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A Mathematical Programming Approach for Optimization of Distribution Networks with Photovoltaic Sources

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Abstract. This paper presents a mathematical formulation for minimizing the active losses of the electric power distribution network with photovoltaic (PV) generation sources. The modeling considers the distribution network reconfiguration (DNR) and the adjustment of the photovoltaic inverter power factor (fp_{pv}). Depending on the used technology, the PV inverters can inject active and reactive power by adjusting its fp_{pv}, controlling the voltage profile, and minimizing network losses. Another way to minimize losses and improve the voltage profile is by performing the DNR. The DNR is performed by opening/closing switches to form a new network structure, reducing power losses while satisfying operation constraints, and improves the voltage profile. This mathematical formulation involves continuous and discrete variables, constraints, nonlinear, and non-convex objective functions. To solve this problem, the use of a mathematical programming approach was proposed. The problem aims to find the network configuration and the fp_{pv} of the PV inverters adjustment that minimize the losses of active power and improve the voltage profile. The tests were performed in the 33-bus modified system using AMPL software and Knitro solver. The results showed that the methodology is effective and the combination of PV inverters control and DNR can reduce network losses by more than 85% in some situations of load and generation.

Keywords: distributed generation, mathematical programming, distribution network reconfiguration, photovoltaic inverter.

1 Introduction

Due to the high dependency and the high consumption of electrical energy, it is essential that the energy distribution networks (DN) operate efficiently, reliably, and economically to ensure the basic parameters of energy quality. With the insertion of new sources of power generation, the attention to the need to maintain the quality of the electric power supply must be even greater. The use of Photovoltaic (PV) generation has been gaining great prominence among the sources used in Distributed Generation (DG).

The energy produced by the photovoltaic panels is made available to the load or to the DN by means of the inverters [1]. Normally, the inverters of the photovoltaic systems are adjusted aiming only at the supply of active power, operating with a unit power factor (pf_{pv}). This can change the power factor (*pf*) at the connection points of the DN during peak generation periods [2], as part of the demand for active power being supplied by the PV generation and considering that the DN remains the only source of power reactive for consumer units with PV, the *pf* at the connection point will decrease.

In energy DN, there are technical power losses, mainly due to the Joule effect [3]. Controlling the pf_{pv} of solar inverters can minimize technical losses and improve the voltage profile in the DN. Due to the increased penetration of PV generation, many works involving the control of reactants of photovoltaic inverters have been developed. In the work presented in [4], the authors studied the impacts of high PV penetration in DN. To solve the problem and reduce impacts, an Optimal Power Flow (OPF) was modeled with the objective functions of minimizing technical losses in the DN and energy consumption, by controlling the voltage levels in the bus. In the work presented in [5], reactive power control is applied to control the voltage on the DN buses and minimize electrical losses, this implementation was called Volt / Var control. The work proposes Volt / Var control from the PV inverter and presents the power factor control curve according to ABNT [6]. In [7], a network with reactive control through the solar inverter is analyzed, considering the minimization of losses in the feeders and maintenance of voltage profiles within regulated parameters. For reactive control it is considered that the inverter will be able to operate most of the time close to its nominal capacity and additional conversion losses are considered, which are modeled in the studied algorithm. The present work will explore the control of pf_{pv} together with DNR to minimize active losses.

DNR [8], [9] aims to find the optimal DN topology by opening and closing switches (turning on or off a line) for a given load and generation level. The DNR problem is modeled with the objective of reducing active power losses, considering the networks operational restrictions. DNR shows itself as the most attractive option when seeking to optimize the network and, at the same time, save natural resources, equipment and labor costs, providing technical and financial improvements. In addition, the reconfiguration also promotes improvements in the electric voltage profile, adapting the supply to the levels established by the regulatory agencies. However, DNR is a highly complex combinatorial problem, with a non-convex, non-linear, multimodal characteristic and presents continuous and discrete variables making it a difficult task to be solved, regardless of the technique used. Although economical, the DNR technique requires a solid study of the topology of the resulting system, because there are a large number of possible solutions that are not viable. Due to these characteristics, several computational techniques have been widely used to solve the problem and can be basically divided into two groups: mathematical and approximate programming techniques. In the specialized literature there are a variety of works related to DNR, in [10] - [13] the problem of DNR was solved using mathematical programming. The second group, of the approximate techniques, includes techniques that have stochastic characteristics, resulting in non-conservative solutions. Many approximate techniques have been used to solve the DNR problem [9], [14] - [21]. Each group of techniques has particularities that influence the search process and computational time. In the specialized literature, there is no indication of a specific technique to solve the DNR problem.

In this paper, a model is proposed that involves the DNR and the optimal adjustment of the pf_{pv} of the photovoltaic inverters in order to have an optimization of the DN without investments in reinforcements, with the objective of minimizing the losses of active power of the network through the support active and reactive power of the inverters associated with PV and DNR. The injection of reactive power also helps to regulate the voltage profile and the network pf [22]. In order to achieve a reactive support in the system with PV generation, it is proposed to consider the power injection limits of the inverters [23].

2 Mathematical Modeling

2.1 Search space and radiality

The DNR problem presents several possibilities of reconfiguration 2^n [9], making it very difficult to find the ideal solution in large systems, regardless of the technique. An alternative that can be used to reduce the search space without compromising the solution is to perform an analysis on the system meshes, which is based on the condition given by eq. (1).

$$n_{os} = n_s - n_b + 1 \quad (1)$$

Eq. (1) establishes the number of switches that need to be opened for the system to remain in a radial and connected configuration, where n_{os} is equivalent to the number of meshes in the system when all switches are closed (mesh topology); n_s is the number of keys in the system; and n_b the number of buses in the system. Mesh identification can be performed based on graph theory, as presented in [24]. Thus, it is possible to establish vectors that represent the meshes of the system that are composed by the switches of the respective lines. As an example, consider the hypothetical system in Fig. 1 that has three meshes: $M_1 = [1\ 2\ 3]$; $M_2 = [3\ 4\ 5]$ e $M_3 = [5\ 6\ 7]$. The mesh vectors are formed according to eq. (2). Where: M_n is the key vector of mesh *n*; S_n^i is the key *i* in mesh *n*; and MD_n is the dimension of the vector in the *n* mesh.

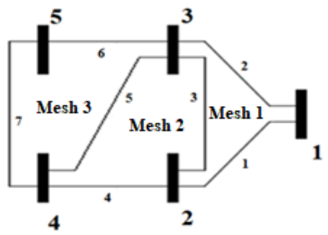

Figure 1. 5-node system.

$$M_n = [S_n^1,\ S_n^2,\ S_n^3, \dots,\ S_n^i\ ]\ \ i =\ 1,2, \dots , MD_n \tag{2}$$

2.2 Reactive power control on grid-connected pv generators

Figure 2 shows an operational scheme for a photovoltaic generator connected to a point in the DN. The network operator remotely monitors the flow of active and reactive power through the photovoltaic inverter and the meter installed in that section of the line. Whenever necessary, the operator remotely sends a command to the photovoltaic inverter, so it can readjust its own power factor pf_{pv} to supply the line with the necessary amount of active and reactive power. The operator's decision can be made with the aid of an optimization tool, as presented in this work. Fig. 3 shows the curve of the photovoltaic inverter.

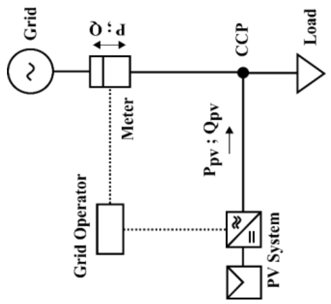

Figure 2. Operational scheme for the PV generator.

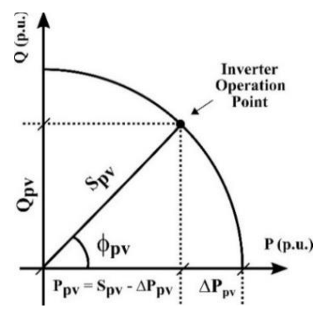

Figure 3. PV inverter capability curve.

According to Fig. 3, the amount of reactive power (Q_{pv}), that the inverter can supply to the grid at any given moment depends on the apparent power available (S_{pv}), which in turn depends on the environment (such as solar irradiance and temperature environment) and electrical parameters (such as installed capacity and efficiency of the inverter). It is worth mentioning that feeding the DN with reactive power implies a reduction of ΔP_{pv} in the active power that would be provided by the inverter if its power factor were unitary. In this work, this capacity of photovoltaic inverters to provide reactive power is explored.

2.3 Proposed mathematical model

The proposed model was modeled as a mixed integer nonlinear problem (MINP), represented by equations (3) to (10) that describe the load flow, after identifying the network meshes. The load flow solution provides the network status variables considering the power balance in the bars inserted in it. The model aims to minimize losses with the introduction of DN operational restrictions and the inclusion of the pf_{pv} control of the photovoltaic inverter. Because the pf_{pv} is a variable that is adjusted, it influences the flows, depending on the variables V, θ and pf_{pv}.

$$Min \rightarrow P_{loss} =\ \textstyle\sum_{NL} S_{km} g_{km}(V_k^2 + V_m^2 - 2V_k V_m cos\theta_{km}) \tag{3}$$

s.a:

$$P_{PV_k} + P_{G_k} - P_{L_k} - \textstyle\sum_{m \in \Omega\kappa} P_{km}\big(V, \theta, pf_{pv}\big) = 0 \tag{4}$$

$$Q_{PV_k} + Q_{G_k} - Q_{L_k} + Q_{sh_k} - \textstyle\sum_{m \in \Omega\kappa} Q_{km}\ \big(V, \theta, pf_{pv}\big) = 0 \tag{5}$$

$$V_k^{min} \leq V_k \leq V_k^{max} \tag{6}$$

$$fp_{pv}^{min} \leq fp_{pv} \leq fp_{pv}^{max} \tag{7}$$

$$S_n^i = 0\ ou\ 1 \quad \text{(binary variable)} \tag{8}$$

$$\textstyle\sum S_n^i = NL - Nt \tag{9}$$

$$\textstyle\sum_i^{MD_n} S_n^i = MD_n - 1 - n_{adj} \quad n =\ 1,2, \dots , N_t \tag{10}$$

Where: P_{loss} is the sum of active losses on the lines; S_{km} is the key between the buses $k\text{–}m$, referring to S_n^i; V_k and V_m are the voltage modules at node k and m, respectively; θ_k and θ_m are the stress angles at node k and m, respectively; P_{PV_k} and Q_{PV_k} correspond, respectively, to the generation of active and reactive power at node k with PV generation; P_{Gk} and Q_{Gk} are the injection of active and reactive power at node k, respectively; P_{L_k} and Q_{L_k} correspond, respectively, to the active and reactive load at node k; $\Sigma_{m\in\Omega\kappa}$ the summation domain is defined by the set of nodes connected to node k of active power and reactive power, as a function of V, θ and pf_{pv}; P_{km} and Q_{km} are active and reactive power flow on the $k-m$ line, respectively; Q_{sh_k} is the reactive power injected by the shunt bus at node k; *NL* is the total number of distribution lines; N_t is the number of loops with all keys closed; n_{adj} is the number of open keys common to the adjacent mesh; and *min* and *max* are the lower and upper limits, respectively.

Equation (3) represents the active power losses of the system for a given configuration. The eq. (4) and (5) represent, respectively, the balance of active and reactive power in the buses and prevent the displacement of the load. Eq. (6) represents the voltage limits on the buses, which must remain between V_k^{min} and V_k^{max} ,that is, they must be between 93% (ninety-three percent) and 105% (one hundred and five percent) of the nominal operating voltage of the system at the connection point. Eq. (7) represents the limits of pf_{pv} of photovoltaic inverters, considering the variation of pf_{pv}0.85 to 1 with variation of PV generation with DNR. Eq. (8) represents the position of the switch (0 open, 1 closed). Eq. (9) determines the number of system keys that must remain closed after the DNR. Finally, eq. (10) represents the number of switches that must remain closed in each mesh to ensure connectivity between the buses, while at the same time undoing loops and ensuring the radiality of the network.

3 Solution Methodology

The modeling of the problem was developed in the AMPL language [25] using the *Branch & Bound* (B&B) technique available in the Knitro solver [26] to assist in the solution of the DNR.

3.1 Branch and bound algorithm

The B&B technique was developed in the 1960s [27] to be applied to problems of discrete and combinatorial optimization, being widely used to solve problems of integer linear programming (ILP). As the DNR problem is a PNLIM problem, the B&B technique will be associated with the MINP problem. The B&B prerogative is to explore a research space through a tree search until it finds the ideal solution to the problem. The performance of the B&B algorithm is related to the search strategy used. In [28], several strategies are presented, such as *Depth-First Search* (DFS), *Breadth-First Search* (BrFS), *Best-Bound Search* (BBS) and *Cyclic Best-First Search* (CBFS), which is a generalization of these three methods.

Another key point in a B&B algorithm is the definition of the branching strategy, which determines how the subproblems will be divided. There are several strategies such as*: Binary Branching* (BB) [28], *Wide Branching* (WB) [28], *Pseudocost Branching* (PCB), *Most Fractional* (MF) [29], *Strong Branching* (SB) [29], among others. These mentioned strategies can be adjusted in Knitro to solve the MINP problem. In this work, we chose to leave the choice of strategies to the Knitro heuristic, which presents this possibility.

4 Simulations and Results

The simulations were performed on an Intel ® Core ™ i7 CPU @ 1.8 GHz computer with 4 GB of RAM and Windows 10 Home - 64 bits. The tests were carried out in the system of 33 buses and 37 lines. The system has 12.66 kV of nominal voltage, supplying a total load of 3,715 kW and 2,315 kVAr. with the voltage set at 1.0 pu in the substation. In [9] more details of the system are presented, such as its layout. The voltages in the other buses of the network were limited between 0.93 to 1.05 pu. This system has 32 interconnection switches, normally closed, 5 opens interconnection switches and five loops. The initial topology of this system considers switches 33 - 34 - 35 - 36 - 37 as open, with active losses of 202.68 kW. Tab. 1 presents the solution of the 33-bus system according to the specialized literature, only for networks without PV penetration. The proposed methodology took 4.67 s to obtain the solution and explored 518 nodes in the B&B tree. In this way, the modeling was validated for DNR.

Table 1. Solution for the 33-bus system.

Methodology	*Open interconnection switches*	*Active power losses (kW)*
[10] and [21]	7, 9, 14, 32, 37	139.55

Proposal	7, 9, 14, 32, 37	139.55

4.1 33-node system with pv generation

Photovoltaic generators were introduced in the original system of 33 buses in several buses. Table 2 shows the maximum PV value introduced in the buses. The PV generation added to the system can provide a total of 3,600 kW considering $fp_{pv} = 1$.

Table 2. Nodes with photovoltaic generation.

#Node	*PV (kVAr)*	*#Node*	*PV (kVAr)*	*#Node*	*PV (kVAr)*	*#Node*	*PV (kVAr)*
3	180	11	100	20	150	29	100
4	150	12	120	21	150	30	200
5	120	14	150	24	450	31	250
7	210	18	70	25	500	32	280
8	250	19	120	26	50		

Considering the modifications presented for the 33-bus system, several simulations were carried out, considering the initial case and four more scenarios: **Initial Case** - initial configuration of the DN without considering the photovoltaic generation; **Case A** - PV generation with pf_{pv} set to 1 and without new reconfiguration (maintaining the ideal network reconfiguration without PV generation); **Case B** - PV generation with pf_{pv}defined in 1 and with a new reconfiguration: **Case C** - pf_{pv} limited in the range of 0.85 to 1.0 and without a new reconfiguration (maintaining the ideal network reconfiguration without generating FV); and **Case D** - pf_{pv} limited in the range of 0.85 to 1.0 and with new reconfiguration.

4.2 33-bus system with 100% and 80% pv generation

Using the proposed methodology and considering 100% of the load and 100% and 80% of the PV generation, the tests described in the previous item were carried out. Tab. 3 shows the results obtained for these scenarios. According to column 1, the analyzes were performed based on the following items: open keys in the scenario; total losses of active power; total active generation (DN); total active photovoltaic generation; total reactive generation (DN), total reactive photovoltaic generation; pu medium tension profile; total CPU time; and the number of buses explored from the B&B tree until the solution is obtained. The next columns present the results for the scenarios considering the PV generation.

1) 100% of PV generation - In a comparative analysis for 100% of PV, it is possible to observe that the system with PV and pf_{pv} fixed in 1 (Case A) presented better results than the original system (Initial case), with 65% improvement in active losses. The system with new reconfiguration and pf_{pv} fixed at 1 (Case B) showed an improvement of 70% in relation to the initial case. Systems with variable pf_{pv} and without reconfiguration (Case C) showed an improvement of 91% in relation to the Initial Case and 72% over Case B. Case D, with reconfiguration and variable pf_{pv}, showed an improvement of 95 % in initial active losses and 40% in case C. The tension profile improved from 0.965 pu in the initial case to 0.994 in case D.

2) 80% of PV generation - Using the proposed methodology and considering 100% of the load and 80% of the photovoltaic generation, the same analyzes as the previous item were carried out. In terms of optimization, the results were similar to the previous test, with only greater losses and a worse voltage profile, which is expected due to the lower photovoltaic generation. Only the reconfiguration of Case D changed with the 80% PV generation.

Analyzing the results of Tab.3, it is possible to verify that the insertion of the PV generation with pf_{pv} unit, by itself, already contributes to the reduction of active power losses in the network and improvement of the voltage profile. And with the addition of the new DN configuration, active losses are further reduced. With the reconfiguration of the network and with 100% of the PV with fp_{pv} unit, the active losses are reduced by 69.39% if compared to the initial case. Optimizing the pf_{pv} of solar inverters, there is a significant reduction in technical losses in the grid and an improvement in the voltage profile, when compared to the cases without the optimization of the pf_{pv}.

It is observed that associating the reconfiguration with the control of the pf_{pv} brings benefits. A point that needs to be analyzed more carefully is the detriment of active power to generate reactive power in photovoltaic generations. For example, in case D, with 100% PV, 488.87 kW of active power are no longer generated to be generated 2305.48 kVAr. With this reduction in active power generated by PV, the attractiveness of installing photovoltaic systems can be reduced. On the other hand, the system needs a reactive control for economic and safe operation of the network, therefore, financial compensation to the consumer may be convenient to inject or absorb reactive power in the electric network.

In all cases, the CPU time did not exceed 20s, showing that the methodology can be applied without great computational effort in similar systems to the 33-bus system. Another important point is that the solutions obtained by the exact technique are conservative, which can not be guaranteed with the use of approximate methods.

Table 3. Results analysis to 100% and 80% of pv generation

	Initial Case	***Case A.***		***Case B***		***Case C***		***Case D***	
		100% PV	***80% PV***	***100% PV***	***80% PV***	***100% PV***	***80% PV***	***100% PV***	***80% PV***
Open Switches	7-9-14-32-37	7-9-14-32-37	7-9-14-32-37	7-9-14-28-30	7-9-14-28-30	7-9-14-32-37	7-9-14-32-37	7-9-13-16-28	7-9-14-17-28
Active Power Losses (kW)	139,55	49,30	53,35	42,71	48,26	12,091	23,093	7,197	18,727
Total Active Generation (kW)	3.854,55	3.764,30	3.768,35	3.757,71	3.763.26	3.727,091	3.738,09	3.722,197	3.733,727
Total PV Active Gen. PV (kW)	0,00	3.600,00	2.880,00	3.600,00	2,880,00	3.122,409	2,497,50	3.111,128	2,488,073
Total Reactive Generation (kVAr)	2.402,30	2.335,77	2.339,12	2.333,14	2.337,80	2.309,91	2.318,09	2.305,48	2.314,17
Total PV Reactive Gen. (kVAr)	0,00	0,00	0,00	0,00	0,00	1.783,59	1.428,73	1.803,78	1.444,01
Average Voltage Profile (pu)	0,965	0,986	0,982	0,986	0,981	0,991	0,986	0,994	0,989
Total CPU time (s)	4.672	0,028	0,028	8,969	15,250	0,027	0,027	19,156	12,984
B&B nodes	518	---	---	735	1.378	---	---	2.353	1.315

Table 4 shows the pf_{pv} values of the performed tests. The methodology optimized the pf_{pv}in cases where it was limited to 0.85 to 1, proving to be an additional control of the network. Most of the values were in the lower limit, which characterizes the need for reactive power to optimize the network.

Table 4. Adjustment of the power factor of the photovoltaic inverters

	PV 100%		**PV 80%**	
Bus	**Case C**	**Case D**	**Case C**	**Case D**
3	8.500	8.500	8.500	8.500
4	8.500	8.500	8.500	8.500
5	8.500	8.500	8.500	8.500
7	8.500	8.668	8.500	8.621
8	9.127	9.060	9.093	9.181
11	8.865	8.920	8.869	8.879
12	8.858	8.912	8.864	8.874
14	8.734	9.060	8.780	8.777
18	9.171	8.500	9.147	8.500
19	8.500	8.500	8.585	8.603
20	9.016	8.991	8.986	9.029
21	9.031	9.010	8.999	9.046
24	8.500	8.500	8.565	8.500
25	8.796	8.500	8.729	8.500
26	8.500	8.694	8.500	8.637
29	8.500	8.500	8.500	8.500
30	8.500	8.500	8.500	8.500
31	8.500	8.500	8.500	8.500
32	8.500	8.500	8.500	8.500

5 Conclusions

In this article, a model was presented using mathematical programming to optimize electricity distribution systems with photovoltaic generation sources. The optimization problem was solved considering DNR and the optimal control of pf_{pv}. The proposed modeling proved to be efficient in the association of AMPL with Knitro. Based on the results, it can be stated: a) the insertion of PV brings great benefits to DN, improving losses and the tension profile; b) only the PV with reconfiguration or control of the pf_{pv} allows to minimize losses and improve the voltage profile. c) combining the optimal pf_{pv} with reconfiguration brings greater benefits than applying just one technique. d) to generate reactive power, it is necessary to suppress a part of the active power that can be generated in the PV. As an extension of this work, it is proposed to obtain the optimal configuration and adjustment of the pf_{pv} with variations in load and generation over a period.

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