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Lagrangian Relaxation Applied to Combinatorial Reverse Auctions for the Electricity Sector: Variation of Sub-gradient Method Parameters

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Abstract. Some optimization problems are $\mathcal{NP}$-hard and they can be intractable when solving for big instances exactly. However, if most of the set of constraints has proper structure, then Lagrangian Relaxation can be a suitable approach. Anyway, the duality gap for the non-convex problem can remain large, if no good upper-bound for the objective function can be found. This paper presents a Lagrangian Relaxation which deals with some problem parameters to provide better objective function bounds, by improving the sub-gradient algorithm. It is applied to solve the Winner Determination Problem (WDP) of a Combinatorial Reverse Auction (CRA) for the Brazilian Electricity Sector, an auction that allows bids on packages of generation power plants. The WDP is formulated as an Integer Optimization Problem to allocate the packages that minimizes the sum of accepted bids. Besides, packing rules are applied to ensure a totally unimodular matrix for the relaxed sub-problem constraints, which is solved by a linear optimization method.

Keywords: Lagrangian Relaxation, Sub-Gradient Method, Combinatorial Reverse Auction

1 Introduction

Nowadays the electricity sector in Brazil contracts energy in a Regulated Contracting Environment through simultaneous reverse auctions. However, the items offered in these auctions can be complementary from the participant's perspective. Thus, a Combinatorial Reverse Auction (CRA) could benefit the sector increasing its efficiency.

Implementing a combinatorial auction is not trivial, as participants are allowed to bid on packages of items, winning the whole package or none of them. The constraints of the problem imply that no item can be allocated in more than one winning package. To avoid the trivial solution in CRA, it is necessary an additional constraint to guarantee the demand's fulfillment. This problem is known as the Winner Determination Problem (WDP) and it can be formulated as a Binary Integer Linear Programming (BILP) problem. Müller [1] classifies most WDPs in the class $\mathcal{NP}$-Hard.

Rothkopf et al. [2] proposed a series of packing rules to achieve tractable instances of the WDP in a combinatorial auction, among then, there is the INTERVAL BIDS rule. This rule implies that packages include only adjacent items, so the packing matrix is totally unimodular and the WDP could be solved with Linear Programming (LP) methods.

However, the additional demand constraint destroys the unimodular property. Albieri et al. [3] mentions that INTERVAL BIDS rule could be used in CRA by relaxing the complicating constraint. Thus, the relaxed sub-problem is solved by LP methods and its result provides bounds for the proposed method. Modifications in the sub-gradient algorithm are used to accelerate the convergence process by generating better Lagrange multipliers (λ).

In this context, this paper aims to evaluate the sensibility of proposed parameters in the sub-gradient algorithm and how it affects the convergence of the Lagrangian multipliers λ.

2 Lagrange Relaxation in Combinatorial Reverse Auction

Each allowed package P_m, $\forall m \in \{1, \dots, M\}$, is formed by combinations of n items, $\forall n \in \{1, \dots, N\}$. The best bid on a package P_m is represented by c_m (\$/year). The matrix $\mathbf{A}_{N\times M}$, $\forall m \in \{1, \dots, M\}$ and $\forall n \in \{1, \dots, N\}$, is the packing matrix and it represents the formation of packages. The vector $\mathbf{b}_{1\times M}$, $\forall m \in \{1, \dots, M\}$, contents the generation capacity of each package. A decision variable x_m, $\forall m \in \{1, \dots, M\}$, is used to identify the winning packages. The WDP formulated as a BILP problem is presented in Equation (1):

$$\begin{aligned} z = \underset{x}{\text{minimize}} \quad & \mathbf{c}^{\mathbf{T}}\,\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\,\mathbf{x} \le \mathbf{e}, \\ & \mathbf{b}\,\mathbf{x} \ge D, \\ & \mathbf{x} \in \mathbb{B}^m \end{aligned} \tag{1}$$

being D the energy demand to be contracted. The first set of constraints ensures that no items are allocated in more than one winning package. The second constraint guarantees the energy demand is reached. The last constraint guarantees that the decision variable x_m belongs to the binary set.

Formulating the WDP as a Linear Programming (LP) problem and relaxing the last constraint makes the problem mathematical tractable if the relaxed solution has integer values.

Müller [1] mentions that to obtain integer solution, it is necessary to restrict the packages in which participants bid in such a way that the constraint matrix, formed by the vertical concatenation of the matrix $\mathbf{A}$ and the vector $\mathbf{b}$, is totally unimodular. The INTERVAL BIDS rule, in which items are organized in a list and participants can bid only in packages of consecutive items, guarantees that the packing matrix $\mathbf{A}$ is totally unimodular. However, the second constraint of the WDP breaks the totally unimodularity of the constraint matrix. In this case, applying the Lagrangian relaxation becomes a good alternative of solution, as it allows to relax the complicating constraint by penalizing its violation in the objectuve function. The Relaxed Winner Determination Problem (RWDP) is formulated in Equation (2):

$$\begin{aligned} z_R = \underset{x_R}{\text{minimize}} \quad & \mathbf{c}^{\mathbf{T}}\,\mathbf{x_R} + \lambda\,(D\ -\ \mathbf{b}\,\mathbf{x_R}) \\ \text{subject to} \quad & \mathbf{A}\,\mathbf{x_R} \le \mathbf{e}, \\ & 0 \le \mathbf{x_R} \le 1 \end{aligned} \tag{2}$$

being $\lambda \ge 0$ the Lagrange multiplier. In this way, whenever a solution $\mathbf{x_R}$ violates the constraint $\mathbf{b}\ \mathbf{x} \ge D$, the value of the objective function is penalized.

The Lagrangian Dual Problem (LDP) seeks to find better values of the Lagrange multipliers in order to improve the Lower Bounds (LB). It is formulated as:

$$z_D = \underset{\lambda \ge 0}{\text{maximize}} \quad z_R \tag{3}$$

There is an interaction between LDP and RWDP problems. In each round k, the LDP problem provides values of λ^k to RWDP, which provides x_R^k to LDP.

The proposed Lagrangian Relaxation aims to reduce the difference of Upper Bound (UB) and LB to be less than a δ. The duality gap can be reduced by adjusting the parameters of the Lagrangian Relaxation. The solution of the RWDP z_R provides the LBs, while feasible solutions of the WDP provides the UBs.

Algorithm 1 presents the logic of the Lagrangian Relaxation proposed. Values for some parameters need to be predefined: λ^1, LB, UB, and the maximum number of rounds. The RWDP is solved providing z_R and x_R and then, LB can be updated. If x_R is feasible in the original problem the UB is updated according to the defined criterion. If the duality gap between the LB and the UB is greater than δ, the sub-gradient method calculates the Lagrangian multiplier for the next round λ^{k+1}. The Lagrangian Relaxation stops either when a good solution is found or when the method reaches the maximum number of rounds.

set a value to Lagrange multiplier λ;
set LB $= -\infty$ and UB $= \infty$;
start rounds;
for *round* $= 1$ *: maximum number of rounds* **do**
    solve RWDP;
    if $z_R > LB$ **then**
        LB $= z_R$;
    end
    if x_R *is feasible in WDP* **then**
        if $z_R < UB$ **then**
            UB $= z_R$;
            $x_F = x_R$ is the best feasible solution found;
        end
    end
    if $UB - LB \leq \delta$ **then**
        break;
    end
    calculate Lagrange multiplier λ with Sub-gradient Method;
end
Best feasible solution found $= x_F$;
Best objective value found $=$ UB;

Algorithm 1: Lagrangian Relaxation Algorithm.

3 Sub-gradient Method

The Sub-gradient Method calculates the Lagrangian multipliers of the next rounds based on the gradient, which shows an improvement direction, and the step size to be taken. The gradient G^k in round k is calculated as shown in Equation (4). If the relaxed constraint is not met, then the gradient is positive and it indicates an increase in the penalization.

$$G^k = D \;-\; \mathbf{b}\;\mathbf{x_R^k} \tag{4}$$

Step size considers the current UB and LB and the value of the gradient. A big difference in the bounds indicates that the problem is far from a good solution and step size should be greater. The step size is calculated as formulated in Equation (5).

$$t^k = \frac{\pi\ (UB - LB)}{\sum (G^k)^2} \tag{5}$$

where $\pi \in [0, 2]$.

Given a value of λ^k, the Lagrangian multiplier of the next round λ^{k+1} is generated according to Equation (6).

$$\lambda^{k+1} = max(0,\;\; \lambda^k + t_k \cdot G^k) \tag{6}$$

Modification 1

Frangioni et al. [4] present a deflection parameter to deflect the gradient in the Equations (5) and (6). This parameter considers the gradient calculated in two subsequent rounds, the current and the former. The parameter $\alpha \in (0, 1]$ can be adjusted to get better convergence. The deflection parameter is calculated as shown in Equation (7). In the first round, the deflection is equal to the gradient.

$$d^k = \alpha\ G^k + (1-\alpha)\ d^{k-1} \tag{7}$$

Modification 2

If the sign of the gradient remains the same in η rounds, the variable π is multiplied by a factor F_1 such that $F_1 \geq 1$. This modification aims to accelerate the convergence process.

Modification 3

Every η consecutive rounds, if the Modification 2 has not been applied, the variable π is multiplied by a factor F_2 such that $0 < F_2 \leq 1$. This modification aims to avoid *zig-zagging* behavior.

Modification 4

Based in some parameters of sub-gradient method, it is proposed a new method to calculate the Lagrangian multiplier, as shown in Equation (8).

$$\lambda^{k+1} = max\left(0,\ \lambda^k\ \left(1 \pm \frac{\pi\ (UB - LB)}{UB}\right)\right) \tag{8}$$

where $\pm$ is indicated by the sign of deflection d^k.

4 Experiments

The Lagrangian relaxation seeks the best value of the penalty (λ) that should be applied to the relaxed constraint to problem provides good bounds. The sub-gradient method determines the values of λ that depends on some parameters like, step size, π, gradient and bounds. In order to determine the effect of those modifications on the sub-gradient results, we ran experiments to compare the convergence for LB and UB. The difference between these bounds indicates the problem's duality gap and the tighter the gap, the closer is the result to optimal.

4.1 Methodology

The products offered in the Combinatorial Reverse Auction (CRA) are energy generation assets. For each asset, a value-price and a capacity generation is calculated based on a normal distribution curve. For each package, a bid price was proposed based on the value-price of their components and applying a variance of $\pm 30\,\%$.

There are 150 energy assets offered that result in 11.325 packages. Only the best bid received for each package is considered. The auctioneer's objective is to met the energy demand and the packing constraints at the lowest possible cost. The energy demand to be contracted is $D = 5.715,65\,\mathrm{MW}$ and correspond to 40 % of the generation capacity of the assets offered.

The initial λ^1 corresponds approximately to the average value of the received bids $[410.000\,\$/\mathrm{year}]$. The maximum number of rounds is set at 300 and π is equal to 1.

A modified sub-gradient method is defined by the application of the modifications 1, 2, 3 on the sub-gradient method. Initially, all parameters (α, $F1$ and $F2$) have a value equal to one. Then, three experiments were conducted varying each parameter to calculate the Lagrangian multipliers using the modified sub-gradient and modification 4. The number of consecutive rounds to vary parameter $F1$ and $F2$ was set $\eta = 5$. The values of the parameters used for each experiment are shown in Table 1.

4.2 Results

The results found for the WDP formulated as BILP by exact methods were $Z^* = 1.673.022.642\,\$/\mathrm{year}$ and a energy capacity of winning packages $\mathbf{b}\ \mathbf{x}^* = 5.715,83\,\mathrm{MW}$. The result shows winning packages with relative

Table 1. Parameters sub-gradient modified

	Exp. 1	Exp. 2	Exp. 3
α	0,5	0,75	1,0
$F1$	1,0	1,50	2,0
$F2$	0,5	0,75	1,0

high values of price ($\approx 330.000\,\$/\mathrm{MW}$) and relative low values of capacity ($\approx 10\,\mathrm{MW}$), which are included to adjust to demand.

Characterization of the problem was performed to understand the results of a relaxed problem according to Lagrangian multiplier variation, as shown in Figure 1.

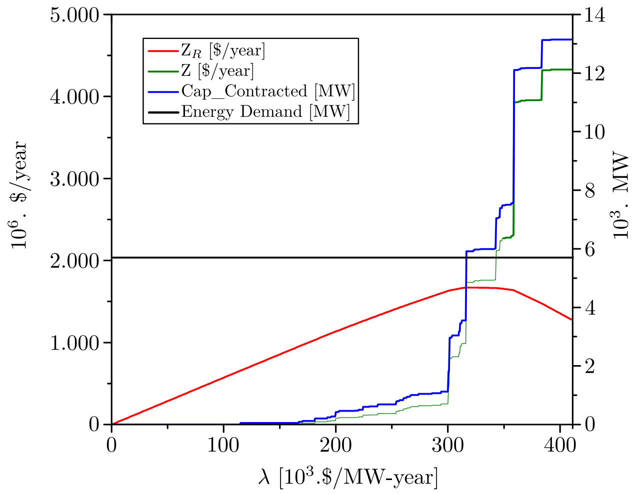

Figure 1. Z_R, Z and contracted capacity based on the result of relaxed problem according to Lagrangian multiplier

As noted in Figure 1, for low λ, the capacity contracted by the relaxed problem does not meet the required demand, because it is cheaper to pay the penalization than to contract capacity. As λ increases, the amount of capacity contracted and Z_R increase until the demand is met. From that point, λ represents a bonus that decreases Z_R. With greater λ, the tendency is to contract greater energy capacity, achieving lower Z_R.

Small variations in λ generate large variations in the contracted capacity and the gradient G. As shown in Figure 2, the sudden reduction of the gradient module implies a sudden increase in the step size. This behavior moves λ away from the convergence value.

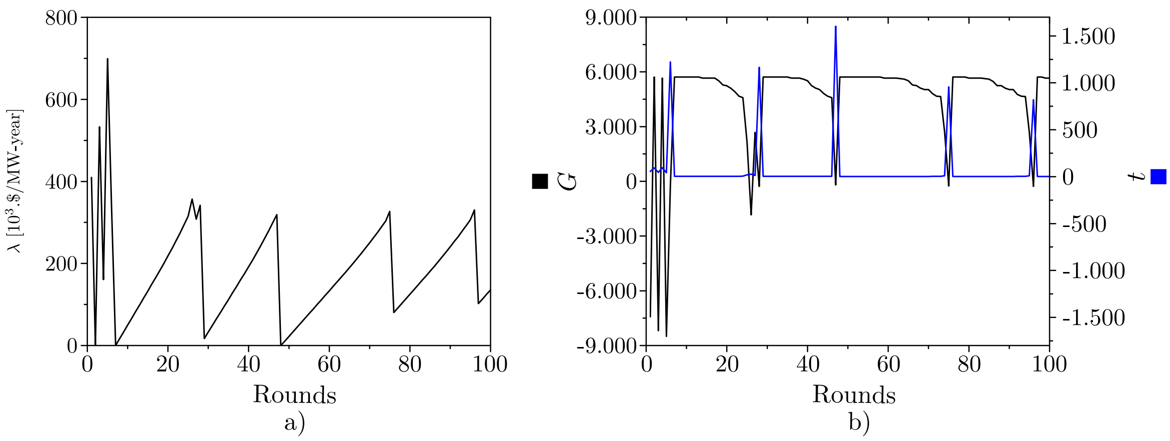

Figure 2. Variation of parameters using Sub-Gradient Method: a) Lagrangian multiplier, b) gradient and step size.

A new formulation to calculate the Lagrangian multiplier is proposed in Equation (8) to avoid the previous step behavior. The λ^{k+1} value is based on percentage of λ^k. This percentage depends on how close the boundaries of the problem are to each other, and the π value, which depends on the parameters $F1$ and $F2$.

The usage of α on a problem with only one relaxed constraint does not present good results. Including the weighing of gradient values of past rounds can mislead the improvement direction of λ. With lower values of α more rounds are necessary to correct this direction. The behavior of λ according to rounds for different values of α is shown in Figure 3.

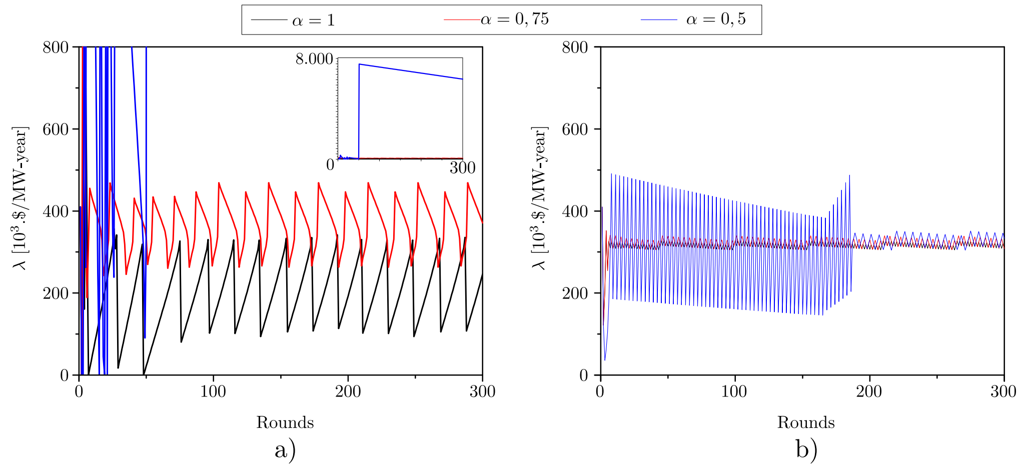

Figure 3. Behaviour of Lagrangian multiplier in rounds varying α: a) modified sub-gradient, b) modification 4.

The increase of π multiplying by a factor of $F1$ accelerates the process of convergence of λ when it is far from the convergence point, with a higher step size. However, the increase of π does not allow a small variation of the Lagrangian multiplier when it is close to the convergence point, generating a *zig-zagging* behavior. The behavior of λ according to rounds for different values of $F1$ is shown in Figure 4. When the convergence is already fast π is not modified and all experiments have the same result, as can be seen in Figure 4.b).

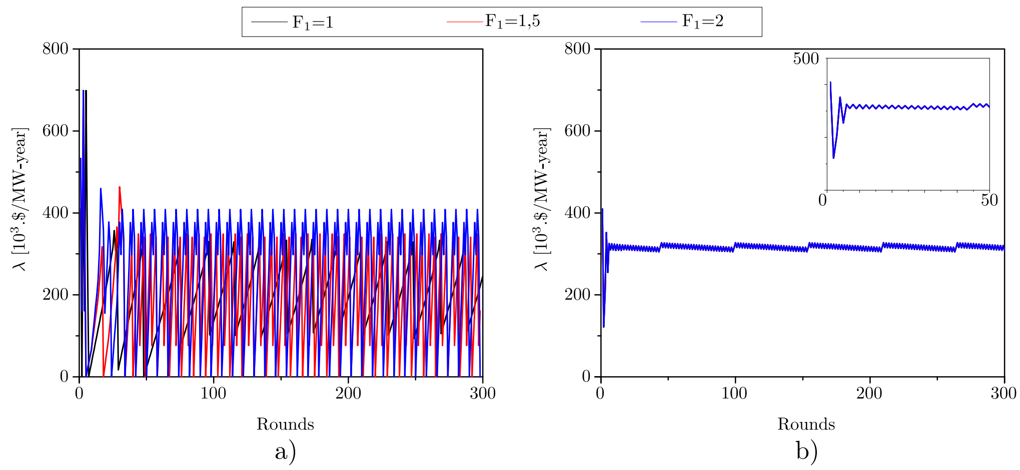

Figure 4. Behaviour of Lagrangian multiplier in rounds varying $F1$: a) modified sub-gradient, b) modification 4.

Multiplying π by a factor of $F2$ decreases the variation of the Lagrangian multiplier. When λ is far from the convergence point it can take more rounds to change the sign of gradient. $F2$ helps to avoid the *zig-zagging* behavior, allowing to reach the limit of λ that changes the gradient's sign. The behavior of λ according to rounds for different values of $F2$ is shown in Figure 5.

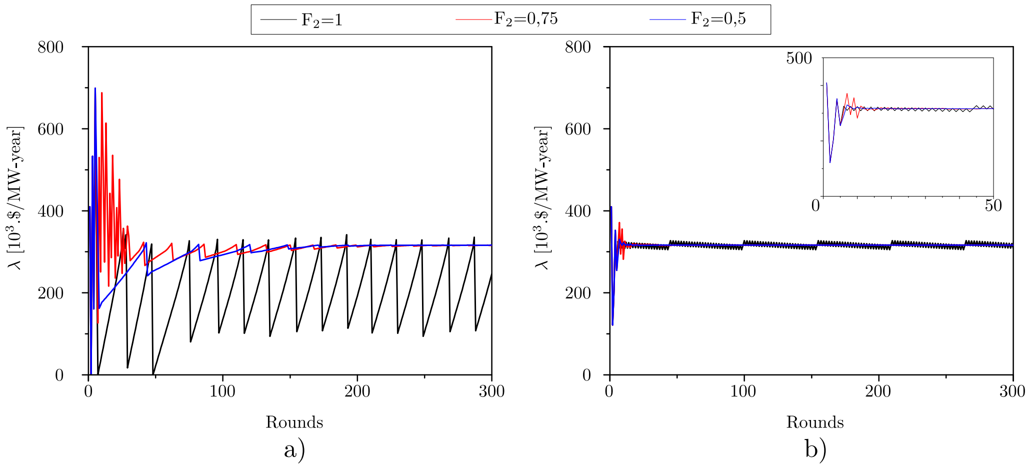

Figure 5. Behaviour of Lagrangian multiplier in rounds varying $F2$: a) modified sub-gradient, b) modification 4.

5 Conclusions

The electricity sector should consider implementing the Lagrange Relaxation to solve the WDP, whenever the demand is an estimated value. This method does not allow that packages with an average value above the Lagrangian multiplier limit win, preventing that more expensive packages are included in the result just for better fulfillment of the demand.

The α parameter did not show any improvement in any of the methods. As there is only one relaxed constraint, the deflection parameter has the same direction in all rounds, changing the module and the orientation. The $F1$ parameter accelerates the convergence of λ values, however without a mechanism that reduces the variation of λ there is a worsening in its convergence value, performing *zig-zagging* with greater amplitudes. The $F2$ parameter shows good results, as it allows reaching the inflection point $\lambda = 316.137,3$. Results of the relaxed problem next to this point indicates the best duality gap found for the case study.

The Sub-Gradient method presents a *zig-zag* profile because of the great variation of the gradient value G when λ is close to its convergence point. In order to detach the Lagrangian multiplier variation with the gradient, Modification 4 was proposed. This new method presents faster convergence for all the experiments, especially with lower values of $F2$.

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