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Vertical Dynamics of a Half-car Computational Model

L. T. Matos[1], N. S. C. Seguins[1], V. S. Pereira[1], D. I. G. Costa[1]

[1]*Dept. of Mechanical Engineering, Federal University of Maranhão*
Avenida dos Portugueses, 1966, 65080-805, São Luís/Maranhão, Brazil
teixeira_matos@hotmail.com, seguins.neilton@gmail.com, vilson.sp@ufma.br, dalmo.costa@ufma.br

Abstract. Vertical dynamics studies of a vehicle can be used to obtain responses to imperfections of a track. Closely related to comfort issues, this analysis is affected mainly by the vertical movement (bounce) and alternating oscillations such as roll and pitch. An off-road vehicle model for student competitions, such as SAE Mini-Baja, aims to challenge students to design and build vehicles that will be put to test in tracks with critical conditions. The main goal of this work is to analyze the vertical dynamics of an off-road vehicle prototype, using dynamic modeling and numerical simulation. A half car 2-D vehicle model is presented in this paper. An analysis of the system's damped forced vibration was performed using a track function as input, simulating the disturbances that the vehicle's suspension suffers during competition. From this type of analysis, it was possible to obtain important data, linked to safety, handling, driver comfort and car structure. The results revealed that factors such as stiffness and damping of the suspension set can generate unwanted vibration frequencies, discomfort to the pilot, in addition to compromising the structure and stability of the vehicle. Additionally, the results obtained are in accordance with the literature.

Keywords: Dynamic modeling, Vertical dynamics, Forced vibration

1 Introduction

A system is classified as dynamic when its state varies with time, and static when there is no such variation. Dynamic systems modeling is associated with product development process activities, linked to the preliminary phases of the project [1]. Dynamic modeling and simulation deals with the representation of physical systems that undergo changes over time, and the knowledge of such changes allows the designer to verify and direct the product to the desired performance with the characteristics that it must present when completed.

The use of dynamic modeling is increasing, being motivated by factors that aim at the continuous improvement of products and processes, the search for competitive advantages such as greater efficiency and lower cost, in addition to enabling the construction of models that are increasingly closer to reality. With the development of computational tools, as hardware, such as high performance computers, and software, such as the various numerical analysis platforms, the use of simulation tools in product development has become more accessible and has had a high dissemination [1].

These computer simulation techniques allow to obtain the response of a vehicle's suspension set when subjected to an excitation condition, such as the profile of a track through which the vehicle travels. This approach requires the use of mathematical models that accurately represent the system studied, according to Vilela [2]. Several researches are carried out in order to improve the suspension of a vehicle. Huang et al. [3], implemented an active suspension control that adapts online to different track profiles through road-adaptive algorithm schemes. Prabakar et al. [4] and Kamalakkannan et al. [5], performed an optimization of the vehicle response to strong track excitations using magnetoreological dampers. Nkomo et al. [6] features a design and implementation of an efficient active suspension controller for a built half-car model. There are also models where it is possible to obtain the track profile precisely, aiming to obtain excitation functions that are closer to reality, as presented by Xue et al. [7].

This paper uses an off-road vehicle from Baja SAE category. This type of vehicle was created in 1976 by the Society of Automotive Engineers (SAE) as an academic competition category, where students are challenged to design and build vehicles to be put to test in with severe conditions. This challenge motivates the constant search for improvements in the development of the project, aiming at better performance, lower cost and greater competitiveness.

In this way, it is understood that the constant search for improvements in prototypes generates a need for studies, aiming to reduce stages of the project when analyzing the behavior of pertinent variables of the prototype model. For example, the vibration frequencies are one of the most important variables of the project, in which a incorrect analysis can lead to catastrophic failures during competition tests. Within this perspective, the present work seeks to analyze the vertical dynamics of an off-road vehicle prototype in the Baja SAE category, with the assistance of dynamic modeling and numerical analysis software.

2 Vertical dynamics

When driving a motorized vehicle some factors can be identified as sources of vibration, such as component looseness and aerodynamic forces, but the excitations caused by the road are the most prominent factor [8].

The vertical dynamics studies the vehicle's responses to the most varied imperfections of the track. It is directly linked to comfort, which is affected by vertical movement, that is, the tendency of the car to jump vertically, also called bounce, and the alternating oscillations between the axes, called roll and pitch, which are characterized by the body's rocking movement around the Z and X axes, respectively, as shown in Fig. 1.

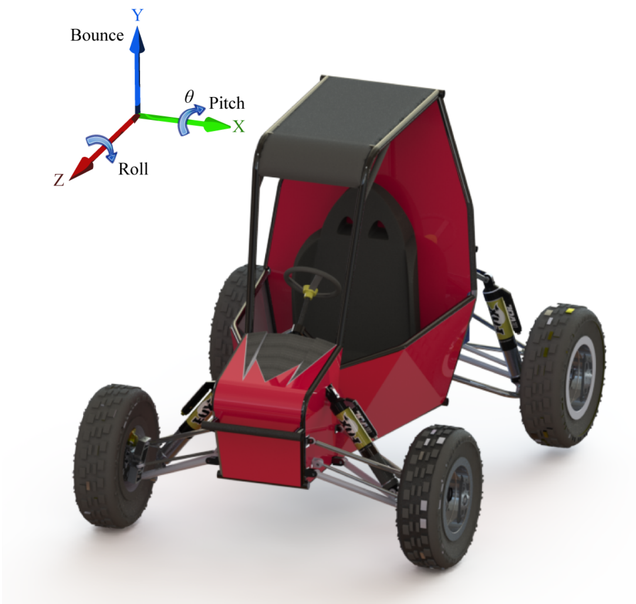


Figure 1. Rotation axes and vertical movement of the vehicle

Vertical dynamics studies the vibration control of sprung and unsprung masses, with the objective of optimizing parameters to minimize the vibrations transmitted to the pilot and to optimize driveability. Duarte et al. [9] states that the study is usually divided into three parts, the excitations to which the vehicle is subjected, the modeling of vehicle's behavior in relation to the excitations and the human perception and tolerance to these vibrations.

The ability of the vehicle suspension system to isolate the vehicle body from vibrations from the road is closely linked to passenger comfort [8]. According to Gillespie [10], the perception of vibration is one of the main criteria used by people to judge the quality of construction and design of a vehicle, however, because it is a subjective criterion, there is a great difficulty in analyzing it. However, as shown by Diniz et al. [11], vibrational tolerance is quantified by the results of acceleration by frequency. The ISO Central Secretary [12] provides a tolerance graph where it is possible to check the time limit in which a person can be subjected to a certain level of vibration without discomfort and loss of mental calculation.

The optimization of parameters related to vertical dynamics is essential to ensure high performance of off-road vehicles in competitions, where designers give up vehicular comfort to obtain gains in handling.

3 Vehicle modeling

A model with 2 degrees of freedom, known in the literature as a half-car, was used. The car was modeled as a rigid body of total mass *M* (sprung mass) and moment of inertia of mass J_0 in relation to its center of gravity (**CG**), supported by springs where k_d and k_t are front and rear stiffness coefficient from tires and suspension, and c_d e c_t represents front and rear damping coefficient, as shown in Fig. 2.

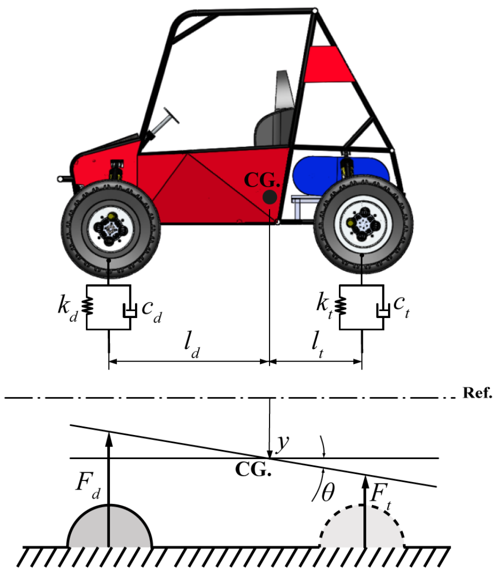


Figure 2. Representation of the vehicle as a rigid body and its free-body diagram

Figure 2 illustrates the modeled system, where F_d and F_t are the forces imposed by the springs, $y(t)$ the vertical displacement suffered by the CG as a function of time, l_d and $l_t$the distances of the applying forces to the **CG** and $\theta(t)$ the rotation of the rigid body as a function of time. For simplification of calculation, it is adopted for small angles $\sin\theta = \theta$. Through the equations of equilibrium forces in the vertical direction ($+\downarrow \sum F_y = m\ddot{y}$), we can write:

$$M\ddot{y} + (c_d + c_t)\dot{y} + (c_t l_t - c_d l_d)\dot{\theta} + (k_d + k_t)y + (k_t l_t - k_d l_d)\theta = k_d y_d + k_t y_t + c_d \dot{y}_d + c_t \dot{y}_t, \tag{1}$$

where y_d e y_t are the input functions that represent the track through which the car will travel. From the moment equation in relation to the **CG** ($\sum M_{CG} = J_0\,\ddot{\theta}$) and adopting the clockwise direction as positive, we have:

$$J_0\ddot{\theta} + (c_t l_t - c_d l_d)\dot{y} + (k_t l_t - k_d l_d)y + (c_d l_d^2 + c_t l_t^2)\dot{\theta} + (k_d l_d^2 + k_t l_t^2)\theta = -k_d l_d y_d + k_t l_t y_t + c_t l_t \dot{y}_t - c_d l_d \dot{y}_d. \tag{2}$$

The system of equations can then be represented in matrix form, given in:

$$\begin{aligned}
\begin{bmatrix} M & 0 \\ 0 & J_0 \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \ddot{\theta} \end{Bmatrix} &+ \begin{bmatrix} c_d + c_t & c_t l_t + c_d l_d \\ c_t l_t - c_d l_d & c_d l_d^2 + c_t l_t^2 \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{\theta} \end{Bmatrix} \\
&+ \begin{bmatrix} k_d + k_t & k_t l_t + k_d l_d \\ k_t l_t - k_d l_d & k_d l_d^2 + k_t l_t^2 \end{bmatrix} \begin{Bmatrix} y \\ \theta \end{Bmatrix} = \begin{bmatrix} k_d & k_t \\ -k_d l_d & k_t l_t \end{bmatrix} \begin{Bmatrix} y_d \\ y_t \end{Bmatrix} \\
&+ \begin{bmatrix} c_d & c_t \\ -c_d l_d & c_t l_t \end{bmatrix} \begin{Bmatrix} \dot{y}_d \\ \dot{y}_t \end{Bmatrix}.
\end{aligned} \tag{3}$$

3.1 Natural frequencies

The frequency of free vibration at which the system will oscillate after receiving a disturbance is called the natural frequency. It is important to be aware of the natural frequencies of the mechanical systems that make up the vehicle, in order to avoid the resonance phenomenon. In this way, vehicle comfort and stability is guaranteed [13].

For a simplified system with two degrees of freedom, there are two normal modes of vibration associated with two natural frequencies. Thus, for the calculation of natural frequencies, the system is considered to be in non-damped free vibration. In this way, the second term of eq. (3) is canceled. Assuming that the solutions to the eqs. (1) and (2) are in the form:

$$y(t) = Y\cos(\omega t + \phi) \quad and \quad \theta(t) = \Theta(\omega t + \phi), \tag{4}$$

we get,

$$\begin{bmatrix} -M\omega^2 + k_d + k_t & -k_d l_d + k_t l_t \\ -k_d l_d + k_t l_t & -J_0\omega^2 + k_d l_d^2 + k_t l_t^2. \end{bmatrix} \begin{Bmatrix} Y \\ \Theta \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{5}$$

Assuming that the determinant of the Y and Θ coefficients must be equal to 0, we find the two natural frequencies of the system, ω_1 and ω_2, through the roots of the resulting equation.

From eq. (5), we can determine the amplitude ratio for $\omega_{1,2}$ by:

$$\frac{Y}{\Theta} = \frac{-(k_t l_t - k_d l_d)}{-M\omega^2 + k_d + k_t}. \tag{6}$$

The amplitude ratio gives the distances from the vehicle's center of gravity to the oscillation center, as illustrated in Fig. 3.

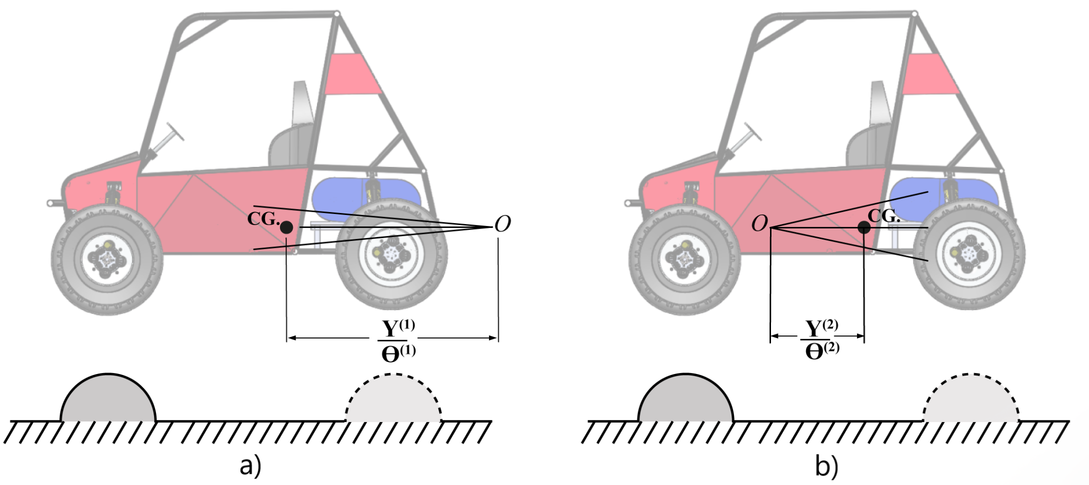


Figure 3. Vehicle amplitude ratios associated with bounce a) and pitch b) motion

According to Gillespie [10], if the center of oscillation *O* is outside the wheelbase, the movement is characterized as bounce and the associated frequency is the bounce frequency Fig. 3 (a). If the center of oscillation is in the wheelbase range, the movement and associated frequency represents the pitch motion, as we see in Fig. 3 (b).

As Merling [14] states, the bounce frequency should not be more than 1.2 times higher than the pitch frequency, keeping the frequencies in the range of 1 to 1.5 Hz, preventing "interference kicks" from occurring by overlapping the movements. However, race cars tend to have natural frequencies in the order of 1.67 and 2.1 Hz, giving up comfort for driveability. The most requested racing cars, on the other hand, tend to have values in the range of 2.1 and 2.9 Hz. Frequency values less than 0.83 Hz cause a very large oscillation of the mass, and the vehicle may overturn.

3.2 Suspension

For the design of a vehicle, it is necessary to experimentally characterize suspension parameters, such as the stiffness of the springs, tires and damping coefficients. These tests are generally performed on testing machines, as performed by Diniz et al. [15]. In these approaches, tire damping is generally disregarded, since its value is minimal compared to the influence of other parameters. Another important factor to consider is the Olley criterion, which states that the rear suspension must have a stiffness of at least 30% greater than the front.

As mentioned by Samant et al. [16], in racing cars the suspension must absorb oscillations around 0.2 to 0.5 seconds after receiving an excitation force. This characteristic is directly related to the damping ratio of the suspension set. Gillespie [10] states that an ideal damping ratio falls between 0.3 and 0.4, because in this range the suspension has low amplification values at resonant frequencies. The damping ratio for the front and rear suspension set can be approximated by:

$$\zeta_{d,t} = \frac{c_{d,t}}{\sqrt{4k_{d,t}(m/2)}}. \tag{7}$$

4 Numerical Results

The simulation carried out in this work consisted of gathering experimental parameters obtained in the literature and applying it to the Baja prototype of the team from Federal University of Maranhão (UFMA), in order to evaluate optimal values for the construction of the vehicle.

The parameters used in the analysis are shown in Tab. 1. Tire inflation pressures and spring stiffness were obtained from tests carried out by Diniz et al. [11]. The chassis geometry and mass data are from the model under development by the UFMA Baja team, in accordance with the competition regulations. To obtain the front and rear damping constants, eq. (7) was used, adopting a damping ratio of 0.4 for the front and rear suspension set.

Table 1. Parameters adopted for modeling

Parameters	Description	Value
k_d	Equivalent front wheel stiffness	9859.46 N/m
k_t	Equivalent rear wheel stiffness	18402.70 N/m
c_d	Front damping coefficient	571.44 Ns/m
c_t	Rear damping coefficient	780.70 Ns/m
M	Mass of half car	103.5 kg
l_d	Front wheel distance to CG	0.708 m
l_t	Rear wheel distance to CG	0.630 m
J_0	Moment of inertia	64.35 kgm^2
V	Vehicle speed	20 km/h
ω_f	Excitation frequency	87.26 rad/s

According to the BAJA SAE regulation, one of the obstacles to be overcome by vehicles are 40 cm diameter trunks placed on the track. Therefore, for the analysis carried out in this work, it was considered as excitation for the front and rear wheels, a sinusoidal pulse with amplitude of 0.4 m, simulating the action of the trunk on the vehicle, as shown in Fig. 4. For the pulse on the rear wheels, a lag of approximately 0.28 s was considered with respect to the front wheels. Time which it takes for the rear wheels to touch the obstacle after the front wheels.

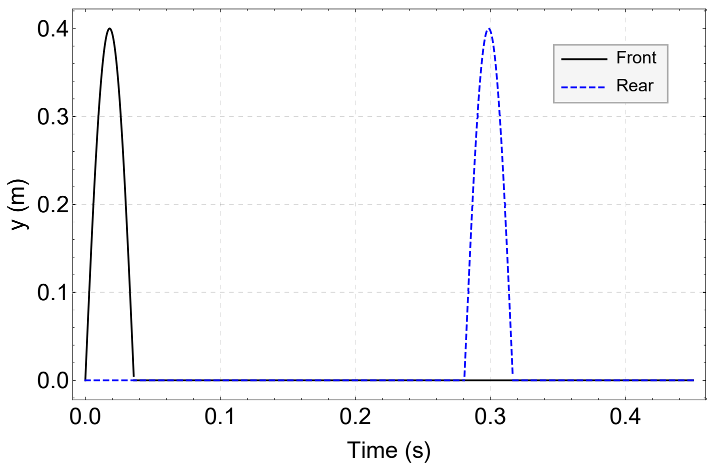
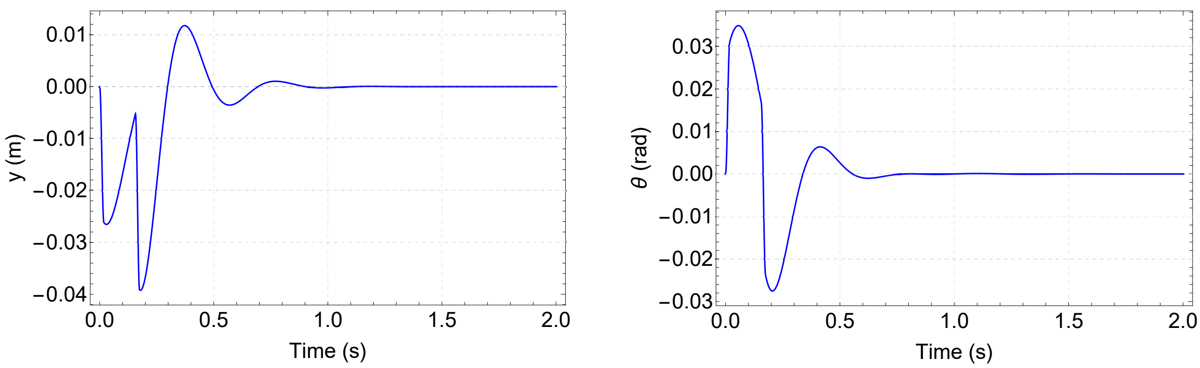


Figure 4. Track excitation function

Figure 5, shows the behavior of bounce and pitch motion of the vehicle, both around its CG. We noticed that initially the CG of the vehicle rotates in the positive direction and vertical displacement in the negative direction, that is, when the vehicle collides with the obstacle, it tilts backwards and its center of gravity moves upwards. This behavior is expected for a vehicle when trying to overcome an obstacle. In addition, we note that after the pulse hits the rear wheel, the suspended mass takes around 0.75 seconds to stabilize.



(a) Vertical displacement response (bounce)

(b) Rotation response (pitch)

Figure 5. Vehicle response to track excitation function

Other information that we can extract from the simplified model are the values obtained through eq. (5), which provides the values of the natural frequencies ω_1 = 2.02 Hz, ω_2 = 2.76 Hz and the amplitude ratios of - 0.40 m and 1.55 m, for pitch and bounce respectively. This shows us that ω_1 is the pitch frequency and ω_2 is the bounce frequency, as shown in Fig. 3. These values are in line with the values adopted for racing vehicles [14].

5 Conclusions

In this work we propose a simplified approach to numerical modeling of the suspension set of a BAJA off-road vehicle. The results obtained show us that the system behaves as expected and the proposed modeling is satisfactory. The values of natural frequencies found allow for better vehicle handling and maintain safe driving passing through obstacles in the competition, despite vehicular comfort being disregarded for these types of vehicles.

This analysis using the simplified half-car model is an initial approach to a more complex model. Experimental responses must be obtained in the future to validate the simulations performed in this work.

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