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Hybrid Method for Adjusting Models of Nonlinear Regression

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Abstract. Regression analysis is widely applied in many fields of science and engineering to predict any variable that is difficult to determine. Generally, Nonlinear regression models are more complex than linear regression. More traditional models require initial parameters to be adjusted, and the procedure for estimating these values is not simple. In this paper, a hybrid method is proposed to meet nonlinear needs. The proposed method consists of two steps. In the first one, two genetic algorithms (GA) were applied for automated parameter prediction. The second step consisted of applying the Levenberg-Marquardt algorithm (GALM) to obtain the appropriate values. The T LBO and Differential Evolution algorithms were tested to estimate the initial parameters. The model selected for the study concerns the population analysis of individuals; and the database describes the forest inventory of the Tectona Grandis planting in southern Minas Gerais. The results showed that both genetic algorithms were efficient to estimate the initial parameters, with equivalent square mean error. Therefore, it is concluded that the proposed hybrid method is effective for estimating the initial parameters.

Keywords: Nonlinear parameterization; Genetic Algorithm; Hybrid method.

1 Introduction

Regression analysis is widely applied in many fields of science and engineering to predict any variable that is difficult to determine. In general, non-linear regression models are more complex than linear regression.

In non-linear regression models, the relationship between the predictor and response variables follows the general form $Y = f(t, \beta)$, where f is a non-linear function with at least one parameter [1] . The main advantages of these models are associated with the biological interpretation identified in some examples of non-linear models. In biological behavior models, usually β_0 is defined as the asymptote (maximum potential of the response variable), β_1 is the biological constant and the rate of growth in the maximum biological potential is β_2 [2]. The disadvantage of non-linear regression models is associated with the requirement of initial parameters for the solution of the algorithm to converge.

The values of the initial parameters are necessary to initiate the convergence of the algorithm and provide a precise adjustment of the model. An inconvenience when using nonlinear regression models [3]. This disadvantage can increase if the initial values provided do not make sense or if a biological standard is not reached. Usually, the adaptation process returns non-convergence results. However, the partial derivative is another flexible option to obtain the initial parameters and this approach generally results in excellent results [2]. In addition, parameter estimation is not as friendly as in linear regression and requires experts to ensure convergence. The non-linear regression method then proves ineffective without estimating initial parameters. [4].

Considering these issues, this research proposes to propose a method of hybrid algorithm to predict the initial parameters for nonlinear regression models applied in a predictive model of survival of equian populations.

This article is organized into sections. Section II will provide an overview of the database, the non-linear regression model and the hybrid method for estimating the initial parameters via Differential Evolution or TLBO. Section III presents the results, which will be discussed in section IV. In section V, the conclusions of this work will be presented.

2 Materials and Methods

2.1 Database

For this study the database of the continuous forest inventory was used in a plantation of *Tectona grandis* in the extreme south of the state of Minas Gerais. Field measurements were taken into account by simple casual sampling where the number of individuals of the species per hectare was collected. As the planting is a controlled environment, the age of the individuals was obtained by the silvicultural register of the forest implantation.

In order to know the mortality of individuals over time, the assessment of survival was carried out in two different years: 2011 and 2016. In possession of the measurements, it was possible to establish the database where each observation contains the age of the population in 2011 ($I1$), the age of the population in 2016 ($I2$) and the number of individuals present in the population in the respective years ($N1$ and $N2$). The database used is composed of 17 instances. Table 1 presents the characteristics of each variable.

Table 1. Description of the database used.

Variable	N	Average	Minimum	Maximum	Standard Deviation
I1	17	2.547647	1.88	3.55	0.636521
I2	17	3.708824	3.03	4.59	0.611131
N1	17	804.0514	636.8769	975	77.44029
N2	17	754.7661	625	839	51.27827

2.2 Nonlinear regression model

Demographic processes, such as birth rates, mortality, fertility, as well as the time when they occur during the population's life cycle, are important and affect the size and composition of populations. However, when dealing with artificial equine populations, all these processes are left out except for mortality. In arboreal individuals, mortality is the result of competition between abiotic resources, mainly light. Dominant and better adapted individuals to the environment are more likely to be maintained for several years in the population.

The measurement of the fitness of a forestry individual to a given environment is costly because it is done by direct measurements in the field. When there is no need to know the specific individual who will prevail in the population, this estimate is made in a generic way and only takes into account the number of individuals remaining. Still, we can model survival with several variables such as precipitation, average temperature, altitude, soil type, among others. However, it is known that the population loses individuals over time, which makes age an easily obtainable variable and produces quality estimates.

[5] made a major contribution to the study of biometrics, linking many population models into a common expression. Among them, it was the development of a model that predicts population size in the future based on their age and initial population, as shown in Equation 1.

$$N_2 = N_1 e^{\beta_0 * (I_2^{\beta_1} - I_1^{\beta_1})} \tag{1}$$

The work of [5] promoted a great accuracy in population estimates, however, the model does not have a linear shape. Despite this, despite the technical difficulties in providing the parameters for linear regression, the model is widely used today. In a specific case, there is no literature that deals with this model for the type of this work.

2.3 Hybrid Metod

Hybridization. The hybrid method proposed by this work consists of estimating the initial parameters of the equation via Differential Evolution or TLBO. After the convergence of evolutionary algorithms, the values obtained will be applied to non-linear adjustment via Levenberg-Marquardt.

Differential evolution. The Differential Evolution (DE) algorithm is a simple and efficient optimization algorithm that was proposed by Rainer Storn and Kenneth Price in 1995 [6]. It is a stochastic search method that initially appeared in order to solve a problem of Chebychev polynomial adjustment.

DE is a method, which besides being easy to implement, performs very well in a large class of problems, as reported by [6]. It is effective for objective functions that are not differentiable or convex and it is easy to find the optimum with small populations.

The Differential Evolution algorithm has few control variables, whose adjustment is simple. It has good convergence properties with an auto adjustment of the adaptation step, that is, as the convergence the steps are smaller.

There are three types of Differential Evolution algorithms, which are: Basic, Variant and Opposition. The Basic DE algorithm for each vector tag $x_i, G, i = 1$ a new vector is generated using the Equation 2.

$$x_{i,G+1} = x_{r_1,G} + F * (x_{r_3,G} - x_{r_2,G}) \tag{2}$$

where r_1, r_2 and r_3 are indexes that are mutually distinct and also different from index i, and F is a constant that determines the size of the step to be taken.

There are still DE algorithms for Variants, which consist of changing the attributes of the cost function; and also by opposition, where a candidate solution and its opposite must be found at the same time.

In this work, the DE algorithm was submitted to a problem that minimizes the residual standard error of the estimate, where the individuals were composed of the value of β_0 and β_1. To calculate the residual standard error, the parameter values were applied to the model (Equation 1) obtaining the estimate and then the error in each observation.

TLBO. The TLBO algorithm is an algorithm inspired by the teaching-learning process and is based on the effect of a teacher's influence on the production of students in a class. The algorithm describes two basic modes of learning: a) through the teacher known as the teacher's phase; b) through interaction with other students (known as the student phase). In this optimization algorithm, a group of students is considered a population and different subjects offered to students are considered different design variables from the optimization problem and the result of a student is analogous to the 'adequacy' value of the optimization problem.

The best solution in the entire population is considered to be a teacher. The design variables are actually the parameters involved in the objective function of the optimization problem provided and the best solution is the best value of the objective function.

Like DE, the TLBO algorithm was subjected to a problem that minimizes the residual standard error of the estimate, where individuals were composed of the value of β_0 and β_1. To calculate the residual standard error, the parameter values were applied to the model (Equation 1) obtaining the estimate and then the error in each observation.

Levenberg-Marquardt. The second stage of the hybrid algorithm is associated with the adjustment of the nonlinear regression model. The initial parameters derived from the best fit of the convergence of evolutionary algorithms are used in this step. We tested the algorithm widely applied in statistical software with an iterative approach for minimal nonlinear regression: Levenberg-Marquardt (LM).

The LM was designed by [7], considering Newton's method in its update function and enhanced by [8], who incorporated the estimated information of the local curvature in the update function. The LM algorithm was implemented as a robust technique by [9]. The author approaches updating the step Δ by his choice, depending on the ratio $\rho(\beta)$ between the actual reduction and the forecast reduction. This relationship is obtained by decomposition in the linear system by measuring the agreement between the linear and the nonlinear function [9].

2.4 Quality analysis

To evaluate the tested method, a simple experiment with 10 repetitions was set up, where the evolutionary computation algorithms were tested and the results used for the non-linear adjustment. The parameter used to measure the quality of the solution was the residual standard error (Syx).

3 Results

The coded DE and TLBO algorithms were powerful to predict the initial parameters of the tested model, where the objective function decreased over the iterations, as expected. The quality of the result proved satisfactory, since the error of the estimate did not exceed 5.48 %. In the form of an initial parameter, the methods were considered robust, with low variation and can be used as a final parameter, since the error presented demonstrates the good quality of the estimates (Table 2).

Table 2. Results obtained after processing evolutionary algorithms.

Algorithm	Repetition	β_0	β_1	Syx (%)
DE	1	0.992584	-0.16315	5.38
DE	2	1.847891	-0.09368	5.35
DE	3	3.01378	-0.05596	5.31
DE	4	3.9811	-0.04436	5.31
DE	5	1.3991	-0.1262	5.34
DE	6	4.5978	-0.03755	5.30
DE	7	0.739658	-0.2144	5.44
DE	8	1.087	-0.1446	5.35
DE	9	2.6045	-0.06143	5.33
DE	10	10	-0.01835	5.28
TLBO	1	0.5014	-0.2317	5.53
TLBO	2	1.1511	-0.1328	5.30
TLBO	3	6.62	-0.0261	5.30
TLBO	4	2.4588	-0.06196	5.36
TLBO	5	3.579	-0.04913	5.31
TLBO	6	12.91	-0.014	5.30
TLBO	7	5.9358	-0.0305	5.31
TLBO	8	11.9256	-0.01539	5.3
TLBO	9	0.2883	-0.3932	5.48
TLBO	10	7.7889	-0.04442	5.42

After obtaining the initial parameters, the LM algorithm was used to perform the non-linear adjustment of the model (Table 3). The quality of the results can be considered satisfactory, since all the initial parameters converge to the same final parameter and consequently produce a single residual standard error of 4.94 $\%$.

The results were promising, the algorithms showed a desirable quality for the initial parameters. In addition, the LM algorithm did not bring major improvements to the residual standard error, in the best cases the reduction of Syx is 6.80

4 Discussion

Some issues that were raised previously about nonlinear regression models were highlighted. These problems are generally related to the database, format of the mathematical model, value of the initial parameters and adjustment method. In forestry science, the correlation between age and survival seems to decrease over time, demonstrating a degenerate pattern of this relationship. Therefore, the adjustment curve process becomes more complex and sometimes unfeasible. The results of this work can reinforce this statement, suggesting an increasing effort for models with a high number of parameters to adjust properly.

In fact, the modeling process involves many aspects to ensure unbiased forecasts. This leads to new methods of artificial intelligence to solve these problems. Traditional algorithms with a low success rate have been rejected in many statistical software and board languages. Therefore, there are knowledge gaps to be explored in the regression analysis.

The most promising contribution of the hybrid method is associated with finding initial parameter values without previous information or manual adjustment. A high success rate was found in the adaptation rate for hybrid methods in the studied database. The Levenberg-Marquardt method is widely used with lower rates of convergence [10]. Despite this, the evolutionary algorithm was robust and significant for the Levenberg-Marquardt

Table 3. Results obtained after processing the LM algorithm.

Algorithm	Repetition	β_0	β_1	Syx (%)
DE	1	0.5863	-1.7836	4.94
DE	2	0.5864	-1.7839	4.94
DE	3	0.5864	-1.7834	4.94
DE	4	0.5864	-1.7833	4.94
DE	5	0.5864	-1.7839	4.94
DE	6	0.5864	-1.7838	4.94
DE	7	0.5864	-1.7838	4.94
DE	8	0.5863	-1.7835	4.94
DE	9	0.5864	-1.7839	4.94
DE	10	0.5865	-1.784	4.94
TLBO	1	0.5865	-1.7848	4.94
TLBO	2	0.5863	-1.7836	4.94
TLBO	3	0.5864	-1.7838	4.94
TLBO	4	0.5864	-1.7838	4.94
TLBO	5	0.5864	-1.7838	4.94
TLBO	6	0.5864	-1.7838	4.94
TLBO	7	0.5864	-1.7838	4.94
TLBO	8	0.5864	-1.7838	4.94
TLBO	9	0.5864	-1.7838	4.94
TLBO	10	0.5864	-1.7838	4.94

convergence.

5 Conclusions

The hybrid method is able to predict the initial value of the parameters for non-linear regression models. The main advantage of applying this hybrid method is the opportunity to adjust nonlinear models as linear models, that is, without worrying about the initial parameters. In general, the Levenberg-Marquardt and genetic algorithms are the best combination, with a high performance and success rate, according to the results of this work. Meanwhile, only the first stage (genetic algorithm) is sufficient to guarantee high quality of the solution for all models with the same reliability observed in classical statistical methods.

The choice of the genetic algorithms for the prediction of the initial parameters requires a good knowledge of the model in question and the predisposition of the data in the selected base. In addition, it is interesting that the process itself does not have a high computational cost, compromising the efficiency of the results.

Through these characteristics, a good prediction of these initial parameters of non-linear models was possible.

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