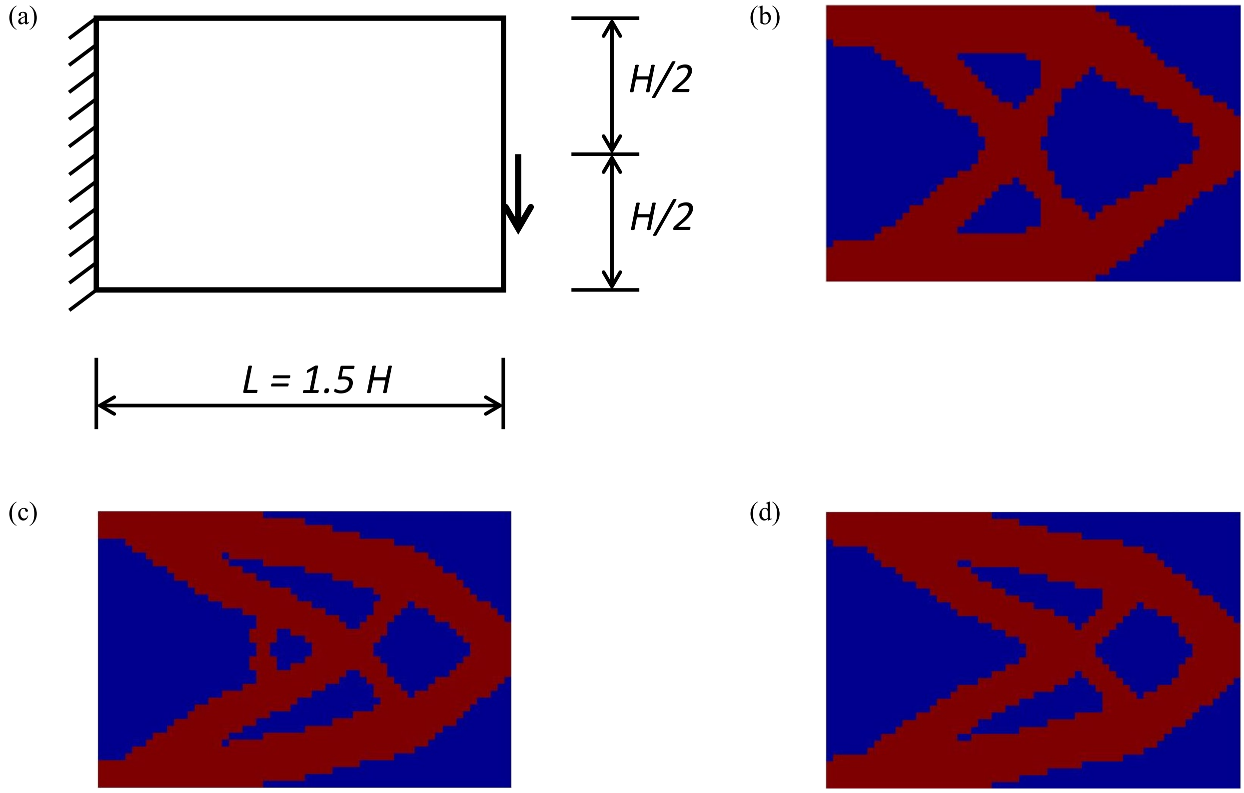

Figure 5. Cantilever beam under forced vibration: (a) structural domain; (b) topology for ω = 0.1 rad/s; (c) topology for ω = 5 rad/s; (d) topology for ω = 10 rad/s.

The topology obtained for ω = 0.1 rad/s is consistent with that obtained for the structure under a static load, which validates the implementation. The results in Fig. 5(c) and 5(d) shows the influence of the frequency over the final topology. All the results present no elements with intermediate densities.

5 Conclusions

In the application of the Heaviside projection to a free vibration problem the solutions presented intermediate densities, which was expected since a relatively low value was adopted for the parameter β. Further studies are needed regarding the use higher values for this parameter. The rate of intermediate densities is affected by the support conditions.

The results obtained for non-damped forced vibration were satisfactory. The final topologies obtained are free of intermediate density. As expected, the final topologies were noticeable different when changing the value of the excitation frequency of the harmonic force.

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