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A COMPARISON OF A NON-LINEAR PENDULUM ABSORBER 2D AND 3D

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Abstract. Designed to reduce the vibrations amplitudes in the horizontal direction of slender structures and industrial components subjected to dynamic excitations, such as wind and earthquakes, a non-linear pendulum control system is assessed in this paper. Firstly, 2D and 3D mathematical models are developed based on the Lagrange's energy equations. The system of non-linear differential equations of motion is numerically solved in time domain. Some parametric studies are performed to understand the role of some key physical characteristics of the control device in its effectiveness in controlling the vibration amplitudes of a structure under certain load scenarios, including the pendulum mass, damping ratio and length. Finally, the results obtained with two perpendicular 2D simplified pendulum model is compared to the results obtained with the 3D pendulum model in order to evaluate the performance in the use of the simplified approach to design this type of passive control system.

Keywords: Passive dynamic control devices, Nonlinear pendulum controller, Coupled oscillators.

1 Introduction

Passive dynamic devices have been used to control undesirable large amplitude motions in structures like bridges, tall buildings and industrial components subjected to environmental actions (like wind and earthquakes) and mechanical vibrations induced by machines [1],[2],[3]. These are efficient and low cost control devices which require low maintenance [1]. Among passive control devices, the nonlinear spherical pendulum is the most adequate to control horizontal vibration amplitudes in planar motion because it has identical natural frequencies in any radial direction, suppressing vibrations of slender and tall structures [4]. According to Battista *et al*. [5], this type of passive controller is the most appropriate to attenuate amplitudes of the low frequency bending mode of tubular tower structures of circular cross section.

The spherical pendulum consists of a mass attached to the tip of a rod linked to the structure by elastic and damping elements [6]. The operating principle of this device is to generate inertia forces that oppose the structure's motion [7].

The performance of a nonlinear pendulum device is influenced by many factors; being therefore somewhat complex to study its dynamic behavior by means of an analytical model. In this way, some studies have used numerical approach to obtain the optimum parameters of the oscillator [2],[8]. Gerges and Vickery [9] reported an increase in the reduction of vibration's amplitudes of the structure with the increase of the pendulum mass, however, the authors used a linear assumption to the mathematical model.

In this paper, a parametric study was performed to understand the effectiveness of the nonlinear pendulum in control the vibration amplitudes of an idealized structure highlighting the role of the pendulum mass, length and damping ratio. The nonlinear 2D and 3D mathematical pendulum models detailed in this work were previously developed by Pinheiro [10] and Pinheiro and Battista [5]. Successful practical application of the 3D model was made by Battista [11] in several telecommunications towers. The numerical results obtained in the presented parametric study serve to show the performance of the pendulum to attenuate the vibration amplitudes of the structure and important conclusions could be drawn. Moreover, the good correlation between the controlled responses yielded by 2D and 3D models in the proposed structure indicate that a pair of simplified 2D mathematical model could be used instead of the 3D model.

2 2D non-linear pendulum

The mathematical model to consider the coupled structure-pendulum is represented by a spring-mass-damper system indicated in Figure 1. The proposed formulation is based on the dynamic equilibrium of the system utilizing the Lagrange's Equation like shown in Eq. (1):

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial E_d}{\partial \dot{q}_i} = Q_i \tag{1}$$

where T is the kinetic energy, V is the potential energy, E_d is the dissipation energy , Q_i is the external force and q_i is the generalized coordinate of the system.

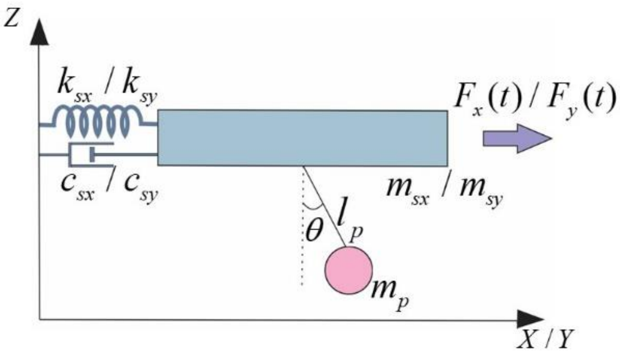

Figure 1. 2D representation of the pendulum-structure system in a generic direction: X or Y

The energy contributions in Eq. (1) can be written as follows in Eqs. (2), (3) and (4):

$$T = \frac{1}{2}m_s\dot{x}(t)^2 + \frac{1}{2}m_p v(t)^2 \tag{2}$$

in which, v is the tangential speed of the pendulum, m_s is the modal mass of the structure, $\dot{x}$ is the velocity of the structure (assuming that the direction X is analyzed) and m_p is the pendulum mass.

$$V = \frac{1}{2}k_s x(t)^2 + m_p g h + \frac{1}{2}k_p\theta(t)^2 \tag{3}$$

$$E_d = \frac{1}{2}c_s\dot{x}(t)^2 + \frac{1}{2}c_p\theta(t)^2 \tag{4}$$

whereas the terms k_s and c_s are the modal stiffness and the modal damping of the structure, respectively, x is the displacement of the structure and θ is the rotation of the pendulum mass. k_p is the pendulum stiffness, c_p is the damping of the pendulum, g is the gravity acceleration and h is the height reached by the pendulum.

Assuming x as the generalized coordinate of the structure and θ as the generalized coordinate of the pendulum and deriving Eqs. (2), (3) and (4) in relation to the structure's DoF, it becomes:

$$\frac{\partial T}{\partial \dot{x}(t)} = m_s\dot{x}(t) + m_p[\dot{x}(t) + l\dot{\theta}(t)\cos\theta(t)] \tag{5}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}(t)} = m_s\ddot{x}(t) + m_p[\ddot{x}(t) + l\ddot{\theta}(t)\cos\theta(t) - l\dot{\theta}(t)^2 sin\theta(t)] \tag{6}$$

$$\frac{\partial T}{\partial x(t)} = 0 \tag{7}$$

$$\frac{\partial V}{\partial x(t)} = k_s x(t) \tag{8}$$

$$\frac{\partial E_d}{\partial x(t)} = c_s x(t) \tag{9}$$

Rearranging Eqs. (5) to (9) according to the Lagrange's formulation, Eq. (1), and considering a generic external force $F(t)$. The structure's motion can be expressed as in Eq. (10):

$$[m_s + m_p]\ddot{x}(t) + c_s\dot{x}(t) + k_s x + m_p l[\ddot{\theta}(t)\cos\theta(t) - \dot{\theta}(t)^2 sin\theta(t)] = F(t) \tag{10}$$

Similarly to the structure, the equation of the pendulum motion can be determined by the partial derivatives of the energy equations in relation to θ , as indicated in Eq. (11):

$$m_p l^2\ddot{\theta}(t) + c_p\dot{\theta}(t) + k_p\theta(t) + m_p glsin\theta(t) + m_p l\ddot{x}(t)\cos\theta(t) = 0 \tag{11}$$

Rewriting Eqs. (10) and (11) in the matrix form:

$$\begin{bmatrix} (m_s + m_p) & m_p l\cos\theta \\ m_p l\cos\theta & m_p l^2 \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_s & m_p l\dot{\theta}sin\theta \\ 0 & c_p \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_s & 0 \\ 0 & k_p \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{Bmatrix} 0 \\ m_p glsin\theta \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix} \tag{12}$$

It's important to note the non-linearity in the differential equation shown in Eq. (12). This non-linearity is presented both in the mass and damping matrices of the system.

3 3D non-linear pendulum

The mathematical model describing the coupling system structure-pendulum in spatial coordinates is given by a 4 DoF model, as shown in Figure 2. Two DoF, x and y , refer to the structural motion in the horizontal plan. The others describe the pendulum motion, where φ is the rotation angle measured from the Z-axis with the rod and θ is the angle formed between the rod projection in *xy* plan with the *x*-axis.

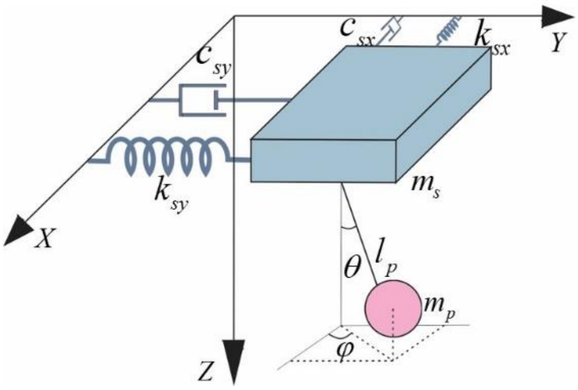

Figure 2. 3D representation of the pendulum-structure system

It is considered that the external forces, F_x and F_y, are actuating in two directions of the horizontal plan of the structure, such as turbulent wind, earthquakes and forces due to vibrations induced by machines and industrial components.

The coordinates of the system are varying at time and can be defined as *x*, *y* and *z*, where *x* and *y* refers to the structural displacement in the directions X and Y, respectively; and the *z* coordinate is relative to the displacements of the pendulum mass in the space as indicated in Eq. (13):

$$\begin{aligned} x &= x(t) + lsin\varphi \cos\theta \\ y &= y(t) + lsin\varphi sin\theta \\ z &= l\cos\varphi \end{aligned} \tag{13}$$

The kinetic energy, potential energy and dissipation energy of the system are given by the Equations (14), (15) and (16), respectively.

$$T = \frac{1}{2}m_{sx}\left(\frac{\partial}{\partial t}x(t)\right)^2 + \frac{1}{2}m_{sy}\left(\frac{\partial}{\partial t}y(t)\right)^2 + \frac{1}{2}m_p v^2 \tag{14}$$

where m_{sx} and m_{sy} are the modal mass of the structure in the X and Y direction, respectively.

$$U = \frac{1}{2}k_{sx}x(t)^2 + \frac{1}{2}k_{sy}y(t)^2 + m_p gh + \frac{1}{2}k_p\varphi(t)^2 \tag{15}$$

in which k_{sx} and k_{sy} are the stiffness in the X and Y direction of the structure, respectively.

$$E_d = \frac{1}{2}c_{sx}\left(\frac{\partial}{\partial t}x(t)\right)^2 + \frac{1}{2}c_{sy}\left(\frac{\partial}{\partial t}y(t)\right)^2 + \frac{1}{2}c_p\left(\frac{\partial}{\partial t}\varphi(t)\right) \tag{16}$$

The terms c_{sx} and c_{sy} in Eq. (16) are the modal damping of the structure in the X direction and Y direction.

Analogously to the 2D nonlinear pendulum proceedings, the energy equations are rewritten in function of the DoFs of the mechanical system indicated in Figure 2 and, then, it is derived in relation to the respective generalized coordinates (x , y , θ and φ).

Rearranging the partial derivatives of energy equations given in Eq. (1) and considering the generic external force actuating on the horizontal plan, the mathematical model to describe the motion of the 3D pendulum mass and the structure in a matrix form can be written as follows:

$$\begin{aligned} &\begin{bmatrix} (m_{sx}+m_p) & 0 & bcos\theta & -acos\theta \\ 0 & (m_{sy}+m_p) & bsin\theta & acos\theta \\ bcos\theta & bsin\theta & m_p l^2 & 0 \\ -asin\theta & acos\theta & 0 & m_p alsin\varphi \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\varphi} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_{sx} & 0 & -a\dot{\varphi}cos\theta & -a\dot{\theta}cos\theta \\ 0 & c_{sy} & -a\dot{\varphi}sin\theta & -a\dot{\theta}sin\theta \\ 0 & 0 & c_p & -alcos\varphi\dot{\theta} \\ 0 & 0 & 2al\dot{\theta}cos\varphi & alsin\varphi \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \\ \dot{\theta} \end{Bmatrix} + \\ &\begin{bmatrix} k_{sx} & 0 & 0 & 0 \\ 0 & k_{sy} & 0 & 0 \\ 0 & 0 & k_p & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \\ \varphi \\ \theta \end{Bmatrix} + \begin{Bmatrix} -2b\dot{\varphi}\dot{\theta}sin\theta \\ 2b\dot{\varphi}\dot{\theta}sin\theta \\ ag \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_x(t) \\ F_y(t) \\ 0 \\ 0 \end{Bmatrix} \end{aligned} \tag{17}$$

where, $a = m_p lsin\varphi$ and $b = m_p l\cos\varphi$.

Such as presented in Eq. (12), it is clear the strong non-linearity between the DoFs of the system in Eq. (17). It shows the interaction between the motions of the pendulum mass and the structure.

4 Description of the structure and excitation force

To evaluate the performance of the nonlinear vibration control proposed in this paper, the mathematical model equations are solved numerically with the Runge-Kutta method. As result, it provides the controlled response of the structure in the direction analyzed (X or Y) and the rotation of the pendulum mass in relation to Z axis, for the model 2D. In the 3D model, the results of solving numerically the mathematical equations provide the controlled response of the structure in the directions X and Y, besides, the rotation of the pendulum in relation to Z-axis and the plane *xy*.

The data of the idealized structure proposed to assess the effectiveness of the oscillator are indicated in Table

1. Moreover, the excitation force actuating on the structure is modelled as a sinusoidal force resonant with the structure.

$$F(t) = F_0 sin(2\pi f_e t) \tag{18}$$

where, F_0 is the static portion of the periodic load, as shown in Table 1 and f_e is the excitation frequency.

Table 1. Modal properties of the structure and data of force acting

X-direction		Y-direction	
Modal Properties	Value	Modal Properties	Value
m_{sx} [ton]	2500.0	m_{sy} [ton]	625.0
ξ_{sx} [%]	5.0	ξ_{sy} [%]	5.0
f_{sx} [Hz]	1.0	f_{sy} [Hz]	1.0
F_{0x} [kN]	50.0	F_{0y} [kN]	12.5

To compare the two numerical models proposed, a parametric study was carried out in the next section. It was considered that the 2D and 3D pendulum device are attached on the idealized structure. The response of the structure applying the mathematical model of the 3D non-linear pendulum in X and Y coupled directions was compared with the response of the structure applying the 2D non-linear mathematical model in X and Y direction separately. In addition, the parameters of the pendulum such as its mass, damping ratio and length of the rod were investigated with emphasis on the role of the reduction factor, defined here as the ratio between the controlled and uncontrolled response of the structure in terms of its maximum displacements.

5 Parametric study

5.1 Pendulum mass

In order to evaluate the performance of the pendulum in reducing the vibrations amplitudes in both planar horizontal directions in relation to its mass variation, the length of the rod $(l = 0.25m)$ and the damping ratio $(\xi_p = 0.01)$ were kept constant. The results achieved show a great reduction in the structure's vibrations amplitudes considering mass ratios (m_p / m_{sx}) of just 0.5-1.0% between the pendulum mass and the modal mass of the structure in X-direction; this is shown in Figure 3:

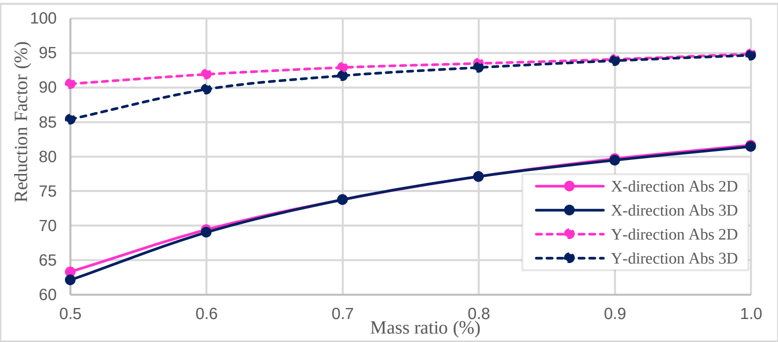

Figure 3. Effectiveness of the pendulum considering its mass variation

It is important to highlight the asymptotic behavior of the curves showed in Figure 3. The higher the mass of the pendulum, the greater is its effectiveness in reducing the vibration amplitudes up to a certain value of the mass ratio. However, the available space for the installation of the control device and the available budget are parameters to be considered in the design of the pendulum.

5.2 Pendulum lenght

The greatest reduction in the vibration amplitudes of the structure are obtained when the pendulum is resonant with the structure. Thus, considering that the length of the control device has an important role in the natural frequency of the pendulum device (ω_p), a previous analysis of the range values of this variable were performed considering the expression (19) deduced by Pinheiro [10]:

$$\omega_p = \sqrt{\frac{k_p / m_p + gl}{l^2}} \tag{19}$$

The above equation is determined regarding the 2D mathematical model. Nevertheless, it does not take into account the damping and non-linearity of the pendulum. Furthermore, considering that the pendulum's stiffness is low $(k_p \cong 0)$, the natural frequency of the pendulum can be approximated by the natural frequency of the simple pendulum.

The reduction factor of the structure's vibration amplitudes in relation to the length of the pendulum rod is shown in Figure 4. The pendulum mass and damping ratio are $0.5\% m_{sx}$ and 1.0%, respectively. It can be observed good correlation between the 2D and 3D mathematical model of the controller. It is seen then that Eq. (19) may well be used to estimate the natural frequency of the 3D pendulum.

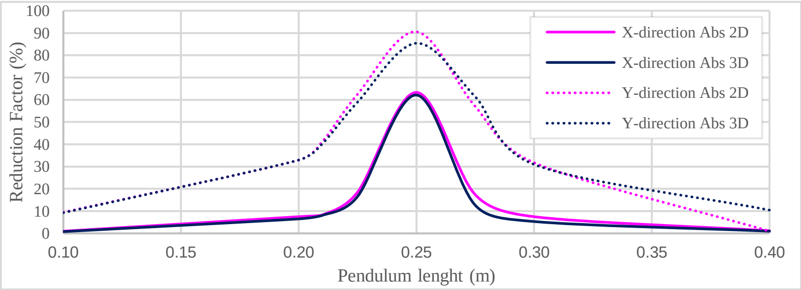

Figure 4. Effectiveness of the pendulum considering its length variation

5.3 Pendulum damping

To determine the influence of the damping ratio of the pendulum in the vibration amplitudes of the structure, the mass $(m_p = 0.5\% m_{sx})$ of the pendulum and the length of the rod $(l = 0.25m)$ are fixed. The optimum value was determined for a damping ratio less than 1%, both in the X and Y direction and in the two models analyzed, as shown in Figure 5. However, in the range of values analyzed, the 2D and 3D mathematical model presented a significant difference in the reduction factor in Y-direction, especially for low damping ratio.

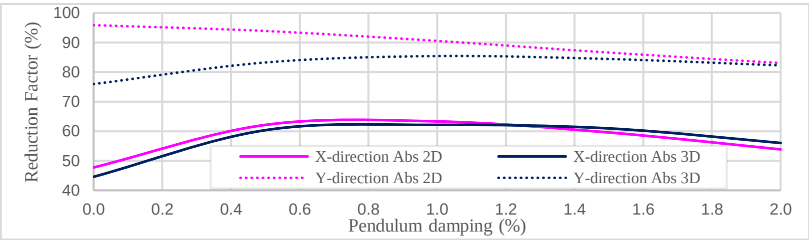

Figure 5. Effectiveness of the pendulum considering its damping variation

It can be seen clearly in Figure 6 the reduction of the maximum amplitude of the angular variation of the pendulum with the increase of the damping in a quasi linear relation. The higher the damping ratio the smaller is the pendulum's amplitude of motion, in such manner that for high damping ratios the pendulum tends to be locked to the structure becoming ineffective. Moreover, Figure 6 indicates the nonlinear behavior of the pendulum control since the maximum amplitudes of rotation reached by the pendulum is larger than 0.2 rad and the assumption of linearized equations would contain enormous error [2].

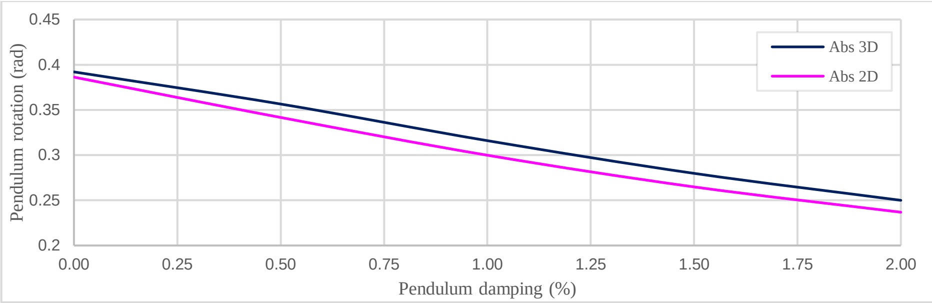

Figure 6. Rotation of the pendulum considering its damping variation

6 Conclusions

In this paper, two mathematical models are presented for studying the reduction of vibration amplitudes of an idealized structure when a nonlinear pendulum controller is attached to it. The results indicate the effectiveness of the control device for mass ratios (m_p / m_{sx}) of just 0.5-1.0% between the pendulum mass and the modal mass of the structure in X-direction. The results also indicate that the reduction factor is extremely sensitive to the pendulum's rod length variation. For both 2D and 3D models the greatest reductions were achieved when the simplified 2D pendulum model frequency was set in resonance with the structure frequency. The role played by the pendulum's damping ratio is related to the possibility of the pendulum locking for high damping ratio values, turning it a useless device. Therefore, it is recommended to provide an almost frictionless pivot to the pendulum rod. Finally, the results obtained to the idealized structure using the 2D and 3D models are in most cases similar and confirms the perspective of obtaining the dynamic responses of the structure with 3D pendulum control devices using the much more simple 2D mathematical pendulum model.

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References

[1] Q. An; Z. Chen; Q. Ren; H. Liu; Yan, X. Control of human-induced vibration of an innovative CSBS-CSCFS. *Journal of Constructional Steel Research*, vol. 115, pp. 359-371, 2015.
[2] S. Fallahpasand; M. Dardel; M. H. Pashaei; H. R. M Daniali. Investigation and optimization of nonlinear pendulum vibration absorber for horizontal vibration suppression of damped system. *The Structure Design of Tall and Special Buildings*, vol. 24, pp. 873-89, 2015.
[3] P. Xiang; A. Nishitani. Structural vibration control with the implementation of a pendulum-type nontraditional tuned mass damper system. *Journal of Vibration and Control*, vol. 23(19), pp. 3128-3146, 2015.
[4] T. Ikeda; Y. Harata; A. Takeeda. Nonlinear responses of spherical pendulum vibration absorbers in towerlike 2DOF structures. *Nonlinear Dyn*, vol. 88, pp. 2915-2932, 2017.
[5] D. Sado; J. Freudlich; A. Dudanowicz. The Dynamics of a coupled mechanical system with spherical pendulum. Vibrations in Physical Systems, vol. 26, pp. 309-316, 2016.
[6] R. C. Battista; M. S. Pfeil; E. M. L. Carvalho; W. D. Varela. Double controller of wind induced bending oscillations in telecom towers. Smart Structures and Systems, vol. 21, n.1, pp. 99-111, 2018.
[7] M. A. S. Pinheiro; R. C. Battista. Efficiency of a spatial pendulum in vibration control. ASAEE (Associação Sul Amaericana de Engenharia Estrutural). In: *XXXV Jornadas Sulamericanas de Engenharia Estrutural*, Rio de Janeiro, 2012.
[8] A. J. Roffel; S. Narasimhan; M. ASCE; T. Haskett. Performance of pendulum tuned mass dampers in reducing the responses of flexible structures. *Journal of Structure Engineering*, vol.139, pp.1-13, 2013.
[9] R. R. Gerges; B. J. Vickery. Optimum design of pendulum-type tuned mass dampers. *The Structural Design of Tall and Special Buildings*, vol. 14, pp. 353-368, 2005.
[10] M. A. S. Pinheiro. Non-linear pendulum absorber of the horizontal vibrations in slender towers. D*Sc Thesis*, Universidade Federal do Rio de Janeiro, 1997.
[11] R. C. Battista. Design and commissioning of mechanical devices to attenuate the wind induced oscillations of telecommunication towers (*in Portuguese*), PEC5211, Fundação COPPETEC, Rio de Janeiro, 2004.