XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Optimization of post tensioned unidirectional flat slabs

Caroline Galvão Toscano[1], Antônio Macário Cartaxo de Melo[1], Evandro Parente Júnior[1]. João Pedro Alexandre Silva Mota[1]. Matheus da Silva Menezes[2]

[1]*Laboratório de Mecânica Computacional e Visualização, Departamento de Engenharia Estrutural e Construção Civil, Centro de Tecnologia, Universidade Federal do Ceará*
Fortaleza-CE, CEP 60440-900, Brasil
caroltoscano@alu.ufc.br, macario@ufc.br, evandro@ufc.br, jpsm@alu.ufc.br
[2]*Departamento de Ciências Naturais, Matemática e Estatística, Centro de Ciências Exatas e Naturais, Universidade Federal Rural do Semi-Árido*
Mossoró-RN, CEP 59625-900, Brasil
matheus@ufersa.edu.br

Abstract. The structural solution for slabs of a building is a very important factor and significantly affects the total construction cost. The use of prestressed tendons, in particular its unbonded type, has become increasingly popular, as well as its application to flat slabs because it allows larger spans, greather architectural flexibility and construction speed and less labor required. Although the prestressed flat slabs have a higher consumption of concrete in relation to ribbed slabs, they are an attractive solution due to their simple formwork. For an efficient design, it is necessary to pay attention to the thickness of the slab, the number and layout of the tendons. In this sense, numerical optimization techniques are appropriated tools to search for an optimal design of flat slabs. This work presents an optimization model for post tensioned unidirectional flat slab with unbonded tendons according to the recommendations of NBR 6118:2014, seeking the minimization of material and labor costs. The design variables are the slab thickness, the number of tendons and the eccentricities that characterize their profiles in both directions, all of them are of the discrete type. A grid model is applied in the structural analysis, where the prestressed tendons are treated as equivalent loads. The optimization problem is solved using an open source genetic algorithm, which is successfully applied to discrete structural optimization problems. The results of the proposed optimization model are compared with others obtained using the conventional design approach found in the literature and show an excellent cost reduction.

Keywords: Flat slabs, Prestressed concrete, Structural optimization, Genetic algorithm, Grid model.

1 Introduction

Buildings of multiple floors for residential or commercial uses represent a large part of the civil construction market. The search for safety, durable and economical solutions for the construction systems of these buildings, especially for the slabs, has become a constant challenge for industry and academia[1].

The search for improvements in construction, economic processes and the need for structures with large spans enabled the development of the slabs without the presence of beams. Such structures, called emergencies, can be executed in either reinforced or prestressed concrete.

According to Cauduro [2], the use of prestressed structures in residential buildings in Brazil had a significant increase since 1997, a year in which emerged the greased and plasticized strands in the national market. This is due to the fact that the adherent system, previously used for the prestressed slabs, presents a reasonably complex technology and higher cost when compared to reinforced concrete slabs supported on beams. Thus, the system was restricted to large commercial buildings, bridges and viaducts, with the members of the largest extension being precisely prestressed.

The application of the post tensioning system in slabs presents a series of benefits when compared to the conventional reinforced concrete system, among which the following stand out [3]: greater architectural freedom due to the possibility of overcoming large spans or heavily loaded spans while maintaining a great slenderness in the slab ; greater floor area due to fewer columns; greater speed in the stripping of the formwork system;

reduction and even elimination of deflections and cracking in the slabs; greater resistance to punching shear in flat slabs and slabs with column capitals and drop panels, obtained by the proper placement of prestressing tendons in the regions over the columns. Especially for unbonded tendons, Emerick [3] highlights the quick installation, reduction of friction losses, elimination of the grout injection, protection against corrosion and possibility of new post tensioning as advantages.

The design practice of reinforced and prestressed concrete structures follows the same iterative process, in which trial solutions based on designer's experience and intuition are analysed with respect to ultimate and service conditions and an acceptable one is choose. However, this method does not ensure the solution is the best, having the potential to use optimization strategies taking into account a given performance that is intended to be achieved and satisfying all requirements simultaneously.

The definition of the optimization algorithm is problem dependent, e.g., on the nature of the design variables. Considering engineering projects, several variables assume integer values or a group of discrete values. In this way, the class of evolutionary algorithms that uses techniques inspired by evolutionary biology has been successfully applied to this type of problem, especially Genetic Algorithms (GA's). Among the works can be cited Bezerra [4], Sobrinho [1] and Mota [5], in which GAs are used to optimize reinforced and prestressed reinforced concrete slabs.

Structural analysis is essential for determining the efforts and displacements of structures, and the analysis of the prestressed flat slab pavement is more complex, so it is possible to perform simplifications that facilitate the understanding of its behavior. Modeling of flat slabs through grid idealization is efficient according to the literature([6–12]).

In this context, the use of optimization techniques for the design of prestressed flat slab is a good solution for the maximum use of material properties, thus reducing consumption and, therefore, the cost of the structure. Numerous studies have been published addressing the optimization of the design of prestressed flat slab ([13–19]).

In this article, the grid model was developed using FAST (Finite Element Analysis Tool) software to study the ideal design of unidirectional prestressed flat slab. The slab used by Colonese [20] was chosen to perform the numerical analysis and optimization that aims to minimize the cost, using a genetic algorithm in the BIOS (Biologically Inspired Optimization System) software.

2 Structural analysis

With the evolution of computational resources, numerical methods, like as the Finite Element Method (FEM), have been widely used for structural analysis of slabs. The code NBR6118:2014 [21] permittes the method of equivalent frames for the analysis of flat slab with regular geometry [11]. The ACI Committee 318-19 [22] removed the detailed provisions for the direct design method and equivalent frame method but they are still permitted.

The grid idealization is a simplified method in which the floor as a whole is replaced by an equivalent bidirectional grid composed of bar elements. The grid method is simpler and more efficient than the use of plate finite elements. For flat slabs, this method presents good results when compared to the other methods, becoming one of the most used procedures currently [12]. Therefore, the grid method will be used to analyze the prestressed slab subject to static loads and linear elastic behavior for the material.

Convergence studies should be carried out based on the variation of the mesh, using the grid mesh that best represents the slab, maintaining special attention in the support regions, and further refinement of the mesh in these regions can be done. Hambly [23] recommends that the spacing between bars of the grid does not exceed a quarter of the span.

The mechanical properties, as a longitudinal and transversal modulus of elasticity, are estimated according to the requirements of the design codes and the characteristics of the material.

After choosing the mesh, it is necessary to define the geometric properties of the grid in order to represent the slab in the best possible [10]. Flexural and torsional stiffness in both directions is considered to be concentrated in the bars of the equivalent grid. The values adopted must be such that the slab and the equivalent grid, subject to the same loading, present the same displacements and internal efforts. Therefore, the moment of inertia is given by

$$I_f = \frac{bh^3}{12} \tag{1}$$

where b is the width of the strip represented by the bar and h is the slab thickness. According to Hambly [23], it is recommended to use torsional inertia equal to twice the flexural inertia, considering *uncracked-elastic* range, i.e., gross section.

2.1 Loads

In this work, we chose the concept of tributary area for the nodes, where each load applied to the slab at a distance less than or equal to half the length of the grid bar, in both directions, is taken directly to the node.

With respect to the prestressing force the strands embedded in the band of a grid model element are considered as equivalent loads along this element [24]. According to the profile of the tendons, there will be alternation of points of maximum and minimum elevation, with inflection points appearing on the tendon, usually in the vicinity of the supports. Thus, the inflection of the tendon will cause the inversion of the load at some point inside the grid element that is close to the support. Therefore, in order to ensure the representation of the equivalent prestressing load on the entire floor, this load is converted into point loads at the nodes.

3 Optimization model

The purpose of the optimization model is to minimize the cost of the slab, satisfying the serviceability and ultimate limit states recommended by NBR6118:2014 [21]. The objective function is the total cost considering the material and labor, which consists of the cost of concrete, nonprestressed and prestressed bars. The optimization problem can be defined as:

$$\begin{aligned} &\text{minimize:} \\ &\quad C(\mathbf{X}) = c_c \cdot V_c + c_s \cdot q_s + c_{cb} \cdot q_{cb} \\ &\text{subject to:} \\ &\quad g_i(\mathbf{X}) \leq 0 \end{aligned} \tag{2}$$

where: $C(\mathbf{X})$ is objective function $(\mathrm{R\$/m^2})$; c_c is cost of unit volume of concrete $(\mathrm{R\$/m^3})$; V_c is volume of concrete per unit area (m^3/m^2); c_s is cost of unit mass of steel CA-50 e CA-60 $(\mathrm{R\$/kg})$; q_s is average mass value of steel CA-50 and CA-60 per unit area $(\mathrm{kg/m^2})$; c_{cb} is cost of unit mass of prestressing tendon. $(\mathrm{R\$/kg})$; q_{cb} is average mass value of prestressing tendon per unit area $(\mathrm{kg/m^2})$; $g_i(\mathbf{X})$ is a normalized constraint function (detailed in Table 2); $\mathbf{X}$ is the vector of the design variables (h,n_{cbxp}, n_{cbxc}, n_{cby}, e_{x1}, e_{y1}, e_{x2}, e_{y2}...).

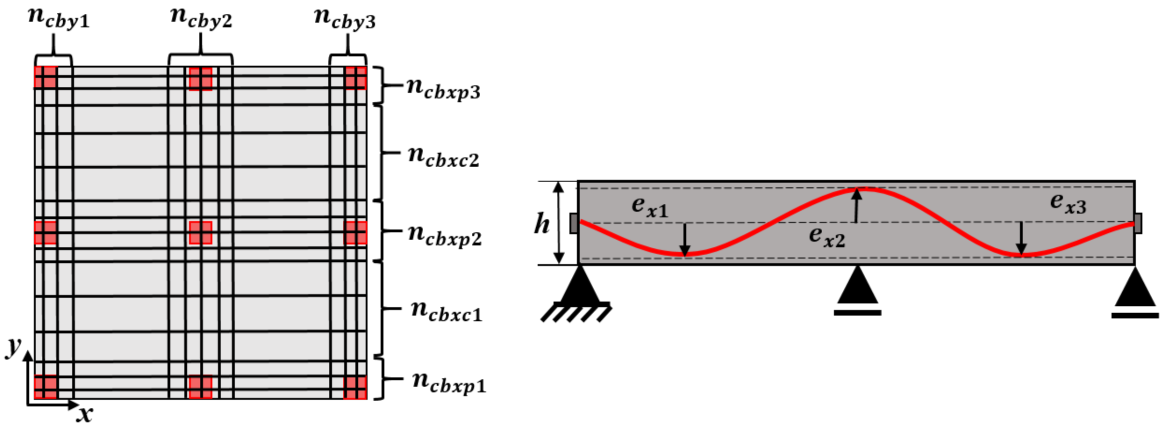

Figure 1. Design variables

The design variables $\mathbf{X}$ are defined in Table 1. The variable (h) is discrete that meets the limits of the code and considers the design practices. Based on the minimum thickness imposed by code (ABNT, 2014), the slab thickness varies from 16 to 40 cm with increment fo 1 cm. The tendons in each direction can be organized in tracks, distributed or in a mixed layout. In this work, the tendons are concentrated in the column bands in the y-direction and have a mixed layout in the x-direction. The eccentricity of the tendon is a discrete variable that represents a percentage of the distance from the center of gravity of the section to maximum eccentricity permitted by the concrete cover in support and center of span sections (see Figure 1).

Table 1. Design variables

h = thickness of the concrete slab

n_{cbxp} = total number of prestressing tendons in a set of 2 grid bars in the X-direction in the column bands

n_{cbxc} = total number of prestressing tendons in a set of 2 grid bars in the X direction in the central bands

n_{cby} = number of prestressing tendons concentrated in the column bands in the Y-direction

e_{xi}= eccentricities defining the tendon profile in the X-direction

e_{yj} = eccentricities defining the tendon profile in the Y-direction

It is worth noting that the model does not address the bracing structure, therefore, independent structures should be considered for this purpose. Constraints implemented in the model are shown in Table 2.

Table 2. Constraints specified by the Brazilian code NBR6118:2014 [21]

Constraint	Clause	Limit	Normalized constraint functions
1. Slab strip treated as prestressed	20.3.2.1	$\sigma_{min} \leq \sigma_{med}$	$g_1(\mathbf{X})$
2. Tensile stresses in concrete on the underside	3.2.2	$\sigma_{t,inf} \leq 0,7 f_{ct,m}$	$g_2(\mathbf{X})$
3. Tensile stresses in concrete on the upper face	3.2.2	$\sigma_{t,sup} \leq 0,7 f_{ct,m}$	$g_3(\mathbf{X})$
4. Guarantee that at one or more points of the cross section, the normal tension is zero, with no traction in the rest of the section	3.2.5	$-0,7 f_{ckj} \leq \sigma_{cqp} \leq 0$	$g_4(\mathbf{X})$ and $g_5(\mathbf{X})$
5. Excessive deformations of the structure in service	13.3	$\delta_{desl} \leq \delta_{lim}$	$g_6(\mathbf{X})$ and $g_7(\mathbf{X})$
6. Tensile stress during prestressing	17.2.4.3.2	$\sigma_{t,max} \leq 1,2 f_{ct,m}$	$g_8(\mathbf{X})$
7. Compression stress at prestressing	17.2.4.3.2	$0,7 f_{ck} \leq \sigma_{c,max}$	$g_9(\mathbf{X})$
8. To avoid using transverse armor	19.4.1	$V_{Sd} \leq V_{Rd1}$	$g_{10}(\mathbf{X})$
9. Diagonal compression stress on critical surface C	19.5.3.1	$\tau_{Sd} \leq \tau_{Rd2}$	$g_{11}(\mathbf{X})$
10. Diagonal compression stress on critical surface C'	19.5.3.2	$\tau_{Sd} \leq \tau_{Rd1}$	$g_{12}(\mathbf{X})$

Where: σ_{min} is eqqual to 1 Mpa; σ_{med} represents the compressive stress in the section of the slab where the tendons are distributed; $\sigma_{t,inf}$ and $\sigma_{t,sup}$ represent the tensile stresses on the bottom and top of the slab; $f_{ct,m}$ is concrete's average tensile strength; σ_{cqp} is the stress for the almost permanent combinations of loads; $f_{ck,j}$ is the specified compressive strength of concrete; δ_{desl} is the displacement in each span of the structure; δ_{lim} is $L/250$ for displacament and $L/350$ for vibrations (where L is the lower length of the slab); $\sigma_{t,max}$ is the maximum tensile stress at prestressing; $\sigma_{c,max}$ is the maximum compressive stress at prestressing; V_{Sd} is the design shear force; V_{Rd1} is the design shear strength for elements without shear reinforcement;With respect to punching shear, τ_{Sd} represents the design shear stress, and τ_{Rd1} and τ_{Rd2} represent the allowable shear stresses.

For the calculation of the required reinforcement, the position of the neutral axis is determined and the section balance is checked. However, if the prestressing tendons is not sufficient to balance the requesting moment, the necessary nonprestressed steel (A_s) is calculated. According to NBR6118:2014 [21] the prestressed slab must be dimensioned for the Ultimate Limit State for flexural rupture, and minimum amount of nonprestressed steel ($A_{s,min}$) must be adopted to guarantee the ductility of the concrete section ($A_s \geq A_{s,min}$).

In this article, the adopted minimum concrete cover to be measured from the geometric center of the cross section of the reinforcement to the external fiber, upper or lower, was 35 mm for prestressed tendons and 30 mm for nonprestressed reinforcement (to be possible a comparison with the values obtained Colonese [20]). CA-50 and CA-60 steel rebars are used for concrete reinforcement. The prestressed reinforcement used are unbonded seven-wire tendon, with a characteristic tensile strength value equal to 1900 MPa (CP 190 RB). Unit costs were made available by the company IMPACTO PROTENSÃO, including material and labor costs (Table 3).

Table 3. Cost of structural materials including labor

Material	Cost
Concrete ($f_{ck} = 30MPa$)	$\mathrm{R\$358.16/m^3}$
CA-50 and CA-60 steel	$\mathrm{R\$5.48/kg}$
Prestressing tendons	$\mathrm{R\$14.32/kg}$

The design variables can be separated into two categories. The first category corresponds to the variables that directly contribute to the objective (cost) function (h, n_{cbx}, n_{cby}), while the second category corresponds to variables that define the profile of the prestressing tendons (e_{1x}, e_{1y}, e_{2x}, e_{2y}...). In this work, the tendon profile is defined by the eccentricity at the main points. These variables do not directly contribute to the cost, since the lengths of the tendons are measured in straight line in the top view. In fact the length of the tendons is profile dependent, but this effect is insignificante. Its contribution is to reduce the tensile stresses, allowing the choice of smaller slab thicknesss and fewer tendons. The number of variables in the second category depends on the number of spans in the floor.

4 Application

A prestressed flat slab designed by Colonese [20] following the conventional process was chosen for comparison with the solution obtained from the proposed optimization formulation. The square flat slab of 7.6 m × 7.6 m is supported on four columns in its corners as shown in Figure 2. The prestressing tendons have a uniform distribution (26 tendons) in the x-direction and are concentrated on the columns (13 tendons in each column band) in the y-direction.

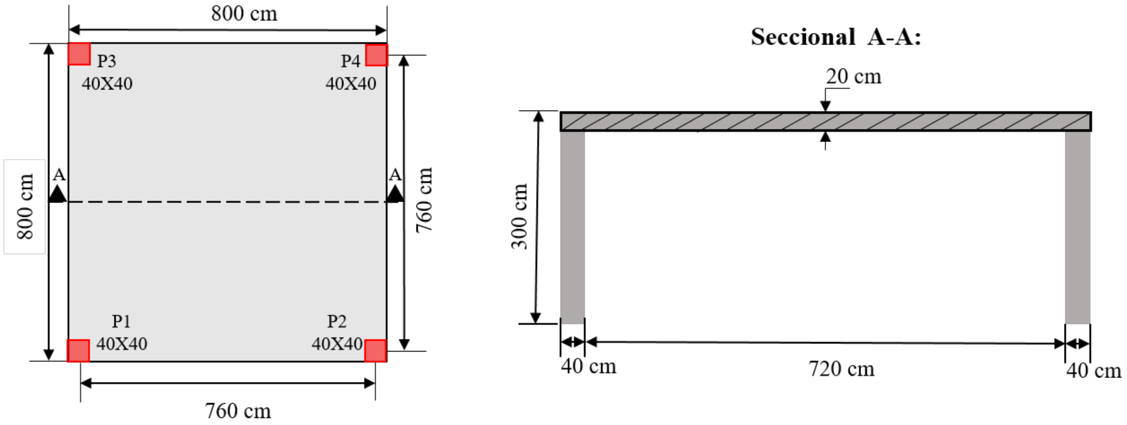

Figure 2. One-bay flat slab

For analysis, three bands in the x-direction are considered, the bands of the columns with 35% of the span and the center of the slab with 30% of the span. For the y-direction, a band of $80cm$ was considered for the tendons concentrated on the columns. The adopted grid spacement was 50 x 50. Parameters must be assigned to the structural model such as loads, stress loss factors and the strength of the concrete. Table 4 presents the values of Colonese [20], used here to make the comparison feasible.

Table 4. Adoped parameters Colonese [20]

Loads		Prestressing losses			Concrete
Permanent	Accidental	Immediate	Deferred	Totals	f_{ck}
$1\mathrm{kN/m^2}$	$1\mathrm{kN/m^2}$	10%	10%	20%	$30\mathrm{MPa}$

To solve the optimization problem via GA's, a calibration process should be used of its parameters, to seek the best performance of the algorithm. In this work, 5 optimizations were carried out, with the population consisting of 50 individuals, with 100 being the number of generations. The adopted crossing rate was 90%, consequently, an elitism rate of 10%, with a mutation probability of 15%. The best result found in the optimization is presented in Table 5 and the critical constraint values in Table 6.

Table 5. Optimal solution and cost comparison

Optimal solution	h	n_{cbxp}	n_{cbxc}	n_{cby}	e_{x1}	e_{y1}
	$20\mathrm{cm}$	4	3	8	100%	100%

	Objective function	Concrete	Steel reinforcement	Tendons
This work	$\mathrm{R\$154.81/m^2}$	$\mathrm{R\$71.63/m^2}$	$\mathrm{R\$19.62/m^2}$	$\mathrm{R\$63.56/m^2}$
Colonese [20]	$\mathrm{R\$170.73/m^2}$	$\mathrm{R\$71.63/m^2}$	$\mathrm{R\$20.39/m^2}$	$\mathrm{R\$78.70/m^2}$

Note that the eccentricities were maximum in both x and y-directions, showing the maximum utilization of the prestressing tendon for the number of adopted tendons. It is also noticed that the slab thickness and the amount of tendons in the x-direction remained the same, but there was a reduction of 38.5% in the amount of prestressing tendons in the y-direction.

Table 6. Comparison of critical constraints

Constraints	G_{5y}	G_6	G_{12x}
This work	$\mathbf{-0,04}$	$\mathbf{-0,08}$	$\mathbf{-0,03}$
Colonese [20]	$-4,52$	$-0,22$	$-0,03$

As a discrete optimization model was adopted in this work, no constraint is active. However, it is noted that the constraint related to punching proved to be the most critical. In addition, it is possible to highlight the constraints related to the limit state of decompression and excessive deformations which also presented critical values.

5 Conclusions

From the case studied, it was observed that the optimization model managed to obtain a solution that presenting savings of 9.32% in relation to the cost of the slab calculated using the conventional design approch. Other parameter that can be observed is the percentage of reduction in the most critical constraints and their relationship with the amount of material used. The punching constraint in the x-direction already had a critical value in the conventional design, making it impossible to change the slab thickness and the number of tendons. In the y-direction, the model was able to reduce the number of tendons, reducing the constraint value related to the decompression limit state in the y direction by 99%.

Acknowledgements. This work was carried out with the support of the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brazil (CAPES) - Financing Code 001.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] Sobrinho, F. R., 2018. Otimização de pavimento de lajes nervuradas protendidas. Dissertação de mestrado, Universidade Federal do Ceará, Fortaleza, Brasil.

[2] Cauduro, E. L., 2002. *Manual para a boa execução de estruturas protendidas usando cordoalhas de aço engraxadas e plastificadas*. Belgo Mineira.

[3] Emerick, A. A., 2002. *Projeto e Execução de Lajes Protendidas*. Brasília.

[4] Bezerra, E. M. F., 2017. Otimização multiobjetivo de lajes nervuradas em concreto armado. Dissertação de mestrado, Universidade Federal do Ceará, Fortaleza, Brasil.

[5] Mota, J. P. A. S., 2020. Otimização integrada de pavimentos de lajes nervuradas protendidas. Dissertação de mestrado em andamento, Universidade Federal do Ceará, Fortaleza, Brasil.

[6] Almeida Filho, F. M., 2002. Estruturas de pisos de edifícios com a utilização de cordoalhas engraxadas. Dissertação de mestrado, Escola de Engenharia de São Carlos - USP, São Carlos, Brasil.

[7] Hennrichs, C. A., 2003. Estudo sobre a modelagem da lajes planas de concreto armado. Dissertação de mestrado, Universidade Federal de Santa Catarina, Florianópolis, Brasil.

[8] Mello, A. L. V., 2005. Estruturas de pisos de edifícios com a utilização de cordoalhas engraxadas. Dissertação de mestrado, Escola de Engenharia de São Carlos - USP, São Carlos, Brasil.

[9] Reis, E. M., 2007. Análise de pavimentos de edifícios utilizando a analogia de grelha. Dissertação de mestrado, Universidade Federal de Santa Catarina, Florianópolis, Brasil.

[10] Dornelles, F. L., 2009. Estudo sobre a modelagem da protensão em lajes lisas com o uso de analogia de grelhas. Dissertação de mestrado, Universidade Federal de Santa Catarina, Florianópolis, Brasil.

[11] Carneiro, A. L., 2015. Análise e dimensionamento de lajes lisas protendidas sem aderência. Dissertação de mestrado, Universidade Federal do Espírito Santo, Vitória, Brasil.

[12] Nobre, K. M. A., 2017. Comparação entre métodos de análise de lajes protendidas. Dissertação de mestrado, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brasil.

[13] MacRae, A. J. & Cohn, M. Z., 1987. Optimization of prestressed concrete flat plates. *Journal of Structural Engineering*, vol. 113, pp. 943–957.

[14] Kuyucular, A., 1991. Prestressing optimization of concrete slabs. *Journal of Structural Engineering*, vol. 117, pp. 235–254.

[15] Lounis, Z. & Cohn, M. Z., 1993. Multiobjective optimization of prestressed concrete structures. *Journal of Structural Engineering*, vol. 119, pp. 794–808.

[16] El Semelawy, M., Nassef, A. O., & El Damatty, A. A., 2012. Multiobjective optimization of prestressed concrete structures. *Journal of Structural Engineering*, vol. 39, pp. 5758–5766.

[17] Talaei, A. S., Nasrollahi, A., & Ghayekhloo, M., 2016. An automated approach for optimal design of prestressed concrete slabs using PSOHS. *KSCE Journal of Civil Engineering*, vol. 21, pp. 782–791.

[18] Mohammed, A. H., Tayşi, N., Nassani, D. E., & Hussein, A. K., 2017. Finite element analysis and optimization of bonded post-tensioned concrete slabs. *Cogent Engineering*, vol. 4, pp. 1–16.

[19] Mohammed, A. H., Nassani, D. E., Tayşi, N., & Hussein, A. K., 2018. Nonlinear finite element model for the optimization of post-tensioned one-way concrete slab. *KSCE Journal of Civil Engineering*, vol. 22, pp. 2519–2527.

[20] Colonese, S., 2008. Comparação entre métodos de análise para lajes lisas protendidas com cordoalhas engraxadas: Estudo de casos. Dissertação de mestrado, Universidade Estadual do Norte Fluminense Darcy Ribeiro, Campo dos Goytacazes, Brasil.

[21] ABNT, 2014. *NBR 6118: Projeto de estruturas de concreto – Procedimentos*. Rio de Janeiro, Brasil.

[22] AMERICAN CONCRETE INSTITUTE, 2019. *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (318R-19)*. Farmington Hills, MI.

[23] Hambly, E. C., 1976. *Bridge Deck Behaviour*. Chapman and Hall Ltd.

[24] Aalalmi, B., 2014. *Post-Tensioned Buildings: Design and Construction*. PT-Structures, first edition.