XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

A Particle Swarm Optimizer with Sensitivity Analysis for the Optimal Placement and Sizing of Distributed Generators in Radial Distribution Systems

Diego José da Silva[1], Gringo Tavares Galina Barbosa[2], Edmarcio Antonio Belati[3], Eduardo Werley S. Ângelos[4]

[1] *d.jose@ufabc.edu.br*
[2] *gringo.galina@ufabc.edu.br*
[3] *edmarcio.belati@ufabc.edu.br*
[4] *eduardo.werley@ufabc.edu.br*

Federal University of ABC - UFABC
Avenida dos Estados 5001, Santa Teresinha, 09210-580, Santo André, SP, Brazil

Abstract. In recent years, the penetration of Distributed Generation (DG) has rapidly increased worldwide, mainly due to the liberalization of electricity markets, restrictions on system expansion and environmental concerns. Technological advances in small generators and energy storage devices have also accelerated the process. This paper proposes a methodology based on Particle Swarm Optimization (PSO) and Sensitivity Analysis (SA), namely (PSO-SA), to identify the optimal placement and sizing of DG in radial distribution system (RDs). For improved efficiency, a limited set of candidate buses for placement is defined through SA on the Lagrange multipliers. A comparison against a regular PSO and other similar approaches is made. Numerical results show the effectiveness and robustness of the method in providing good solutions with reduced power losses, when compared to other techniques.

Keywords: Distributed Generation, Particle Swarm Algorithm, Sensitivity Analysis, Loss Minimization.

1 Introduction

In the last few years, the penetration of Distributed Generation (DG) has gain special attention, mainly due to liberalization of markets, restrictions on system expansion and environmental concerns (Hung et al. [1]). Furthermore, the placement of DG in RDs can contribute to technical, economic, and environmental advantages El-Ela et al. [2]. Optimal Placement and Sizing of Distributed Generation (OPDG) is considered a Mixed-integer Nonlinear Programming (MINLP), involving continuous and discrete variables (Coelho et al. [3]). According to Hemdan and Kurrat [4], the minimization of power losses is the main goal of the most published works.

Several approaches have been proposed to solve OPDG, using deterministic or approximate methods. In Acharya et al. [5], an analytical method based on the exact loss formula is used. The objective function is designed to reduce power losses, and sensitivity factors are applied to reduce the number of solutions in the search space. In Kaur et al. [6], a Sitting Planning Model together with Sensitivity Analysis (SA) is used to reduce the search space, and a procedure based on Integrated Sequential Quadratic Programming and a Branch and Bound algorithm is employed to find the optimal capacities of DGs. On the other hand, the study conducted by Khalesi et al. [7] uses Dynamic Programming and a multi-objective model in order to reduce power losses and reinforce the voltage profile after DG placement. Time-varying loads are taken into consideration. The results show that DGs provide technical and financial benefits if allocated in correct locations with adequate sizes.

According to Gil Mena and Martín García [8], Computational Intelligence (CI) techniques are viable alternatives for solving the OPDG. In Sultana et al. [9], a method based on Grey Wolf Optimizer (GWO) is used to place DGs and minimize reactive power losses. In Ali et al. [10], the OPDG problem is solved using Aint Lion Optimization Algorithm (ALOA), where candidate buses for DG placement are chosen using Loss Sensitivity Factors (LSFs). Recently, Ullah et al. [11] propose a modified PSO strategy, named Phasor Particle Swarm Optimization (PPSO), seeking to minimizing energy loss in RDs, considering different load levels.

In this context, we propose a methodology based on PSO and SA, namely Particle Swarm Optimization with

Sensitivity Analysis (PSO-SA), to carry out the optimal placement and sizing of DG in RDs, where SA indicates the most suitable buses for placement, and PSO optimizes the sizes and capacities. The objective of the combination of the algorithms is not only to improve the performance of the regular PSO but also provide less disperse solutions in the search space. The method is tested on a 70-bus system and its effectiveness is demonstrated through the comparison against a regular PSO and other stochastic approaches.

2 Problem Formulation

The OPDG problem consists in determining the location and size of DGs in order to reduce network losses and improve voltage profile, also satisfying technical and operational constraints. In this work, the objective function is designed to minimize the total active power losses, as in eq. (1):

$$\min \quad \sum_{l=1}^{N_L} P_{losses,l} \tag{1}$$

where $P_{losses,l} = g_{km}(V_k^2 + V_m^2 - 2V_k V_m \cos\theta_{km})$ are the active power losses in branch l. The voltage magnitudes in buses k and m are given by V_k and V_m; g_{km} is the conductance between buses k and m; θ_{km} is the corresponding angular difference; and N_L is the number of lines of the system.

The objective function eq. (1) is subject to the constraints given as follows:

Load flow constraints

Equations (2) and (3) represent the active and reactive power balance, respectively.

$$P_{DG_k} - P_{l_k} = V_k \sum_{m \in K} V_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) \tag{2}$$

$$Q_{DG_k} - Q_{l_k} = V_k \sum_{m \in K} V_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}) \tag{3}$$

where P_{DG_k} and Q_{DG_k} are, respectively, the active and reactive power generated at bus k; P_{l_k} and Q_{l_k} are the active and reactive power load in the same bus; and G_{km} and B_{km} represent the real and imaginary parts of the k-m element of the network admittance matrix ($Y = G_{km} + jB_{km}$).

Bus voltage constraints

Inequality (4) represents the minimum and maximum limits imposed on voltage V_k.

$$V_k^{min} \leq V_k \leq V_k^{max} \tag{4}$$

DG constraints

Inequalities (5) and (6) express the minimum and maximum limits of the active and reactive power injected at location k by DGs.

$$P_{DG}^{min} \leq P_{DG_k} \leq P_{DG}^{max} \tag{5}$$

$$P_{DG}^{min} \leq Q_{DG_k} \leq P_{DG}^{max} \tag{6}$$

3 Solution Approach

3.1 Parcticle Swarm Optimization

Originally designed by Kennedy and Eberhart [12], PSO is an evolutionary algorithm that intents to mimic the behavior of bird flocking, fish schooling or other swarming phenomena. Each particle $X_i = (X_{i1}, X_{i2}, \cdots, X_{in})$ in PSO is a candidate solution evolving in the search space with velocity (or rate of change) $V_i = (V_{i1}, V_{i2}, \cdots V_{in})$. During the iterative process, the position and velocity of a particle i are given by (7) and (8), respectively:

$$V_i^{t+1} = wV_i^t + rand_1 C_1(P_i - X_i^t) + rand_2 C_2(P_g - X_i^t) \tag{7}$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{8}$$

where $P_i = (P_{i1}, P_{i2}, \cdots, P_{in})$ is the best position of particle i as of the current iteration, and the current best solution of the swarm is denoted by $P_g = (P_{g1}, P_{g2}, \cdots, P_{gn})$. The term w is the *inertia weight*, which is assigned to the particle's previous velocity (V_i^t); $rand_1$ and $rand_2$ are two random vectors in the range [0;1]; C_1 is the constant acceleration of cognitive learning and C_2 is the constant acceleration of the social learning.

3.2 Sensitivity Analysis

Sensitivity Analysis is used in this work to generate a ranking of buses for DG placement in order to limit the search space. The process is based on the Lagrange multipliers, which can be used to estimate modifications in a given objective function when there is a change in the problem constraint. For example, consider problem (9):

$$\min \quad L(x) \tag{9}$$

$$s.a \quad g_i(x) = b_i \qquad i = 1, \cdots, m \tag{10}$$

where g_i can be interpreted as a constraint on the available resources modeled in equation i. In this work, we propose to analyze the behavior of the optimal value of the objective function L for a variation in g_i. Let us assume that the optimal value x^* is a function of the resources, g. According to Helmult [13], the Lagrange multipliers are given by (11):

$$\frac{\partial L(x^*(g))}{\partial(g_i)} = \lambda_i \tag{11}$$

where λ_i is the multiplier associated with the equality constraint i. In other words, λ_i is the marginal change of the objective function after a small variation in g_i, representing the increase (or decrease) in the objective function by an unit increase (or decrease) of resources.

In RDs, where the minimization of active losses is usually the main goal, the Lagrange multipliers can provide the changes in the objective function when an unit-change of active power occurs in the power balance constraint. Due to the non-linear characteristic of the problem, the greater the disturbance, the greater the error in the estimates. In this work we are interested in listing buses that can contribute to the minimization of power losses by increasing the generated active power. Thus, the Lagrange multipliers associated with active-power balance constraints are used to rank candidate buses for DG placement and reduce the search space.

3.3 PSO-SA Algorithm

The flowchart of the proposed algorithm is presented in Fig. 1. The inputs are the network data, the number of DGs (N_{DG}) and the configuration parameters of PSO (p, population size and maximum iteration, C_1 and C_2, w, and V_i). An initial topology preprocessing is recommended to improve the Load flow (LF) efficiency. After the limited set of candidate buses is chosen using SA, PSO optimizes the solution vector (X), composed by the location and sizes of the DGs. The process ends when a maximum number of iterations is reached.

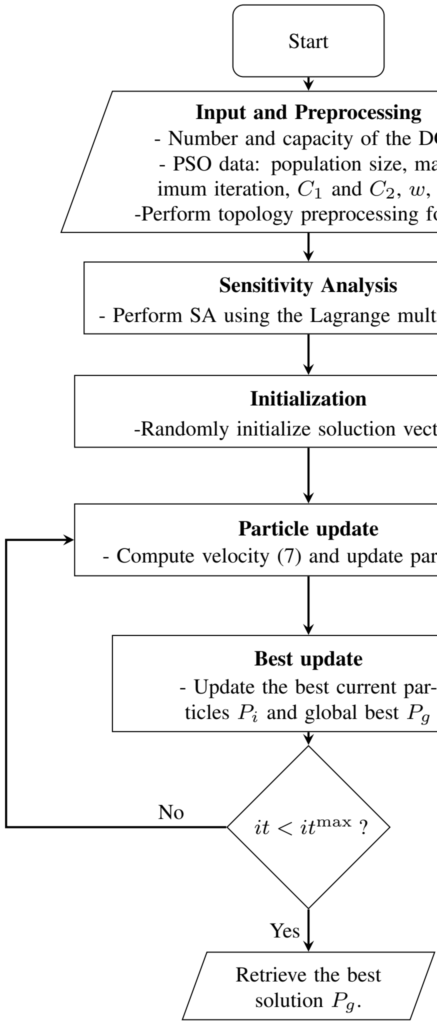

Figure 1. Flowchart of the proposed approach.

4 Test results and discussion

The proposed method was implemented in Matlab® and tested on a 70-bus distribution system [14] with seven laterals. Its effectiveness in allocating DGs to minimize total losses is compared against a regular PSO (without SA) and other OPDG techniques. The parameters were obtained after a series of performance tests with different variations and the best result are obtained with: number of particles = 70; maximum number of iterations = 200; C_1 = 1.8; C_2 = 1.2; w_{max} = 0.9; w_{min} = 0.4; V_{max} = 7; and V_{min} = -7. In Fig. 2 presents the analysis the proposed method for a larger set of instances with different characteristics. In all tests, the placement of 3 DGs is considered, as in [15]. The Back Foward Sweep (BFS) method [16] is used to obtain the LF solutions. For each algorithm, the best solution over 100 runs is considered.

The Lagrange multipliers obtained by performing SA on the 70-bus system are listed in Table 1. The percentage values with respect to the bus with the highest sensitivity (bus 66) are also shown. In order to illustrate the benefits of removing nonsignificant buses from the OPDG problem, let us assume that, in a system with N_b buses, n_λ buses are excluded. The number of possible solutions for placement can be evaluated by:

$$n_s = \frac{(N_b - n_\lambda)!}{(N_b - n_\lambda - N_{DG})!\,N_{DG}!} \tag{12}$$

In the tests with PSO-AS, we consider that the nodes with percentage sensitivities less than 1% are ignored, which results, for the 70-bus case, in the elimination of 26 buses. The total number of solutions for this system, according to (12), is 52,394, considering N_{DG}=3 and N_b= 69 (ignoring the substation). For n_λ = 26, the

Table 1. Lagrange multipliers of the active-power balance constraint (70-bus network)

Bus	λ_i	%	Bus	λ_i	%	Bus	λ_i	%	Bus	λ_i	%
66	-0.1701	100	19	-0.0721	42.38	13	-0.0311	18.31	33	-0.0005	0.3
65	-0.169	99.33	57	-0.0628	36.95	16	-0.0311	18.31	40	-0.0004	0.25
64	-0.1652	97.12	56	-0.0551	32.4	7	-0.0157	9.24	41	-0.0004	0.25
63	-0.1644	96.67	69	-0.0539	31.73	12	-0.0157	9.24	39	-0.0003	0.22
62	-0.1639	96.34	70	-0.0539	31.73	51	-0.0043	2.53	31	-0.0003	0.18
61	-0.1494	87.86	67	-0.0477	28.08	50	-0.0039	2.3	32	-0.0003	0.18
60	-0.1388	81.59	68	-0.0477	28.08	46	-0.0016	1	38	-0.0001	0.12
59	-0.1299	76.38	55	-0.0472	27.78	47	-0.0016	1	48	-0.0001	0.12
58	-0.1073	63.11	54	-0.0416	24.46	36	-0.0015	0.93	5	-0.0001	0.08
28	-0.0753	44.27	10	-0.0367	21.63	43	-0.0015	0.9	30	-0.0001	0.07
27	-0.0752	44.25	15	-0.0367	21.63	44	-0.0015	0.9	3	-0.00005	0.03
26	-0.0751	44.17	18	-0.0367	21.63	45	-0.0015	0.9	4	-0.00005	0.03
25	-0.0748	43.99	52	-0.0349	20.53	35	-0.0014	0.87	29	-0.00005	0.03
24	-0.0745	43.82	53	-0.0349	20.53	42	-0.0012	0.71	2	-0.00002	0.02
22	-0.0744	43.73	9	-0.0348	20.5	6	-0.0011	0.66	1	-0.00001	0.01
23	-0.0744	43.73	14	-0.0348	20.5	11	-0.0011	0.66	37	-0.00006	0.04
21	-0.0735	43.21	17	-0.0348	20.5	49	-0.001	0.63			
20	-0.0729	42.89	8	-0.0311	18.31	34	-0.0009	0.55			

possible solutions reduce to 12,341. Therefore, a reduction of 76,4% in the search space is achieved by eliminating buses with small contributions to the objective function.

Table 2 shows the results obtained by PSO-AS, as well by a regular PSO, the Modified Teaching-Learning Based Optimization Algorithm (MTLBO) [8], the Grey Wolf Optimizer (GWO) [9] and the Water Cycle Algorithm (WCA) [2]. The optimal DG sizes and buses determined by PSO-SA are 526 kW, 380 kW and 1.718 kW at buses 12, 19 and 62 respectively. It can be noticed that the total active power losses found by PSO-AS (2.624 kW) are lower than the other methods.

Figure 3 presents a statistical analysis of PSO and PSO-SA, where the central mark denotes the median over 100 simulations, and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. Outliers are indicated by the '+' symbol. The results indicate PSO-SA achieves better solutions more frequently than PSO, which is one of its main advantages. This behavior is mainly due to the application of SA to elect buses for placing DGs, which improves the search efficiency.

Finally, Figure 4 shows the voltage profile of the system before and after DG placement. The voltage levels are slightly improved, with the voltage magnitude at bus (66) increasing from 0.90 (p.u.) to 0.97 (p.u.).

Table 2. Simulation result for the 70-bus system

	Base-case	**Distributed Generation**				
		GWO [9]	MTLBO [8]	WCA [2]	**PSO**	**PSO-SA**
Losses (kW)	225.01	74.9	69.5	71.5	**69.4**	**69.4**
Loss reduction %		66.7	69.1	68.2	**69.1**	**69.1**
DGs Location/kW		17 (700) 61 (2.000)	11 (493) 18 (378) 61 (1.672)	23 (438) 61 (775) 62 (1.105)	**12 (526)** **19 (380)** **62 (1.718)**	**12 (526)** **19 (380)** **62 (1.718)**
Total active generation (kW)		2.700	2.543	2.318	**2.624**	**2.624**
Voltage at substation (p.u.)	1.0	1.0	1.0	1.0	**1.0**	**1.0**
Voltage at bus 66 (p.u.)	0.90	0.98	0.98	0.98	**0.97**	**0.97**

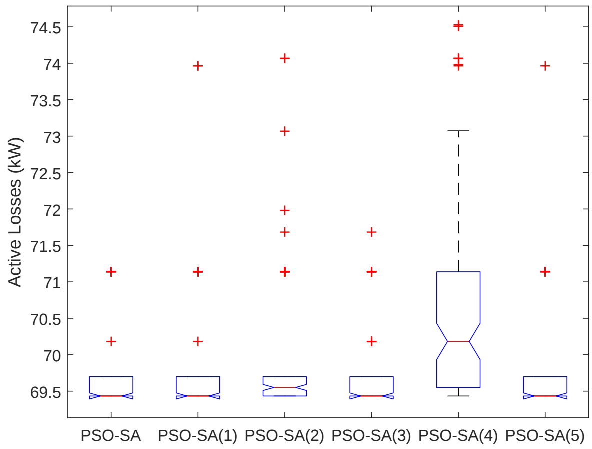

Figure 2. Boxplot of 100 simulations, 70-bus system (a larger set of instances with different characteristics of PSO-SA).

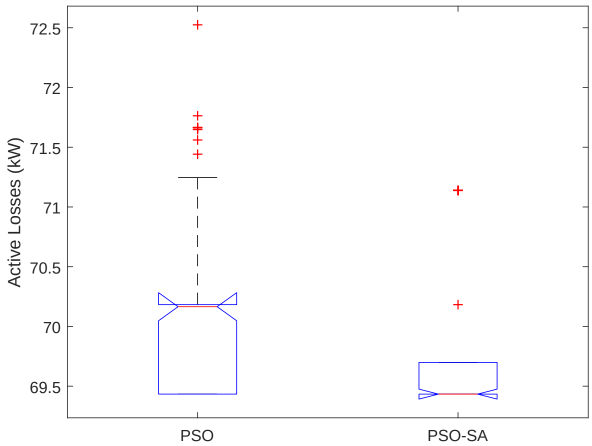

Figure 3. Boxplot of 100 simulations, 70-bus system (the central mark indicates the median, and the '+' symbol denotes the outliers).

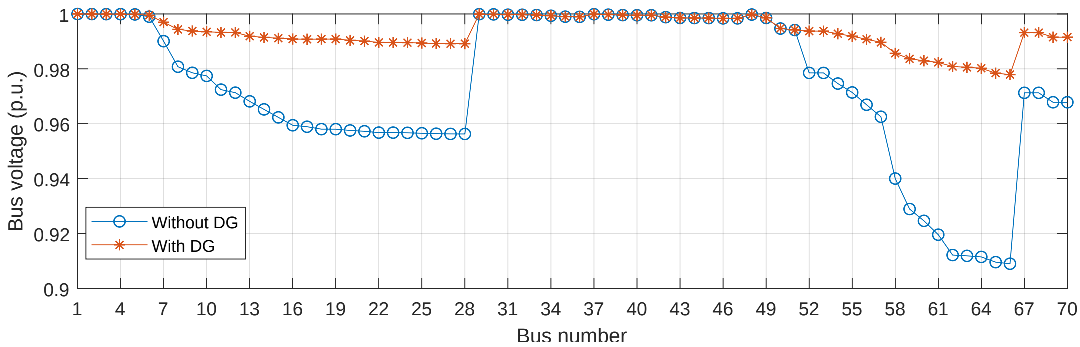

Figure 4. Voltage profile in the 70-bus system before and after DG placement.

5 Conclusions

In this work, the PSO-SA method for the solution of the OPDG problem is presented. The SA is applied in the first phase in order to find the candidate buses for DGs placement, and the PSO is then employed to determine the best DGs sizes. The method has been tested on a 70-bus distribution system, showing excellent performance in solving the OPDG when compared to other meta-heuristics. The optimal performance of the algorithm is mainly due to the application of SA to elected the buses to place DG, which enhance the robustness of the PSO algorithm this is being the main contribution of the proposed work. Finally, the method can be applied to other MINLP problems.

Acknowledgements.

This research work was supported by FAPESP, under Grant 2018/03015-2, INERGE, CAPES-Brazil.

Authorship statement.

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property of the authors.

References

[1] Hung, D. Q., Mithulananthan, N., & Bansal, R. C., 2010. Analytical expressions for dg allocation in primary distribution networks. *IEEE Transactions on Energy Conversion*, vol. 25, n. 3, pp. 814–820.

[2] El-Ela, A., El-Sehiemy, R., & Abbas, A., 2018. Optimal placement and sizing of distributed generation and capacitor banks in distribution systems using water cycle algorithm. *IEEE Systems Journal*, vol. 12, pp. 3629.

[3] Coelho, F. C., da Silva Junior, I. C., Dias, B. H., & Peres, W. B., 2018. Optimal distributed generation allocation using a new metaheuristic. *J Control Autom Electr Syst*, vol. 29, n. 1, pp. 91–98.

[4] Hemdan, N. G. & Kurrat, M., 2011. Efficient integration of distributed generation for meeting the increased load demand. *International Journal of Electrical Power & Energy Systems*, vol. 33, n. 9, pp. 1572 – 1583.

[5] Acharya, N., Mahat, P., & Mithulananthan, N., 2006. An analytical approach for DG allocation in primary distribution network. *International Journal of Electrical Power & Energy Systems*, vol. 28, n. 10, pp. 669 – 678.

[6] Kaur, S., Kumbhar, G., & Sharma, J., 2014. A MINLP technique for optimal placement of multiple DG units in distribution systems. *International Journal of Electrical Power & Energy Systems*, vol. 63, pp. 609 – 617.

[7] Khalesi, N., Rezaei, N., & Haghifam, M.-R., 2011. DG allocation with application of dynamic programming for loss reduction and reliability improvement. *International Journal of Electrical Power & Energy Systems*, vol. 33, n. 2, pp. 288 – 295.

[8] Gil Mena, A. J. & Martín García, J. A., 2015. An efficient approach for the siting and sizing problem of distributed generation. *International Journal of Electrical Power & Energy Systems*, vol. 69, pp. 167 – 172.

[9] Sultana, U., Khairuddin, A. B., Mokhtar, A., Zareen, N., & Sultana, B., 2016. Grey wolf optimizer based placement and sizing of multiple distributed generation in the distribution system. *Energy*, vol. 111, pp. 525 – 536.

[10] Ali, E., Elazim], S. A., & Abdelaziz, A., 2017. Ant lion optimization algorithm for optimal location and sizing of renewable distributed generations. *Renewable Energy*, vol. 101, pp. 1311 – 1324.

[11] Ullah, Z., Wang, S., & Radosavljević, J., 2019. A novel method based on ppso for optimal placement and sizing of distributed generation. *IEEJ Transactions on Electrical and Electronic Engineering*, vol. 14, pp. 1754–1763.

[12] Kennedy, J. & Eberhart, R., 1995. Proceedings of icnn'95 - international conference on neural networks. In *Optimization of the occlusion strategy in visual tracking*, pp. 1942–1948, Washington DC, USA.

[13] Helmult, F., 1978. *Programação não-linear: Análises e métodos*. Edgard Blucher:São Paulo, 1a edição edition. ISBN 1000241731212.

[14] Baran, M. E. & Wu, F. F., 1989. Optimal capacitor placement on radial distribution systems. *IEEE Transactions on Power Delivery*, vol. 4, n. 1, pp. 725–734.

[15] Mahmoud, K., Yorino, N., & Ahmed, A., 2016. Optimal distributed generation allocation in distribution systems for loss minimization. *IEEE Transactions on Power Systems*, vol. 31, n. 2, pp. 960–969.

[16] Rupa, J. A. M. & Ganesh, S., 2014. Power flow analysis for radial distribution system using backward/forward sweep method. *International Journal of Electrical and Computer Engineering*, vol. 8, n. 10, pp. 1628 – 1632.