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# Coupling technique between adjacent structures for vibration control using inerter element

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**Abstract.** Vibration control techniques in structures are widely studied as a way to guarantee reliability and comfort for its users. In addition, structural control techniques ensure that complex structures, with a high degree of vibration restriction, can be designed. A technique that has been gaining space in recent years is of the so called structural coupling. In this technique, adjacent structures are connected by control devices, whether passive, active, semi-active or hybrid. Thus, one structure exerts forces of control over the other, reducing the amplitude of vibrations of each individual structure and the coupled system. Studies on structural coupling as proposed by Pérez Peña [1] use viscofluid and viscoelastic dampers (passive systems) in the connection between buildings and demonstrate the efficiency of this technique. As an alternative to these dampers, more recently, the inert elements have appeared. The use of inertial dampers, which use inertial mass rotating forces, emerged in the early 2000s, when Smith [2] defined the term “inerter” as a mechanical two-node (two-terminal) and one-port device with the property of that the equal and opposite force applied at nodes is proportional to the relative acceleration between the terminals. In the structural coupling technique, the inerter devices are still little explored. Thus, the aim of this work is to verify the efficiency of the application of inerter devices in the connection between adjacent buildings. The buildings are modeled as shear frame structures. The position, quantity and mechanical properties of the connection devices are optimized by the particle swarm optimization algorithm. The initial results indicate that the use of inerters in the connection between adjacent buildings requires caution, since the addition of inertia to the system decreases its natural frequencies, which can be harmful to the coupled system.

**Keywords:** Structural control, Structural coupling, Passive Control, Inerter Element.

## 1 Introduction

With the increase in population demand in large urban centers, cities began a process of vertical construction. The problem with excessive vibrations in buildings started to become an obvious problem and studies on methods of controlling these vibrations have risen. In addition, seismic actions have already registered thousands of deaths and tens of millions of dollars in material losses. Thus, the vulnerability of society to these natural disasters is noted, which makes the protection of civil structures, in order to protect human life and material losses, a world priority.

Structural control is a way of getting solved the problem of excessive vibrations. In general, structural control is a collection of techniques applied to reduce the structural damage caused by excessive vibrations and, mainly, to avoid the structure collapse. Control devices can be classified as: passive, active, semi-active and hybrid. These devices, when installed in the building, have the purpose of adding forces or absorbing energy from the structure's vibration, thus reducing its dynamic responses [3-6].

A vibration control technique initially suggested by Klein *et al.* (1972) [7], known as structural coupling, had the primary objective of preventing pounding between two nearby structures during seismic action. Subsequent studies have shown that this technique is also efficient in controlling the amplitude of vibrations of the coupled structures, reducing the structural damage caused by seismic actions and strong winds. Structural coupling is based on connecting two or more adjacent structures through link devices, using one or more of the various control devices. Thus, one structure exerts a control force over the other, reducing the dynamic response of each structure individually and of the coupled system [8-17].

Recently, an inertial control device, known as inerter, started to be used in the technique of structural coupling. Inerters are mechanical devices with two nodes (two terminals) in which the force applied to them is proportional to the relative acceleration between these terminals [2]. These devices have demonstrated great efficiency in controlling vibrations in several branches of engineering, without the need to add high weights to the structures, in addition to having reduced dimensions [18-26].

Thus, the aim of this work is to evaluate the efficiency of the structural coupling technique with and without the use of an inerter element in the connection. The connection device will be passive and its position, quantity and mechanical properties are optimized through the particle swarm optimization algorithm (PSO). Adjacent buildings are modeled as shear frame structures with multiple degrees of freedom.

## 2 Mathematical modeling for multi degree of freedom (MDOF) coupled systems

The Fig. 1 shows the model of adjacent shear frame structures and the mass-spring-damper model. The mathematical formulation described is based on the work of [1, 27-29].

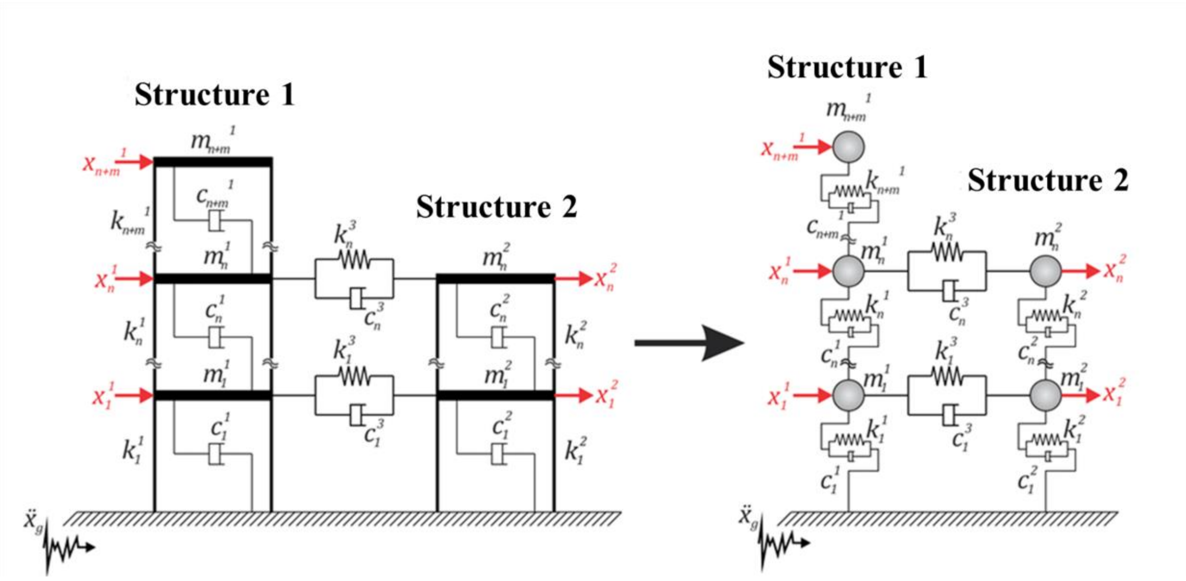


Figure 1. Coupled adjacent structures system

The larger structure (structure 1) has $n + m$ floors and the smaller structure (structure 2) contains $n$ floors, with $j$ referring to the building ($j = 1, 2$). The mass, damping and stiffness properties of each floor $i$ are, respectively, $m_i^j$, $c_i^j$ and $k_i^j$. The viscoelastic dampers that connect the structures on the floor $n$ have the values of stiffness and damping equal to $k_n^3$ and $c_n^3$, respectively. Lastly, $\ddot{X}_g$ is the acceleration of the ground. The values $x_{n+m}^1(t)$ and $x_n^2(t)$ are the displacements of the two models in the time domain. The velocities and accelerations are, respectively, $\dot{x}_{n+m}^1(t)$, $\dot{x}_n^2(t)$, $\ddot{x}_{n+m}^1(t)$ and $\ddot{x}_n^2(t)$.

The equation of motion for a system of coupled structures is given by:

$$\mathbf{M}_{ee}\ddot{\mathbf{x}}_{ee}(t) + \mathbf{C}_{ee}\dot{\mathbf{x}}_{ee}(t) + \mathbf{K}_{ee}\mathbf{x}_{ee}(t) = \mathbf{f}(t) \tag{1}$$

In which: $\mathbf{M}_{ee}$, $\mathbf{C}_{ee}$ and $\mathbf{K}_{ee}$ are the mass, damping and stiffness matrices of the coupled system, respectively; $\ddot{\mathbf{x}}_{ee}(t)$, $\dot{\mathbf{x}}_{ee}(t)$ and $\mathbf{x}_{ee}(t)$ are the vectors that contain the values of accelerations, velocities and displacements, respectively, in both structure; $\mathbf{f}(t)$ is the external force vector. The matrices and vectors are as follows:

$$\mathbf{M}_{ee} = \begin{bmatrix} \mathbf{m}^1_{(n+m,n+m)} & 0 \\ 0 & \mathbf{m}^2_{(n,n)} \end{bmatrix} \tag{2}$$

$$\mathbf{K}_{ee} = \mathbf{K} + \mathbf{K^3} = \begin{bmatrix} \mathbf{k}^1_{(n+m,n+m)} & 0 \\ 0 & \mathbf{k}^2_{(n,n)} \end{bmatrix} + \begin{bmatrix} \mathbf{k}^3_{(n+m,n+m)} & \text{-}\mathbf{k}^3_{(n+m,n)} \\ \text{-}\mathbf{k}^3_{(n,n+m)} & \mathbf{k}^3_{(n,n)} \end{bmatrix} \tag{3}$$

$$\mathbf{C}_{ee} = \mathbf{C} + \mathbf{C^3} = \begin{bmatrix} \mathbf{c}^1_{(n+m,n+m)} & 0 \\ 0 & \mathbf{c}^2_{(n,n)} \end{bmatrix} + \begin{bmatrix} \mathbf{c}^3_{(n+m,n+m)} & \text{-}\mathbf{c}^3_{(n+m,n)} \\ \text{-}\mathbf{c}^3_{(n,n+m)} & \mathbf{c}^3_{(n,n)} \end{bmatrix} \tag{4}$$

$$\mathbf{x}_{ee}(t) = \{x_1^1 \cdots x_{n+m}^1 \quad x_1^2 \cdots x_n^2\}^T \tag{5}$$

$$\dot{\mathbf{x}}_{ee}(t) = \{\dot{x}_1^1 \cdots \dot{x}_{n+m}^1 \quad \dot{x}_1^2 \cdots \dot{x}_n^2\}^T \tag{6}$$

$$\ddot{\mathbf{x}}_{ee}(t) = \{\ddot{x}_1^1 \cdots \ddot{x}_{n+m}^1 \quad \ddot{x}_1^2 \cdots \ddot{x}_n^2\}^T \tag{7}$$

$$\mathbf{f}(t) = \mathbf{T}_w \ddot{\mathbf{x}}_g(t) = -\mathbf{M}_{ee}\{\mathbf{1}\}_{n+m+n}\, \ddot{\mathbf{x}}_{\mathrm{g}}(t) \tag{8}$$

In which: $\mathbf{m}^1$, $\mathbf{m}^2$, $\mathbf{k}^1$, $\mathbf{k}^2$, $\mathbf{c}^1$ and $\mathbf{c}^2$ are the mass, stiffness and damping matrices of structures 1 and 2, respectively. The matrices $\mathbf{K}^3$ and $\mathbf{C}^3$ include the values of stiffness and damping, respectively, of the connecting element between the coupled structures.

### 2.1 MDOF coupled systems with inerter element

The inerter is an element in which the equal and opposite force applied to the nodes is proportional to the relative acceleration between the terminals. The equation of an ideal inerter is given by:

$$F = b(\ddot{x}_2 - \ddot{x}_1) \tag{9}$$

where, $F$ is the force applied at the terminals; $b$ is called inertance with units of kilograms, $\ddot{x}_1$ and $\ddot{x}_2$ are the accelerations at each terminal.

In this formulation of coupled buildings, it is now considered that there is also an inerter element with inertance $b_n^3$ in the connection between the structures, as presented in Fig. 2.

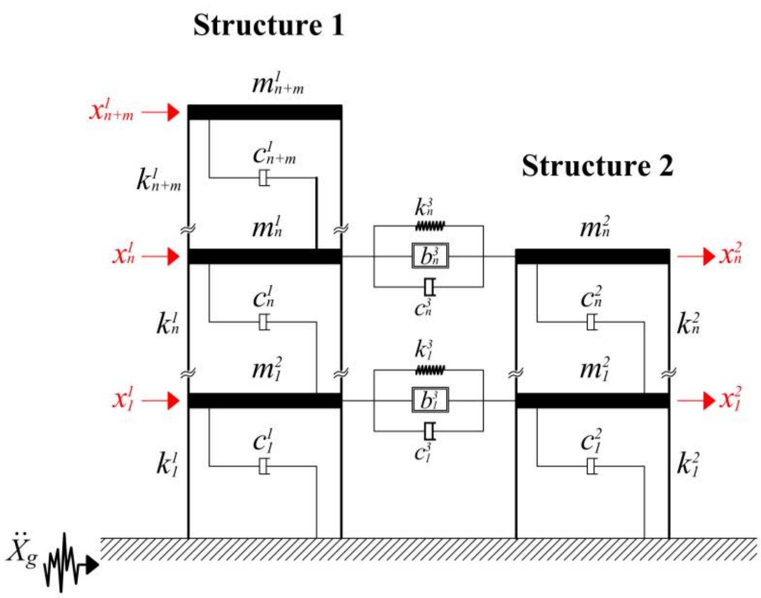


Figure 2. Coupled adjacent structures system with inerter element

For this configuration, the stiffness matrices $\mathbf{K}_{ee}$ and damping $\mathbf{C}_{ee}$ for the coupled system are the same as described in item 2. When an inerter element is considered in the connection, the mass matrix for the coupled system is changed. Considering Eq. (1), the mass matrix coupled $\mathbf{M}_{ee}$ is given by:

$$\mathbf{M}_{ee} = \mathbf{M} + \mathbf{B}^3 = \begin{bmatrix} \mathbf{m}^1_{(n+m,n+m)} & 0 \\ 0 & \mathbf{m}^2_{(n,n)} \end{bmatrix} + \begin{bmatrix} \mathbf{b}^3_{(n+m,n+m)} & -\mathbf{b}^3_{(n+m,n)} \\ -\mathbf{b}^3_{(n,n+m)} & \mathbf{b}^3_{(n,n)} \end{bmatrix} \tag{10}$$

Matrix $\mathbf{B}^3$ contains the inertance values of the connecting element between the coupled structures. The displacement, velocity and acceleration vectors are also not modified.

# 3 Particle swarm optimization

To obtain the optimal positions, quantity and mechanical properties of the connection devices of the buildings, the algorithm known as Particle swarm optimization (PSO) was used. Easy to implement, the algorithm was developed by Kennedy and Eberhart (1995) [30] and is widely used in structural optimization studies due to its shorter calculation time when compared to other methods. In the method, the particles (individuals) that represent the optimization parameters move through defined regions searching a place to minimize the objective function.

The objective function of Eq. (13) applied in the optimization is based on the studies of [1, 17, 28] and is formed by two parts. The first aims to minimize the square of the maximum relative displacements between the floors of the two adjacent buildings, according to Eq. (11). The second, presented in Eq. (12), aims to minimize the value of the sum of squares of these displacements.

$$\mathrm{f}_{obj1} = \max\{\max(\{\Delta\}^1)^2 \quad \max(\{\Delta\}^2)^2\} \tag{11}$$

$$\mathrm{f}_{obj2} = \sum_{i=1}^{n+m}\left(\{\Delta\}_i^1\right)^2 + \sum_{i=1}^{n+m}\left(\{\Delta\}_i^2\right)^2 \tag{12}$$

$$\mathrm{f}_{objtotal} = \mathrm{f}_{obj1} + \mathrm{f}_{obj2} \tag{13}$$

The relative displacements between floors are calculated as follows:

$$\begin{cases} \{\Delta\}_i^j = \max(\mathbf{x}_i^j) \\ \{\Delta\}_i^j = \max(\mathbf{x}_i^j - \mathbf{x}_{i-1}^j) \qquad 1 < i \le n_{floors} \end{cases} \tag{14}$$

Where: $\{\Delta\}^j$ is the vector that contains the relative displacements of each building ($1 \le j \le 2$) and $\mathbf{x}_i^j$ is the absolute displacement vector in time, calculated in each floor.

# 4 Numerical analysis

The numerical analysis was performed using algorithms developed in MATLAB®. The structures used for this study contain 8 floors (Structure 1) and 4 floors (Structure 2) and are modeled as shear frame structures. Both will be subjected to horizontal accelerations at the base from the 1940 El Centro, 1994 Northridge and 1995 Kobe earthquakes. The structures have the following characteristics: mass per floor $m_j^i$ = 30.000 kg, floor height $H_j^i$ = 3,0 m and floor stiffness $k_j^i$ = 12,58 MN/m.

Initially, an optimization analysis will be performed through the PSO to determine the position, quantity and optimal mechanical properties of the connection elements for the system without the inerter element (Analysis 1), as shown in Fig. 1 and for the system with the inerter element (Analysis 2), as shown in Fig. 2. In this way, it is possible to compare the influence of the inerter element in the optimization of the connection elements. Afterwards, a comparison will be made of the maximum displacement, velocity and acceleration responses of systems with and without the inerter element, in order to assess the efficiency of this element in reducing responses.

## 4.1 Results and discussions

The fundamental frequencies of structures 1 and 2, for the uncoupled system are shown in Table 1. In Table 2, it is possible to view the optimization results for the system without the inerter element, for the three earthquakes. The results of the optimization for the system with the inerter element are shown in Table 3.

Table 1. Fundamental frequencies of uncoupled system

| Structure | Frequency (Hz) |
|---|---|
| 1 (8 Floors) | 0.601 |
| 2 (4 Floors) | 1.132 |

Table 2. Optimization results – Analysis 1

| Earthquake | Optimal position | $c_3$ (Ns/m) | $k_3$ (N/m) | $f_{objtotal}$ (m$^2$) |
|---|---|---|---|---|
| El Centro | Floor 4 | 2.033E+05 | 0.00 | 0.0068 |
| Kobe | Floor 4 | 3.135E+05 | 0.00 | 0.0714 |
| Northridge | Floor 4 | 2.046E+05 | 6.729E+06 | 0.0172 |

Table 3. Optimization results – Analysis 2

| Earthquake | Optimal position | $c_n^3$ (Ns/m) | $k_n^3$ (N/m) | $b_n^3$ (kg) | $f_{objtotal}$ (m$^2$) |
|---|---|---|---|---|---|
| El Centro | Floors 2 and 4 | 1.363E+05 | 0.00 | 1523.57 | 0.00656 |
| Kobe | Floors 1 and 4 | 3.008E+05 | 0.00 | 3646.26 | 0.06933 |
| Northridge | Floors 3 and 4 | 8.989E+04 | 4.178E+06 | 2310.55 | 0.01628 |

In Analysis 1, considering the El Centro and Kobe earthquakes, it was determined the use of viscofluid dampers, in which the stiffness constant was zero. Thus, the natural frequencies of the system do not change [1, 14]. For the Northridge earthquake, the results indicated the use of a viscoelastic damper. In this case, the fundamental frequencies of the system change to 0.619 (+3.00%) and 1.201 (+6.10%) compared to the uncoupled system (Table 1). It can be seen that earthquakes with different characteristics affect the optimum mechanical properties of the connection device.

In Analysis 2, for the three earthquakes, it is necessary to install two devices in the coupled system in the positions indicated in Table 3. For the El Centro earthquake there is a 33% reduction in the damping coefficient in the devices. In the Kobe earthquake, the damping coefficient was 4% lower than that obtained in the analysis without the inerter device. Finally, for the Northridge earthquake, attenuations of 56% in the damping coefficient and 38% in the stiffness coefficient were achieved.

As inertia and stiffness have been added to the structures, the natural frequencies of the system change. Thus, Table 4 shows the first two natural frequencies of the coupled system ($\omega_1$ and $\omega_2$). The negative sign indicates reduction.

Table 4. Coefficients in constitutive relations

| Earthquake | $\omega_1$ (Hz) | Difference (%) | $\omega_2$ (Hz) | Difference (%) |
|---|---|---|---|---|
| El Centro | 0.599 | -0.33 | 1.115 | -1.50 |
| Kobe | 0.597 | -0.67 | 1.100 | -2.83 |
| Northridge | 0.690 | 14.81 | 1.494 | 31.98 |

The difference in system frequencies indicates that the second frequency is more affected by the increase in inertia.

The results of absolute maximum displacement, velocity and acceleration for the three earthquakes and structures 1 and 2 of Analysis 1, are shown in Table 5. The values with sub-index “coup” are the answers for the coupled system. Table 6 shows the absolute maximum results for Analysis 2.

Analysis 1 shows that the structural coupling technique was efficient in reducing the absolute maximum responses for the three earthquakes, reaching a 62% reduction in displacements for the El Centro earthquake, 50% for the Kobe earthquake and 73% for the of Northridge earthquake. The reductions in the responses for the three earthquakes were more significant for the more rigid structure (structure 2).

In comparison to Analysis 1, the coupled system in Analysis 2 achieved practically the same behavior. The advantage of the system with inerter is to reduce the point force generated by the connection device, since, for the three earthquakes, it was possible to reduce the damping and stiffness coefficients of the connection device.

Table 5. Absolute maximum results for Analysis 1

| Earthquake | Struct. | Absolute Maximum Response | | | | | | Reduction | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x$ (m) | $x_{coup}$ (m) | $\dot{x}$ (m/s) | $\dot{x}_{coup}$ (m/s) | $\ddot{x}$ (m/s²) | $\ddot{x}_{coup}$ (m/s²) | $x$ (%) | $\dot{x}$ (%) | $\ddot{x}$ (%) |
| El Centro | 1 | 0.208 | 0.135 | 0.856 | 0.689 | 6.316 | 5.122 | 35.09 | 19.53 | 18.92 |
| | 2 | 0.202 | 0.077 | 1.371 | 0.663 | 11.725 | 4.926 | 61.86 | 51.61 | 57.99 |
| Kobe | 1 | 0.652 | 0.501 | 3.373 | 2.625 | 18.505 | 16.91 | 23.15 | 22.17 | 8.62 |
| | 2 | 0.517 | 0.261 | 4.074 | 1.883 | 28.591 | 12.95 | 49.61 | 53.78 | 54.71 |
| Northridge | 1 | 0.329 | 0.242 | 1.404 | 1.118 | 12.602 | 11.950 | 26.33 | 20.33 | 5.17 |
| | 2 | 0.404 | 0.108 | 3.478 | 0.994 | 31.488 | 13.603 | 73.16 | 71.41 | 56.80 |

Table 6. Absolute maximum results for Analysis 2

| Earthquake | Struct. | Absolute Maximum Response | | | | | | Reduction | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $x$ (m) | $x_{coup}$ (m) | $\dot{x}$ (m/s) | $\dot{x}_{coup}$ (m/s) | $\ddot{x}$ (m/s²) | $\ddot{x}_{coup}$ (m/s²) | $x$ (%) | $\dot{x}$ (%) | $\ddot{x}$ (%) |
| El Centro | 1 | 0.208 | 0.137 | 0.856 | 0.702 | 6.316 | 4.998 | 34.16 | 18.02 | 20.88 |
| | 2 | 0.202 | 0.077 | 1.371 | 0.684 | 11.725 | 4.745 | 61.96 | 50.12 | 59.53 |
| Kobe | 1 | 0.652 | 0.494 | 3.373 | 2.614 | 18.505 | 16.73 | 24.20 | 22.52 | 9.57 |
| | 2 | 0.517 | 0.261 | 4.074 | 1.859 | 28.591 | 12.76 | 49.43 | 54.38 | 55.39 |
| Northridge | 1 | 0.329 | 0.245 | 1.404 | 1.125 | 12.602 | 10.686 | 25.30 | 19.88 | 15.20 |
| | 2 | 0.404 | 0.109 | 3.478 | 1.008 | 31.488 | 15.473 | 72.95 | 71.02 | 50.86 |

# 5 Conclusions

A study on the structural coupling technique with the use of an inerter element was carried out in this work. Two adjacent structures, modeled as shear frames, with different heights were used. The structures were subjected to accelerations based on three different earthquake data. Two analysis were carried out, one analysis considering the connection between the structures without the inerter element and a second analysis considering it.

The optimization responses indicated that the parameters of the connection element between the structures are highly dependent on the characteristics of the external excitation. This resulted in different mechanical properties of the connection devices for each earthquake and even resulting in different types of dampers.

With the use of the inerter element in the connection, reductions in the damping coefficient and stiffness of the dampers were observed, in addition to the use of two devices in the coupled system. This reduces the point forces of one structure on the other.

The structural coupling technique proved to be efficient in reducing the maximum responses of both structures. Reductions of up to 73% were obtained in displacements, 71% in velocities and 60% in accelerations. The use of the inerter element (Analysis 2) did not bring significant percentages of reduction in the maximum responses, in comparison with Analysis 1. However, as it is a relatively simple device with small dimensions, it can be a solution when there is little space between a building, as it apparently reduces the dimensions of the dampers without interfering with the performance of the coupling technique

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