

# SHUNT CONTROL ON CANTILEVER BEAM BY NEURAL NETWORK: OBJECTIVE FUNCTION

Venicio S. Araujo<sup>1</sup>, Guilherme S. Prado<sup>1</sup>, Cayo C. Silva<sup>1</sup>, Heinsten F. L. Santos<sup>1</sup>

<sup>1</sup>Universidade Federal de Mato Grosso Avenida dos Estudandes, no 5.055, Bairro Sagrada Familia, 78736900 - Rondonópolis/MT, Brasil  
venicio.araujo@aluno.ufr.edu.br  
guilhermeprado@ufmt.br  
cayocespedes@hotmail.com  
heinsten.leal@cur.ufmt.br

**Abstract.** Piezoelectric materials have been extensively studied in recent years for the development of electromechanical harvesting devices. Usually connected to a structure, these kinds of materials convert kinetic energy into electric energy, and your electronic parameters interact directly to the vibrations of the system they are coupled on. Therefore, this work aims at comparing the use of genetic algorithms and artificial neural network techniques in the implement of shunt control in a structural set of a cantilever beam coupled to a piezoelectric layer in the piezo-beam configuration. For the architecture of the genetic algorithm and the neural network, was used a software with finite element model implemented and the comparisons were made analyzing the computational demand of the algorithms and their respective responses when both of them were defined on the task of finding the best combination between the parameters of resistance and inductance of the piezoelectric patch that result in the best damping to the structure. The comparison between the techniques had a focus on the use of the objective function of the system by them, parameter used as a metric to gauge the aggregate computational demand, and the damping provided with the respective configurations suggested by the two techniques. The results show that the neural network after training completes your execution in order of 102 seconds, much faster than the genetic algorithm, presenting a response with an average gain in damping of 23,24dB, but, even though faster, this technique demands much more iterations than the genetic algorithm, due to its nature of parallel computations, and additional care for the input data, that need a pre-processing not seen in the genetic algorithm technique.

**Keywords:** Shunt Control, Neural Network, Genetic Algorithm, Objective Function, Smart Structures

## 1 Introduction

The field of smart structures, or structures with integrated sensors and actuators, has arisen to offer improved vibration control in applications where passive techniques are either insufficient or impractical. The introduction of these materials with small volume, low weight and ease of structural integration, made piezoelectric sensors and actuators has been the overwhelming transducer of choice for smart structures. (Aphale et al. [1]). It is well known that there are a number of difficulties associated with the control of flexible structures, the foremost of which are: variable resonance frequencies; high system order; and highly resonant dynamics. Traditional control system design techniques such as LQG, H<sub>2</sub> and H<sub>∞</sub> commonly appear in research works and have been well documented. Unfortunately, the direct application of such techniques has the tendency to produce control systems of high order and possibly poor stability margins.

The design of controllers for smart structures is a challenging problem due to the presence of non-linearities in the structural system and actuators, the limited availability of control force, and the non-availability of accurate mathematical models. (Rao et al. [2]). In this scenario, vibration control of smart structures using neural networks has thus been receiving attention for their advantages in self-learning, fault tolerance, and parallel processing.

Works in this area has become common. We can cited some few example, as the study of Finite element analysis and design of actively controlled piezoelectric smart structures (Xu and Koko [3]), the proposition of a fuzzy-logic algorithm to the vibration suppression of a clamp-free beam with piezoelectric sensor/actuator. (Zeinoun and Khorrami [4]), or the developed of a nonlinear feed forward controller for smart structures, that showed

that the neural network is essentially a transversal filter with a nonlinear hidden layer between the input and output. (Snyder et al. [5]).

## 2 Equation of Motion for Beam Structure

For the study of the structure was adopted the classic beam model (piezoceramic-host).(Santos [6]). In this case was considered only the presence of deflection, disregarding the shear, thus Bernoulli Euler theory can be developed the equation of motion for the structure. With the applied theory of the Bernoulli can be developed the equation of motion for the structure, this form the structure-patches-circuits coupled equations of motion can be written as

$$\begin{bmatrix} M & 0 \\ 0 & M_q \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{D}_p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & C_q \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{D}_p \end{Bmatrix} + \begin{bmatrix} K_m & -\bar{K}_{me} \\ -\bar{K}_{me}^t & \bar{K}_e \end{bmatrix} \begin{Bmatrix} u \\ D_p \end{Bmatrix} = \begin{Bmatrix} F \\ F_q \end{Bmatrix}, \quad (1)$$

where  $M_q$  is the inertial vector due to the presence of resistance and inductance,  $u$  and  $D_p$  are the vectors global mechanical displacement and electric displacement dofs.  $M$ ,  $K_m$ ,  $\bar{K}_{me}$ ,  $\bar{K}$  are the mass and mechanical, piezoelectric and dielectric stiffness matrices and  $F$  is the mechanical excitation force vector.  $C_q$  and  $F_q$  are the matrix of the damping and the vector of force dues the presence of resistance and inductance, but how in this work is studded only the output mechanical the value of the vector of electric voltage is equal zero.

## 3 Finite Element Model of Piezoelectric Beams

The structure is a fixed beam of the aluminum of dimension 220 mm in length, width of 25mm and thickness of 3 mm, the piezoelectric has a variable length, width of 25mm and thickness of 0.5 mm, as we can see in the Figure 1. The extension piezoceramics are made of PZT-5H material whose properties are:  $\bar{C}_{11}^D = 97.767$  GPa,  $\rho = 7500Kg.m^3$ , piezoelectric coupling constants  $\bar{h}_{31} = 1.3520 \times 10^9$  N.C<sup>-1</sup>, and dielectric constants  $\bar{\beta}_{33}^E = 57.830 \times 10^6 m.F^{-1}$ . For the beam has:  $\rho = 2700Kg.m^{-3}$  and  $E = 70 \times 10^9 MPa$ . (Santos [6]).

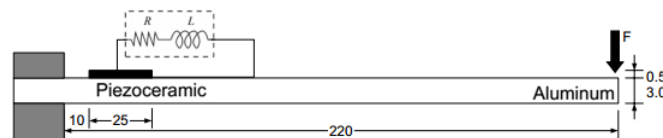


Figure 1. Representation of cantilever beam with bonded extension piezoceramic patch.

The optimization had the focus in resistance and inductance values of the circuit, wherever the resistance (R) is responsible in damping by means of Joule effect and the inductance (L) is responsible to control resonant frequency of the structure, this form had use a shunt circuit, the representation for this system can be seen in Figure 2.

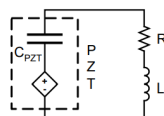


Figure 2. Configuration of the RLC circuit together with the piezoelectric.

### 3.1 Analysis of the equations for harmonic vibrations

For analysis of harmonic vibration, the proposed model (Santos [6]) is used to evaluate the mobility (velocity/force) frequency response function of the base structure. The resistive (R) or resonant (RL) shunt circuit affects both the passive control performance. In this way, it became necessary to use the circuit that will dissipate the energy or to storage for later use.

How this work analyze a purely mechanical excitation, such as  $F_q = 0$  and  $F = bf e^{j\omega t}$ , the amplitude of a displacement output  $y = c_p u$  can be written as  $y = G(\omega) f$ , where the FRF  $G(\omega)$  is

$$G(\omega) = c_p \{ (-\omega^2 M + K_m - \bar{K}_{me} (\omega^2 M_q + i\omega C_q + \bar{K}_e)^{-1} \bar{K}_{me}^t) \}^{-1} b \quad (2)$$

Analyzing the equation 2 it can be noted that the resistance and the inductance have the capacity to change the rigidity properties of the piezoelectric material, in this way it will be applied to the case types i) open-circuit when

$R_c$  tending to infinity and ii) short-circuit when  $L_c = R_c = 0$ . For the open circuit it has

$$G^{oc}(\omega) = c_p \{-\omega^2 M + K_m\}^{-1} b \quad (3)$$

To the closed circuit

$$G^{sc}(\omega) = c_p \{-\omega^2 M + [K_m - \bar{K}_{me} \bar{K}_e^{-1} \bar{K}_{me}^t]\}^{-1} b \quad (4)$$

You may note that no structural modification is observed in the open circuit box, whereas in the case of a short circuit, the rigidity of the piezoelectric patches is reduced.

### 3.2 Vibration Control using piezoelectric actuators and state feedback

This way is necessary to rewrite the motion equations in the form of state space, containing the displacements and modal velocities of the piezoelectric patches and their derivatives of time.

$$\dot{z} = \hat{A}z + \hat{B}V_c + \hat{B}_f f, \quad y = \hat{C}_y z, \quad (5)$$

where

$$z = \begin{bmatrix} \alpha \\ q_p \\ \dot{\alpha} \\ \dot{q}_p \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -\Omega^2 & K_p & -\Lambda & 0 \\ L_c^{-1} K_p^t & -\Omega_e^2 & 0 & -\Lambda_e \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ L_c^{-1} \end{bmatrix}, \quad \hat{B}_f = \begin{bmatrix} 0 \\ 0 \\ b_\phi \\ 0 \end{bmatrix}, \quad \hat{C}_y = [c_\phi \quad 0 \quad 0 \quad 0]. \quad (6)$$

The modal displacements are such that  $u = \phi \alpha$  and, for mass normalized vibration modes,  $\Omega^2 = \phi^t K_m \phi$  and  $\Lambda = \phi^t C \phi$ .  $\Omega$  is a diagonal matrix which elements are the undamped natural frequencies of the structure with piezoelectric patches in open-circuit.  $\Omega_e^2 = L_c^{-1} \bar{K}_e$  and  $\Lambda_e = L_c^{-1} R_c$  are both diagonal matrices which elements stand, respectively, for the squared natural frequencies of the electric circuits and the ratio between the resistances and inductances. The electromechanical coupling stiffness matrix projected in the undamped modal basis is defined as  $K_p = \phi^t \bar{K}_{me}$ . Input  $b$  and output  $c_y$  distribution vectors are also defined, with modal projections  $b_\phi = \phi^t b$  and  $c_\phi = c_y \phi$ , and  $f$  is a vector of the amplitudes of each mechanical force applied to the structure (Santos [12]). A linear state feedback for the applied voltages  $V_c$  is assumed such that  $V_c = -gz = -g_{dm} \alpha - g_{de} q_p - g_{vm} \dot{\alpha} - g_{ve} \dot{q}_p$ , where  $g$  is a matrix of control gains for each state variable. Therefore, the state space equation (5) becomes

$$\dot{z} = (\hat{A} - \hat{B}g)z + \hat{B}_f f, \quad y = \hat{C}_y z. \quad (7)$$

For a single-input mechanical excitation  $f$ , the closed-loop or controlled amplitude of a single displacement output  $y$  can be written such that  $\tilde{y} = G_h(\omega) \tilde{f}$ , where the FRF  $G_h(\omega)$  is

$$G_h(\omega) = \hat{C}_y (j\omega I - \hat{A} + \hat{B}g)^{-1} \hat{B}_f, \quad (8)$$

which can also be derived from the second order equations of motion projected into the undamped modal basis leading to

$$G_h(\omega) = c_\phi \{-\omega^2 I + j\omega(\Lambda + K_p D_{cc}^{-1} g_{vm}) + [\Omega^2 + K_p D_{cc}^{-1} (g_{dm} - K_p^t)]\}^{-1} b_\phi, \quad (9)$$

where the closed-loop dynamic stiffness of the electric circuit  $D_{cc}$  is

$$D_{cc} = -\omega^2 L_c + j\omega(R_c + g_{ve}) + (\bar{K}_e + g_{de}). \quad (10)$$

In this work, the control gain  $g$  is calculated using the standard optimal LQR control theory applied to a single-input/single-output case, that is with only one active-passive patch-circuit pair for the control to minimize the vibration amplitude at one specific location of the structure, such that the following objective function is minimized

$$J = \frac{1}{2} \int_0^\infty (\dot{y}^2 + rV_c^2) dt, \quad (11)$$

where  $\dot{y}$  is the velocity at one location of interest and  $V_c$  is the control voltage applied to the active-passive shunt circuit in all cases following an iterative routine proposed in (Trindade et al. [7]).

## 4 Optimization

Optimize is the act of creation the more favorable conditions for the development of something. When it is spoken in optimization of a circuit, we are looking for the best configuration of your passive elements to guarantee that this circuit do the best way what it was made to do, in case of a shunt circuit, we search for the best configuration to provide dispersion of energy and, consequently, damping of a structure.

Once we are looking for this dispersion of energy using a R-L parallel circuit, we are looking for the best configuration of resistance and inductance of this circuit to provide the minor mechanical or the bigger electricity response for the system, since these concepts are linked, as demonstrated previously.

Artificial intelligence has been widely exploited in recent years to aid in the solution of engineering problems, the mostly used techniques are the genetic algorithm and the artificial neural network, used in a similar research for shunt active power filter control. (Qasim and Khadkikar [8]).

### 4.1 Genetic Algorithm

The Genetic Algorithm (GA) can be described as a family of computational models that are inspired by the evolution of species for problem solving. They incorporate potential solutions to a problem in species of chromosomes, or population, that pass through segregation and data crossing, applying a selection nature that seeks to filter the results that best match the solution of the problem, selecting them positively for future crossings and penalizing those that flee the possible solution of the problem, thus forcing the structure to converge to a single value that meets the nature of the formulation.

A genetic algorithm works in defined space of possible results, in this way, it is necessary to establish boundaries for the code, such boundaries have been defined through the use of a parallel work of structural vibration control using piezoelectric (Santos [6]), which provides us a equation to reach the resistance and inductance values considered optimal to the piezoelectric circuit, these values were defined in the algorithm as the expected average results and the boundaries were a made to restrain a thousand of possible results, called as population, centered in these optimal values. The optimal values used were resistance of  $208.96 \text{ Ohm}$  and inductance of  $121.71 \text{ H}$ . In order of fair comparison with Neural Network, the same space of average results were used for both structures.

### 4.2 Artificial Neural Network

Artificial Neural Networks (ANN) are a set of computational techniques to obtain answers that present a mathematical model corresponding to the neural structure of living organisms, being able to learn and to improve their results with multiple trainings and applications (Araujo et al. [9]). Neural networks have a large number of highly interconnected processing elements (nodes) that demonstrate the ability to learn and generalize from training patterns or data. They, like humans, can perform pattern-matching tasks, while traditional computer architecture, however, is inefficient at these tasks. On the contrary, the latter is faster at algorithmic computational tasks. Neural networks, like fuzzy logic control/decision systems, are excellent at developing human-made systems that can perform the same type of information processing than our brain performs. (Lin and Lee [10]).

This model provides a lot of works in the area of optimization and structure control as the adaptive control in smart structures (Rao et al. [2]) or the identification and control of dynamic systems using recurrent fuzzy neural networks (Lee and Teng [11]), can be also cited your usage for system identification and control on a smart structure (Lee [12]).

### 4.3 Ann's Training

For this paper, to create the target data, the values of Resistance and Inductance were randomly varied by a normal distribution centered on analytical optimal values, defined to the problem of the piezo-beam structure, in order to produce individuals for the Training. The difference between the max amplitude answers of the system coupled with theses values of resistance and inductance and the respective amplitude for the system in open and short circuit condition were saved as the Input data. The algorithm seeks to maximize the value for input, since this means seeking the maximum amplitude gain from the initial condition. The respective values for resistance and inductance were provided for this condition.

## 5 Results

The results for the configuration of resistance and inductance, in Figure 2, suggested by the Artificial Neural Network and the Genetic Algorithm are shown in a comparison of frequency responses with both short circuit,

$R = L = 0$  and open circuit  $R = \infty$  and  $L = 0$ , in the Figure 3.

Verifying the results presented, can be observed a change of stiffness, without affecting the other vibration modes. The suggested configuration of resistance and inductance shows no effect on the another frequencies of the structure, keeping restricted to the control arbitrated. This condition demonstrates a precision control of damping by both adopted methods once only the first vibration mode was chosen for damping.

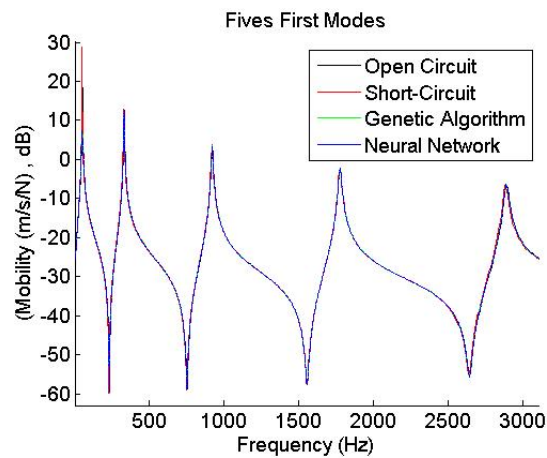


Figure 3. Frequency Responses for Five First Modes of Resonance of Cantilever Beam

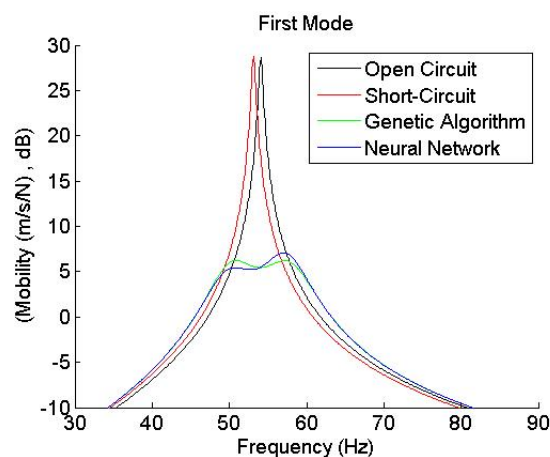


Figure 4. Comparison of Frequency Responses for First Mode of Resonance of Cantilever Beam

The Figure 4 shows the frequency response for the first mode of vibration. On that, can be seen an average damping gain of  $23.24dB$  for the ANN and  $24.27dB$  for the GA, between open circuit and controlled structure. Although close in amplitude, ANN and GA shows a little larger difference of results when is compared the time of executions and the iterations demanded to provide the right configurations.

The results for the time's execution of the Genetic Algorithm method proceed was measured for ten experimentally tests. The values for this method shows up a wide range, having its minimum in 1397.63s and maximum in 1693.20s, with a global variation of 295.57s average, or 5 minutes.

The results for the Neural Network, shows up a more behaved range. The values for the minimum and maximum time of execution are, respectively, 54.18s and 49.92s, keeping restricted to a global variation of 4.26s, much smaller than the one presented by the genetic algorithm, proving to be more reliable in this feature.

The Figure 5 and 6 shows, respectively, the values of iterations demanded for both techniques to provide the configuration of Resistance and Inductance for the beam. Once again, a better behavior is seen for the Artificial Neural Network, demonstrating both a smaller number of iterations, averaging 1330 iterations, and a smaller range of iterations ranging from 1587 to 1125 total iterations to achieve the desired configuration. The Genetic Algorithm presents an average number of iterations of 61122, almost 45 times the demanded for the ANN, with the global variation of 8016 iterations.

Thus is noted a relative advantage for the Artificial Neural Network technique, with an average execution time of 52.81s in contrast to 1646.43s of the Genetic Algorithm, however this scenario is reversed when comparing the damping range produced by both techniques.

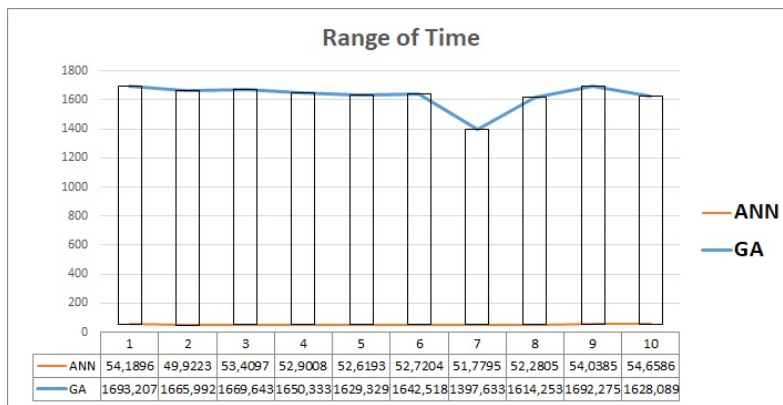


Figure 5. Range of Time Execution

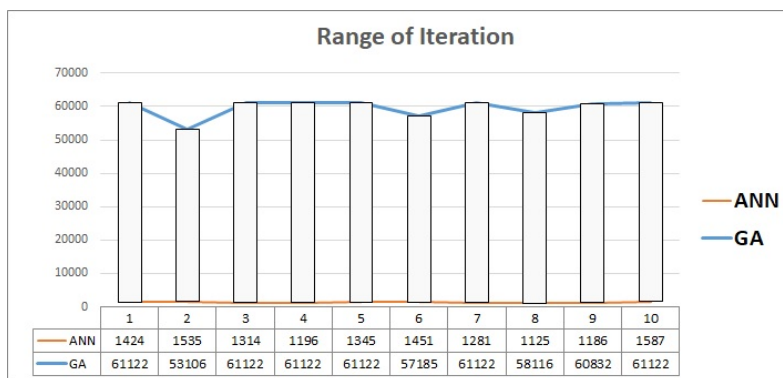


Figure 6. Range of Iterations

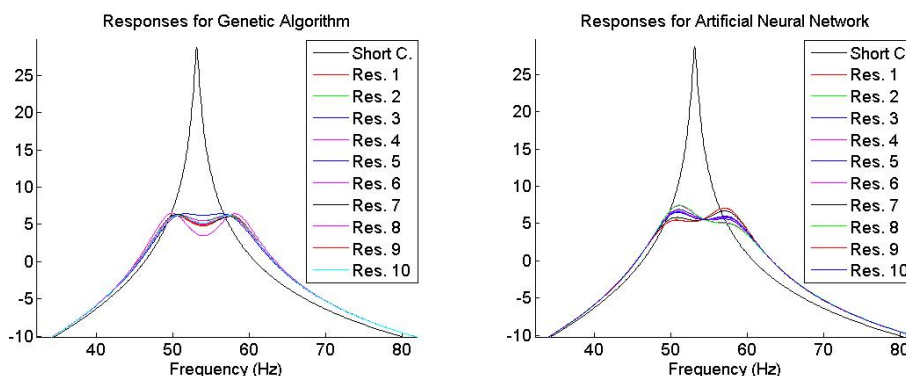


Figure 7. Response of Frequency (a) Genetic Algorithm (b) Artificial Neural Network

The Figure 7 shows the frequency response of the beam for the ten configurations of Resistance and Inductance parameters suggested by both techniques with ten execution. Its seen, even with less predictable behavior, the Genetic Algorithm presents a better global damping than the Neural Network, with the value of 25,23dB, while ANN presents the value of 23,3dB.

Comparing both configurations, can be observed a same nature of control for the structure and the average damping for ANN and GA were, respectively, 23,24dB and 23,64dB. The range of values for the codes was plotted in Figure 8 in order to provide a visual conference of the distinction of responses that both structures presents.

## 6 Conclusion

The results shows that both adopted techniques provided optimized configurations for the structure in a order of damping superior of 20dB, demonstrating the usage of any of these techniques are a interesting alternative for setup of a shunt control system, being the main difference the computational demand associated to each of them, measured by the time execution of the codes, and the number of iterations needed to get the result.

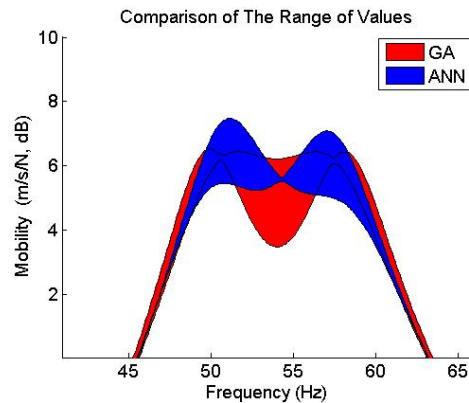


Figure 8. Comparison of The Range of Values

An analysis of performance, with focus in time of processing and attenuation of the amplitude for frequency tuned, shows a better performance for Neural Network compared with Genetic Algorithm, since the ANN presents a damping gain very close to that of GA, working in a much lower computational demand.

The ANN shows up a better choice for the shunt control of smart structures, once this technique demonstrates the answers in order of 50 seconds and has a time of execution less of a tenth than GA's, besides this technique presents a relevant difference between the number of iterations required, demanding almost 50000 iterations fewer than the genetic algorithm to provide close configurations of resistance and inductance, and damping for the beam, what means the ANN is much more adaptive, and justifies the time contrasts.

An additional advantage of this technique is your adaptive learning characteristic, that allows your structure go through complex changes without demand additional computational cost to adapt to the new conditions, being recommended for active vibration control systems or any kind of control that need a quick answer from the controller.

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