XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Brazilian Multiplex Airline Networks in the context of privatization

Fernanda S. Toledo[1], Nelson F. F. Ebecken[1]

[1]*Dept. of Civil Engineering, Federal University of Rio de Janeiro*
Av. Pedro Calmon, 550, 21941-901, Rio de Janeiro/RJ, Brazil
Fernanda.toledo@coc.ufrj.br, nelson@ntt.ufrj.br

Abstract. Multiplex networks are a particular set among multilayer systems, which is represented by multiple layers and each layer consists of a network with one type of interaction. Many real-world systems would be better described as a multiplex network rather than a single-layer network. Air transportation is an example of a complex infrastructure system and the introduction of the weight of interactions and multilayer aspect make their description even more detailed. The concession of main Brazilian airports (a type of privatization) started in 2011, currently, 22 airports are in concession and after the sixth round this number will be doubled. The firsts contracts had problems with huge mandatory investment not compatible with demand, therefore the current and next contracts need to be adjusted, also considering an evaluation of the airports' role in the network to enhance its robustness. This paper aims to characterize structural properties using topological analysis of the Brazilian multiplex Air Transportation Network, comparing the network of 2019 (after the fifth concession round) and 2011 network (before the first concession started) to measure the granted airports versatility and its variation after the private management. The multilayer approach is also compared with an aggregated approach, where airlines compose the multiple layers. Data related to domestic and international passenger traffic of both years were collected at ANAC database and processed using MuxViz software. The proprieties obtained are used to network characterization including node centrality measures, multiplexity, degree and strength. Results show a network concentration during this period, with a demand increase but half operating airlines and less available routes, increasing the demand for larger aircraft. Results can provide an understanding of the versatility of airports, giving an insight into the new investments to consider the airports' network role in the next years.

Keywords: multiplex networks, air transportation, network topology, airport privatization.

1 Introduction

Complex networks have largely been used to model structures of relations in real-world systems, but it was recently that limitations of this approach have been studied. The most important limitation of this application in the real-world system is its multi-layer nature. It means nodes can belong to different layers at the same time, including different neighborhoods for each layer. Networks constructed with empirical data usually present structural features as heavy-tailed degree distributions, small-world propriety and modular structures. Those characteristics are useful implications to information diffusion, robustness considering component failure and other applications (De Domenico et al [1]). For describing multiplex networks, we need to define and quantify interconnectivity between multiple types of connections.

Traditional studies of networks usually consider that edges that connect nodes are static and encapsulates all connections between the two nodes. It is an oversimplification that can lead to misleading results and an inability to address some problems (De Domenico et al [1]). Ignoring the presence of different types of edges, called "multiplexity" hinder the presence and relevance of multiple nodes of transportation, while multiplex networks have the advantage of incorporating multiple channels of connectivity. Multiplex patterns have a fundamental role in dynamical processes and are responsible for new physical phenomena, which are not observable in a single-layer form (Nicosia and Latora [2]).

Multilayer networks are composed of multiple layers that can represent different types of system such as social networks, metabolic networks, transportation systems. Air transportation system can be represented by a network where nodes are operating airports and the edges are direct flights among two airports. Nowadays, we have moved from a sparsely connected transportation system to a redundant one with a capacity to move 2.7 billion passengers in 2011 (Cardillo et al [3]). Scientists have researched properties of airline transportation systems by network theory reveling structural characteristics, but the application of the multilayer approach is yet new. Moreover, to predict and understand dynamical processes, we first need to evaluate the structure of the networks.

In Brazil, airports concessions (a type of privatization) started in 2011 with Natal International Airport (SBSG), followed in 2012 by Brasília (SBBR), Guarulhos (SBGR) and Viracopos Airport (SBKP). In 2013 Galeão (SBGL) and Confins (SBCF) were granted and from 2019 the concessions were made through clusters (Northeast, Southeast and Midwest), which encompass 12 airports, as shown in Tab. 1.

The sixth round is scheduled for 2021 with the North, South and Central clusters, encompassing 22 airports (11% of paid passengers handled in the Brazilian airline industry) and participating in the scope of the Investment Partnerships Program (PPI) and the National Privatization Program (PND). The concessions aimed at improving the quality of service at these airports and accelerate the infrastructure expansion necessary to meet the growth in demand for and also holding major events such as the FIFA World Cup 2014 and the Olympic Games in 2016.

Table 1. Brazilian Airport Concessions

Year	Concession round	Airports
2011	First round	Natal (SBSG)
2012	Second round	Brasília (SBBR), Guarulhos (SBGR) e Viracopos (SBKP)
2013	Third round	Galeão (SBGL), Confins (SBCF)
2017	Fourth round	Fortaleza (SBFZ), Salvador (SBSV), Florianópolis (SBFL), Porto Alegre (SBPA)
2019	Fifth round	**Northeast Cluster**: Recife (SBRF), Maceió (SBMO), Aracajú (SBAR), João Pessoa (SBJP), Campina Grande (SBKG), Juazeiro do Norte (SBJU).
		Southeast Cluster: Vitória (SBVT), Macaé (SBME)
		Midwest Cluster: Cuiabá (SBCY), Sinop (SBSI), Rondonópolis (SBRD), Alta Floresta (SBAT)
2021 *Predicted	Sixth round	**South Cluster**: Curitiba (SBCT), Foz do Iguaçu (SBFI), Londrina (SBLO), Bacacheri (SBBI), Navegantes (SBNF), Joinville (SBJV), Pelotas (SBPK), Uruguaiana (SBUG), Bagé (SBBG).
		Central Cluster: Goiânia (SBGO), Palmas (SBPJ), Teresina (SBTE), Petrolina (SBPL), São Luís (SBSL), Imperatriz (SBIZ).
		North Cluster: Manaus (SBEG), Tabatinga (SBTT), Tefé (SBTF), Rio Branco (SBRB), Cruzeiro do Sul (SBCZ), Porto Velho (SBPV), Boa Vista (SBBV).

The experience of the first rounds of concessions had a basic and clarifying role for changes in the economic regulation model of concession contracts for the 5th round. In this round, there was a flexibilization of the economic regulation, when the ceilings of the fare categories were replaced by a ceiling of average fare revenue per passenger, in addition to the fare deregulation depending on the size of the airport and user proportion. The Supported Proposal instrument was also included, allowing airport operators to propose changes to the Concession Contract to ANAC (National Civil Aviation Agency). The sixth round of concessions offers another opportunity for improvements in the model of economic regulation of granted airports, in the sense of greater flexibility and commitment between users and airport operator. That is the reason it is important to evaluate the development of the concessions in different areas, be it operational efficiency, economic efficiency, service level or the development and role in the decentralization of the Brazilian air network, following developed countries.

The Central cluster of the sixth round has a strategic geographical position and may operate as a domestic hub, connecting to the main routes and in the future as an international one (the case of SBTE and SBSL). The Northern Cluster stands out for SBEG, but all of them have a high potential for air traffic considering the distance and difficult land access in the North Region. While the Southern Cluster is the largest in terms of the number of airports and paid passengers carried (5.6% of the Brazilian market) (ANAC [4]).

In complex networks, different measures depending on the application of interest define centrality in terms of activity, control, communicability and independence (De Domenico [5]). Identifying the most central agents in complex networks is of huge importance as they are responsible for the fastest spread of information, epidemics and failures. Structural proprieties of Air Transportation Multiplex Network evolve as layers are progressively merged until the aggregated model. Given that, important structural features emerge as a consequence of the multilayer character of the system, such as Rich-club effect (in which major airlines are responsible), path redundancy (due to a combination of clustering airlines) or small-worldness (Cardillo et al [3]). Thus, the multilayer approach is the next step to a better and full comprehension of air transportation systems.

The objective of this paper is to evaluate the role of airports granted in the Brazilian Air Transportation Network, applying science of complex networks with a multilayer approach comparing with an aggregated network. In addition, a comparison was made of the centrality of airports in the 2011 network (before the concessions were implemented) and the 2019 network (after the fifth round of concessions). The centrality of the airports of the sixth round of concessions will also be evaluated, to assist in the allocation of resources to increase the strength of the network in case of inactivity of any airport.

2 Multiplex Networks applied to Air Transportation Network

Structural proprieties are characterized for ATNs (Air Transportation Network) in Guimera et al [6]; Colizza et al [7] and dynamical processes in Lacasa et al [8]. Until then, multiplex nature was unexplored. Nicosia et al [2] considered routes of continental airlines (in Africa, Asia, Europe, North America, Oceania, South America). In airline networks, the degree distribution follows a power law with an exponent ranging from 1.8 to 2.3. They found that the majority of airports tend to be connected by a small number of airlines (most of them are active in less than five layers). The outliers are active in 10-30% of layers and a large fraction of layers has 10 active nodes in maximum. Therefore, in the case of continental airlines, a relatively small fraction of nodes is active on at least two layers, just in a few cases the multiplexity is high as 20%. Furthermore, national companies tend to have a large overlap with airlines, while low-cost airlines avoid the overlaps.

European Air Transport Network was studied by Cardillo et al [3] using a multiplex network, they considered each airline as an independent network and evaluated the system resilience on random flight failures in the passenger’s rescheduling. A comparison between a single-plex approach and the multiplex one was used to show the effects of multiplexity on the robustness of the network. They studied if topological properties of multilayer networks can be traced to the properties of the layers or if they emerge from the multiple layers’ existence, comparing the network composed of various layers and the aggregate representation. As a result, they found that topological properties are not usually present in single layers networks, which means they are a consequence and emerge of the multilayer phenomenon of the system, such as the Rich-club effect, path redundancy and small worldness (if low-cost airlines are present). Considering the multiplex character of real systems can contribute to better understanding and also modelling dynamical processes.

Still using airlines in Europe, Mittal and Bhatia [9] also use airlines as layers and data of four major airlines. They proposed calculating bottleness centrality of nodes in a new metric and found 70% of the bottleneck on an airport can be reduced for routing multiple airlines when using a multilayer hierarchy. The value varies from the aggregated network, so considering multilayer can reduce overall bottleneck on nodes and uncover ignored features if using a single layer analysis.

Cinelli [10] found US airports network display a rich-club ordering to degree, eigenvector centrality and metadata. Node metadata used are the number of flights departing from an airport and the number of passengers. They find that arrangement of node metadata is way different from a random reshuffling, therefore significant and difficult to replicate.

Oliveira et al [11] proposed a Multiplex Efficiency Index to quantify fluctuations in network efficiency. They evaluated the evolution of Brazilian domestic air network between 2010 to 2018 in a multiplex perspective but considering non-directed networks. They used monthly and annual networks and concluded that the efficiency index increases with the network concentration in 2018, also four of six privatized airports had a greater or equal value of the index than before privatization. Although, concluded that privatizations not necessarily imply these changes but the expansion of airports, even for public ones.

In general, in Air Transportation Multilayer Network, the layers can be the operating airlines, a set of airports based on their connectivity (if central or peripherical) or domestic and international layers. While the weight of an edge can be the number of available seats in flights, number of passengers, number of flights, cargo handled.

3 Method

To characterize the Brazilian Air Transportation Network (ATN), Complex Networks theory was used, in which the nodes are the operating airports and the edges are the direct flights between them. The multilayer structure was used because of the system characteristics and advantages compared to aggregate network, as the multilayer analysis does not ignore the links between layers, analysing intra-layer and inter-layer edges. The transportation system also has different characteristics compared to other networks as a more efficient spreading process in the multilayer structure [1]. Last researches show that interconnected networks have specific structural and dynamical properties that can not be assumed from isolated networks.

Each layer represented each of the airlines that operate in Brazil, therefore each layer has a different number of nodes and edges. The network studied here is directed and weighted, the weight is the number of passengers transported during all the year between two airports. Two multilayer networks were processed, one using data of 2011 and another for 2019. For this, statistical data available by ANAC were used for the years 2011, before the first concession started, and 2019, after the fifth round of concessions. Data from the main airlines were used, considering airlines that handle at least 0.5% of total air traffic, excluding unproductive flights and flights without paying passengers. For the weight, the sum of paying passengers and free passengers was used.

The data were pre-processed to the appropriate format and processed using the MuxViz software (De Domenico et al [12]). Layer characteristics were obtained as the number of nodes, connections, density, the number of connected components, diameter and mean path length. The correlation regarding node overlapping, edge overlapping, Pearson degree was also obtained and the centrality measures: strength, degree, PageRank, Eigenvector, Hub, Authority, Multiplexity. Centrality is a measure of importance and influence of a node in a network and represents a node versatility in the multilayer approach. We compared the versatility of granted airports in 2019 with the versatility of 2011 and the average value for airports grouped by round of concession.

The fundamental theoretical assumptions and math formalism on which this study is based come from the study of De Domenico et al [1], De Domenico et al [5], De Domenico et al [12]. The tensorial formulation of multilayer networks can overcome limitations, while aggregated information or a separate structure for each layer can lead to misleading results and can also generalize the centrality measures usually adopted. Nodes might (or not) exist in all the layers (measured by multiplexity). Also, there can be interlayer links between nodes and counterparts in different layers (multiplex networks) or between nodes in different layers (general multilayer).

The most famous measure of centrality is PageRank, the ranking measure of a Google search engine. It is a steady-state solution where a random walker jumps to a neighbour (rate r) and teleport to another node (with rate r'). The eigenvector versatility is a ranking of nodes according to their central role. It is one of the most common measures of centrality and is based on an iterative process where the node's score is the sum of the score of its neighbours, which is the largest eigenvalue and corresponding eigenvector of the adjacency matrix. In this context, versatility is a promising description for exploratory analysis in categorized data set (De Domenico et al [5]).

Overlapping is an indication of the correlation between two networks. A mean global node overlapping measure the fraction of node which are common to all layers and a mean global edge overlapping measure the fraction of edges which are common to all layers, both measures the similarity between layers.

4 Results

In accordance with the results, major airlines are designed following the *hub-and-spoke* network structure in which one airport (the hub) is surrounded by low-degree nodes building a starlike graph. In that graph, the average path length measures the average number of steps we need to go from a node to another. In this case, it means the average number of flights a passenger has to take from their origin to their destination. Table 2 shows the results obtained from the multilayer networks of 2011 and 2019. In 2011 eight airlines were responsible for 99% of air traffic, while in 2019 only four airlines could reach this same proportion.

The multilayer air transportation network of 2011 has 8 layers, 153 nodes, 29031 edges and an average degree of 77.43. The 2019 air network have 159 nodes, 20951 edges and an average degree of 26.13. Comparing 2011 and 2019, there is a decrease in the average degree, the same noticed in Wandelt et al [13] using Brazilian dataset between 2002 and 2013. This may be a consequence of efforts to the consolidation of Brazilian air transportation and the privatization of airports.

For the 20 largest hubs in 2019, in general, there is a reduction in the hub score when considering the multilayer network compared with an aggregated network, only Guarulhos (SBGR) had an increase in the hub score in this comparison. SBGR also had the highest authority score when considering multilayers. Hub and authority scores are a centrality measure calculated by the HITS algorithm (a good hub connects to many nodes and a good authority is linked to many hubs), those scores are ranked and compared for both years.

In 2019, it can be seen that the airports of the first concession rounds present a multiplexity of 0.75 or 1 (Tab. 3), therefore, they are present in at least three of the airline's layers. As of the fifth round, the clusters started to contain airports with varied multiplexity, including 0.5 and 0.25. As multiplexity is the fraction of layers where node exists, therefore, it is not defined for aggregate networks.

SBSG was the first airport to be granted, when its terminal opened in May 2014 SBNT was deactivated for civil aviation, therefore both airports are compared in Tab. 3 due to their similar role in the network as they did not operate during the same period and serve the same area. The contracts for the Northeast, Midwest and Southeast Clusters were signed in September 2019, so it is not yet possible to notice management changes or changes in the network due to the concession. Moreover, it is necessary to consider other factors that may influence the network change during this period, such as the increase in demand especially until the 2014 World Cup followed by an economic and political crisis starting at the end of 2014, accentuated in 2016.

Table 2. Comparison of multilayer networks of 2011 and 2019

	Layers	Nodes	Edges	Density	Diameter	Mean Path Length
2019	Azul (AZU)	119	8839	74,3	2788	2,2
	Gol (GLO)	86	6415	74,6	5364	2,1
	Latam (TAM)	77	4952	64,3	7699	2
	Passaredo (PTB)	31	745	24	797	2
2011	Azul (AZU)	44	2370	53,9	4544	1,9
	Gol (GLO)	75	7710	102,8	4148	2,1
	Latam (TAM)	70	7224	103,2	4596	2
	Passaredo (PTB)	70	1621	23,2	744	2,8
	Avianca (ONE)	27	1296	48	6332	1,9
	Pantanal (PTN)	32	703	22	3453	2,2
	Trip (TIB)	99	6473	65,4	1702	2,5
	Webjet (WEB)	24	1634	68,1	3156	1,8

As for the first four concession rounds, most airports remained active in the four airline companies under study (Tab. 2), however, SBSG is not in the Passaredo network, while SBNT was. The same occurred with the SBCF, SBFZ, SBFL, SBPA airports, which had reduced multiplexity because they were no longer in the Passaredo layer. Even with less multiplexity, these airports had an increase in passenger movement (strength) except for SBSG. Granted airports had a degree reduction, but the same happened to public airports, therefore there is no indication of a privatization direct effect. The biggest increases in hub score were from SBGR and SBFZ, which also had the most expressive authority increases. Eigenvector centrality has been reduced in most of these airports, with an increase in SBGR, SBFZ, SBFL and SBSG.

Table 3 also shows the versatility of the airports composing the six concession rounds, the cluster average value and a comparison with 2011 (excluding airports with no available data: SBBI, SBBG in both years and SBRD, SBSI, SBPK and SBUG in 2011). Each cluster has its particular characteristics, the fifth and the sixth rounds have lower degree, PageRank, hub and authority score. It means they are composed of more regional airports that have can have a higher importance in the network depending on the new investments and management.

Although in general airports have an increase in passenger movement, there has been a reduction in the measured centralities. There is an effect of layers concentration as airlines have merged with others or faced a judicial liquidation process as the case of Avianca Brasil (ONE).

Overall, airport centrality values are lower when using the multilayer approach, with a lower eigenvector, hub, authority, Katz, k-core value, but a higher PageRank. The exception is for some airports, such as SBGR and SBSP. Therefore, the results corroborate with the literature that adding layers in an aggregated structure can lead to erroneous identification of the most versatile nodes, overestimating the importance of marginal agents and demonstrating a better forecast of their roles in the congestion of processes (De Domenico et al, 2014). It is also a consequence that nonlinear competition between layers is difficult to be examined *a posteriori*.

In the study of London airports (Cardillo et al, 2013), airports that were more central in the aggregate network and have become less important in the multilayer network because they have many connections distributed on few airlines. Less central airports in the aggregate network can become more versatile as their flights are operated by almost all airlines. The versatility also depends on the contribution of each node to the centrality by layer. In this context, Cardillo et al (2013) consider a versatility measure instead of the centrality measure when using the multilayer approach.

Centrality measures play a crucial role in spreading processes such as the transmission of epidemics to the spread of delays at airports. Therefore, the concept of versatility is important to understand the role of nodes in dynamic scenarios (De Domenico et al, 2015). Analysing the top ten ranking for each measure of versatility, it is noted that the values of hub, authority, eigenvector, are lower when using the multilayer approach compared to aggregate processing. However, as far as PageRank is concerned, multilayer analysis was beneficial for SBKP, SBRP, SBGO, SBSV, SBRJ, SBPA. Given that, PageRank versatility outperforms the standard PageRanks centrality measured in the aggregated network.

Table 3. Characterization of airports in concession and variation compared 2019 to 2011

Node	Strength		Degree		PageRank		Eigenvector		Hub		Authority		Multiplexity	
SBSG	2146762	-10.8%	79	-62.2%	0.06	-30.7%	0.15	18.3%	0.16	18.3%	0.15	17.3%	0.75	-14.3%
SBKP	10609710	45.0%	244	-27.8%	0.56	52.1%	0.04	-75.8%	0.04	-76.4%	0.04	-75.3%	1	14.3%
SBGR	32490462	43.8%	393	-28.7%	1.00	0.0%	1.00	60.3%	1.00	57.7%	1.00	62.0%	1	14.3%
SBBR	15909039	1.9%	229	-44.1%	0.41	-22.6%	0.72	-6.5%	0.75	-3.2%	0.72	-6.5%	1	0.0%
SBGL	9892044	-19.9%	220	-45.7%	0.25	-25.7%	0.31	-51.5%	0.32	-50.2%	0.30	-51.8%	1	0.0%
SBCF	10739764	14.9%	222	-42.9%	0.41	-9.6%	0.44	-20.1%	0.45	-18.1%	0.43	-20.5%	0.75	-25.0%
SBFZ	6547242	24.3%	147	-42.1%	0.18	10.1%	0.39	55.3%	0.41	63.5%	0.38	53.9%	0.75	-25.0%
SBSV	6487071	-24.9%	167	-52.6%	0.22	-40.8%	0.40	-13.7%	0.43	-9.5%	0.40	-14.6%	1	0.0%
SBFL	3489483	18.9%	100	-50.7%	0.10	-8.8%	0.25	10.7%	0.26	13.4%	0.25	9.8%	0.75	-14.3%
SBPA	7731640	4.1%	150	-51.3%	0.25	-12.3%	0.48	-0.2%	0.49	-0.7%	0.47	-0.9%	0.75	-25.0%
1^{st} round	2146762	-10.8%	79.0	-62.2%	0.06	-30.7%	0.15	18.3%	0.16	18.3%	0.15	17.3%	0.75	-14.3%
2^{nd}	19669737	29.6%	288.7	-33.3%	0.66	3.9%	0.59	13.5%	0.60	14.5%	0.58	14.3%	1.00	9.1%
3^{rd}	10315904	-4.9%	221.0	-44.3%	0.33	-16.4%	0.37	-37.0%	0.39	-35.4%	0.37	-37.3%	0.88	-12.5%
4^{th}	6063859	0.0%	141.0	-49.5%	0.19	-19.5%	0.38	6.9%	0.39	9.7%	0.37	6.0%	0.81	-16.1%
5^{th}	1608762	19.0%	63.6	-65.5%	0.07	-41.9%	0.07	-6.5%	0.08	-5.6%	0.07	-7.1%	0.56	-6.9%
6^{th}	1177216	-11.0%	59.0	-66.6%	0.05	-50.4%	0.08	-4.6%	0.08	-2.9%	0.07	-5.1%	0.65	14.7%

Unlike international ATN, Brazilian airlines operate at similar airports. According to Nicosia et (2015) in all the multiplex networks they considered, a small fraction of nodes is noted to be active on at least two layers simultaneously. For continental airlines there is a broad distribution, where the majority couples of layers have less than 1% of common nodes. Therefore, the multiplexity can be as high as 20% just in a few cases. For air transportation network, a competition between airlines are expected in the same area generating a small overlap in the layers. Meanwhile, national companies have a large overlap with other airlines layers, specially to serve a similar set of airports. In a different way, low-cost airlines tend to avoid overlap with other companies' layers.

The topological properties of the Air Transport Network are usually not present in single layers, but a consequence of the system multilayer character. As a significant fraction of nodes is active in multiple layers at the same time, the removal of a few nodes will not result in a massive disruption of the network, which could be an important strategy to slow down the diffusion of information or spreading of an epidemic (Nicosia and Latora, 2015).

Considering node overlapping in the layers, there is 41% of node overlapping between Azul and Gol, 34% between Azul and Latam. Passaredo has only 11-16% node overlapping with other airlines. This is a significant difference comparing the network of 2011, where Passaredo had 11-26% of node overlapping, Azul had 12-24% (24% with Gol), Latam and Gol had 34%. For the other airlines (ONE, TRIP, WEB, PTN) this value is smaller and only TRIP had 12-31% redundancy. Considering edge overlapping, Azul has 26% overlapping with Gol, 20% with Latam. Passaredo has only 0,2-1% edge overlapping, while Gol and Latam have 65%. Comparing with 2011, Passaredo used to have low edge overlapping with other airlines (0-4%). Azul had 10% of edge overlapping with

Gol, 6% with Latam, however, Gol and Latam had 73% of edge redundancy in 2011.

5 Conclusions

According to literature, important structural features emerge as a consequence of the multilayer character of the system, therefore the multiplex approach in real systems can contribute to better understanding and modelling dynamical processes. This paper aimed to evaluate changes in the multilayer Brazilian Air Transportation Network of 2011 (before first airport concession) and 2019, compare the multilayer network with the aggregated network and measure the versality of granted airports.

Comparing 2011 and 2019 Brazilian Air Networks, in 2011 eight airlines were responsible for 99% of air traffic and in 2019 only four airlines were able to handle the same proportion. The results show a network concentration during this period, with airlines merged with others or stopped operating, a decrease in the average degree and increase in the node strength. Moreover, there is also an increase in node and edge overlapping, therefore the need for larger aircraft to move more passengers in fewer routes. Unlike international ATN, Brazilian airlines operate at similar airports. Air transportation in other countries has a maximum of 20% of multiplexity, while in Brazil the minimum multiplexity is 25% in 2019.

When comparing the multilayer approach to the aggregated, nodes that have higher centrality measures in the aggregated network may not be as important in the multilayer in terms of versatility while airports that have a lower centrality than the hubs can be more versatile. In general, there is a reduction in the hub score when considering the multilayer network compared with an aggregated network, only SBGR had an increase in the hub score in this comparison and has the highest authority score.

Airports under concessions consist of main hubs and until the 4th round of concessions only airports with multiplexity higher than 75% were considered, the 5th and 6th round extended to regional airports with multiplexity of 25%. This indicates an opportunity to improve regional airports and enhance their versatility in the network.

Acknowledgements. The authors acknowledge the support and funding of CAPES and FAPERJ.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] De Domenico, M. et al. "Mathematical formulation of multi-layer networks." *Phys. Rev.* X 3, 041022 (2013).
[2] Nicosia, V., Latora, V. (2015). "Measuring and modeling correlations in multiplex networks". *Physical Review E*, 92(3).
[3] Cardillo, A. et al. "Emergence of network features from multiplexity". *Sci. Rep.* 3, 1344 (2013).
[4] ANAC. *Consultation on the economic regulation of airport concessions*. Brasília (2019).
[5] De Domenico, M. et al. "Ranking in interconnected multilayer networks reveals versatile nodes". *Nat. Commun*. 6:6868 (2015).
[6] Guimera, R., Mossa, S., Turtschi, A. & Amaral, L. A. N. "The worldwide air transportation network: Anomalous centrality, community structure, and cities' global roles". *Proc. Nat. Acad. Sci*. (U.S.A.) 102, 7794–7799 (2005).
[7] Colizza, V., Flammini, A., Serrano, M. A., Vespignani, A. "Detecting rich-club ordering in complex networks". *Nature Physics* 2, 110–115 (2006).
[8] Lacasa, L., Cea, M. & Zanin, M. "Jamming transition in air transportation networks". *Physica A* 388, 3948–3954 (2009). networks*." Journal of Complex Networks, 3(2), 159–176.*
[9] Mittal, R.; Bhatia, M. P. S. (2019). "Discovering bottlenecks entities in multi-layer social networks". *Journal of Discrete Mathematical Sciences and Cryptography, 22(2), 241–252.*
[10] Cinelli, M. (2019). "Generalized rich-club ordering in networks". *Journal of Complex Networks.* Vol 7. Issue 5, 702-719.
[11] Oliveira, I.M., Carpi, L.C. & Atman, A.P.F. "The Multiplex Efficiency Index: unveiling the Brazilian air transportation multiplex network—BATMN". *Sci Rep* 10**,** 13339 (2020).
[12] De Domenico, et al. (2014). "MuxViz: a tool for multilayer analysis and visualization of networks". *Journal of Complex Networks, 3(2),* 159–176.
[13] Wandelt, S., Sun, X. & Zhang, J. "Evolution of domestic airport networks: a review and comparative analysis". *Transpometr. B Transp. Dyn.* **7**, 1–17 (2019).