Time domain filter application in wave propagation problems.

A. Frasson[1], F. Queiroz[2] , J. Chacaltana[3], C. Loeffler[4]

[1]*Departamento de Engenharia Elétrica, Universidade Federal do Espírito Santo*
Centro Tecnológico – Av. Fernando Ferrari, 540 – Bairro Goiabeiras – 29075 – 910, Vitoria, ES – Brazil
antonio.frasson@ufes.br
[2]*Departamento de Engenharia Elétrica, Universidade Federal do Espírito Santo*
Centro Tecnológico – Av. Fernando Ferrari, 540 – Bairro Goiabeiras – 29075 – 910, Vitoria, ES – Brazil
felixlgq@gmail.com
[3]*Departamento de Engenharia Ambiental, Universidade Federal do Espírito Santo*
Centro Tecnológico – Av. Fernando Ferrari, 540 – Bairro Goiabeiras – 29075 – 910, Vitoria, ES – Brazil
julio.chacaltana@ufes.br
[4]*Departamento de Engenharia Mecânica, Universidade Federal do Espírito Santo*
Centro Tecnológico – Av. Fernando Ferrari, 540 – Bairro Goiabeiras – 29075 – 910, Vitoria, ES – Brazil
carlos.loeffler@ufes.br

Abstract. Often in wave propagation problems with explicit methods, instabilities at high frequencies appears. In methods with structured meshes, spatial filters can be used, but with unstructured meshes the construction of such filters is more complex. In this work, the high frequencies resulting from instability due to numerical errors are smoothed with the use of filters in the time domain, making a convolution at each time step. Butterworth IIR (infinite impulse response) digital filters of maximum flatness are used.

Keywords: Group delay, Digital filter, Time domain, Finite Differences, Finite Elements.

1 Introduction

For unstructured mesh problems, the use of spatial filters becomes extremely difficult, especially in two or three dimensional problems, since there is no way to characterize a spatial sampling frequency. On the other hand, filters in the time domain add a group delay to the propagation problem at each step of numerical integration, which modifies the propagation speed of the original problem. In this work, tests are presented with the propagation of acoustic waves with finite difference methods using time filtering in each step.

2 Group delay

Every signal when crossing a system is delayed, if a transfer function of a system has non-linear phase, this delay varies with frequency, S. Haykin [1]. A signal translated into frequency in the time domain is shown in eq. (1),

$$x(t) = s(t)cos(2\pi f_0 t), \tag{1}$$

being $x(t)$ the signal in the time domain translated into frequency, $s(t)$ the modulating signal and f_0 the carrier frequency, making the Fourier transformation,

$$X(f) = \frac{S(f - f_0) + S(f + f_0)}{2}, \tag{2}$$

being $X(f)$ the signal in the frequency domain, when it passes through a system that has a transfer function,

$$H(f) = ke^{j\beta(f)}, \tag{3}$$

two types of delays appears, the carrier delay called phase delay, eq.(4),

$$\tau_p = -\frac{1}{2\pi f_0}\beta(f_0), \tag{4}$$

and the delay of the signal called group delay,

$$\tau_g = -\frac{1}{2\pi}\left.\frac{\partial\beta(f)}{\partial f}\right|_{f=f_0}, \tag{5}$$

and the signal, in the time domain, after passing through the system remains;

$$y(t) = s(t - \tau_g)cos[2\pi f_0(t - \tau_p)], \tag{6}$$

being $y(t)$ the time domain signal at the system output.

2.1 The excitation signal used as the source.

The excitation signal used in this work is given by the eq.7,

$$x(t) = -2\pi(0.65 f_{max})^2 t e^{-\pi(0.65 f_{max} t)^2}, \tag{7}$$

being f_{max} it can be considered as the maximum frequency contained in the excitation signal, since the energy contained above that frequency is negligible. The temporal mean of the signal is zero, which avoids problems in temporal integration and the Fourier transform is given in eq. (8),

$$X(f) = \frac{j2\pi f}{0.65 f_{max}} e^{-\pi\left(\frac{f}{0.65 f_{max}}\right)^2}, \tag{8}$$

shown in Fig.1, where one can see the null value of the module at zero frequency and the maximum frequency.

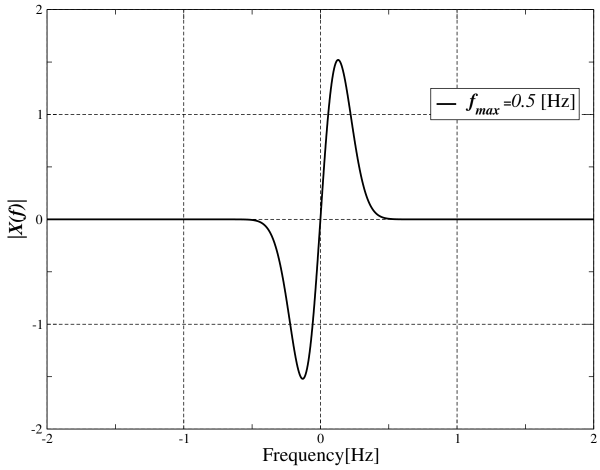

Figure 1. Excitation function spectrum module with $f_{max} = 0.5$[Hz].

2.2 Butterworth filter

The digital filter used was the Butterworth low-pass filter, which is a filter whose transfer function, $H(f)$ has no variation of the module in the passband, avoiding distortion of signal amplitude along the propagation in the domain, S. Haykin [1]. However, the group delay is not constant in the passing band, generating distortions in the signal along with the wave propagation, since different frequencies contained in the signal will propagate with different speeds. This can be avoided by choosing a cutoff frequency higher than the maximum signal frequency. It can be seen in Fig. 2, where a cutoff frequency 8 (eight) times higher than the highest frequency of the signal was chosen as the group delay practically does not vary until 0.5 [Hz], which is the maximum frequency contained in the signals from the examples shown in this work. The sampling frequency must be greater than twice the cutoff frequency of the filter and is given by the inverse of the integration time step of the numerical method used, being; $f_s = \frac{1}{\delta t}$. Figure 3 shows that up to the maximum signal frequency, $f_{max} = 0.5$ [Hz], the variation of the transfer

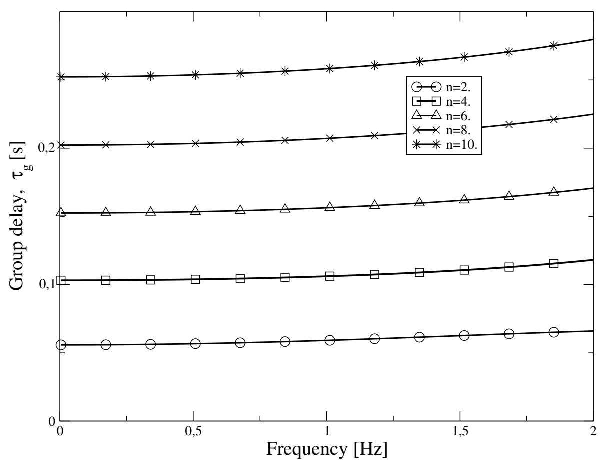

Figure 2. Plot of the group delay of digital Butterworth low-pass filters of orders n=1 through 8, with cutoff frequency $fc = 4$[Hz] and sample frequency $f_s = 80$ [Hz], where n is the filter order.

function module with the frequency is negligible for the different orders of the filter used.

3 Numerical results

In order to carry out the tests, the Finite Differences Method in one dimension was used.

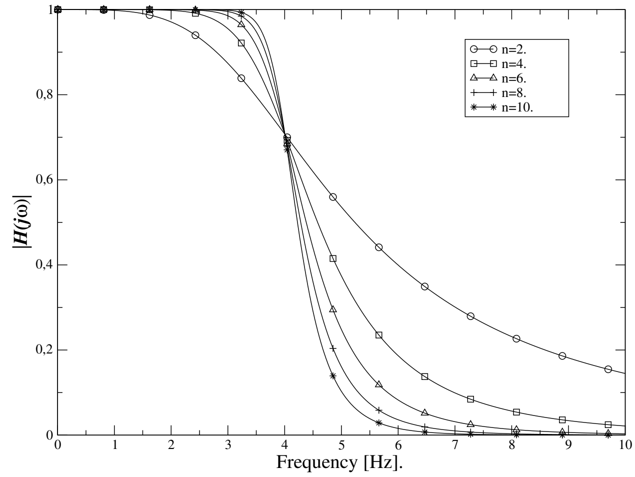

Figure 3. The plot of the transfer function module of digital Butterworth low-pass filters of orders n=1 through 8, with cutoff frequency $fc = 4$[Hz] and sample frequency $f_s = 80$[Hz], where n is the filter order.

3.1 Results of the Finite Differences Method.

Propagation tests were performed on a 20 [m] domain, discretized uniformly by dividing the domain with a discretization distance of $\delta_x = 5.0$. The magic time step was used with the value of $\delta t = \frac{\delta x}{c}$, the original velocity $c = 1.0$[m/s] and the maximum excitation frequency used was $f_{max} = 0.5$[Hz], Allen Tavlove Susan C. Hagness [2]. The speed was modified by using the filter and its expected value was calculated in the form,

$$c_f = \frac{\delta x}{\delta t + \tau_g}, \tag{9}$$

the value of τ_g was taken by the average of the group delays within the frequency range up to the value of f_{max}, and the measured speed c_m was calculated based on the passage of the signal in two domain points. The excitement is at 10[m], the points at 3 [m] and 8 [m] were set for speed measurement purposes. Values of measured speed, calculated speed and errors with using digital Butterworth filters of various orders Tabela 1.

Table 1. Values of measured and calculated speed with using digital Butterworth filters of various orders.

Filter order	Calculated speed c_f	Measured speed c_m	$Error$
n=2	0,065213[m/s]	0,066468[m/s]	1,8881%
n=4	0,036764[m/s]	0,037085[m/s]	0,86609%
n=6	0,025192[m/s]	0,025384[m/s]	0,75654%
n=8	0,019114[m/s]	0,019252[m/s]	0,71766%
n=10	0,015386[m/s]	0,015501[m/s]	0,73874%

In Fig. 4 are the signals using a filter and without using a filter at the instant 90[s], and the excitation has a delay of 5[s], $c = 0.2$ [m / s], $c_m = 0.054675$ [m/s], $c_f = 0.054109$ [m/s], Error = 1.03384%. The sampling frequency used is $f_s = 20$ [Hz], the filter is of order 6, has cutoff frequency $f_c = 4$[Hz] and the maximum signal frequency is $f_{max} = 0.5$[Hz].

4 Conclusions

Time-domain filtering in one dimension works well and introduces a group delay at each time step, as can be seen from the numerical results presented. With the Finite Element Method in two dimensions, O. C. Zienkiewicz

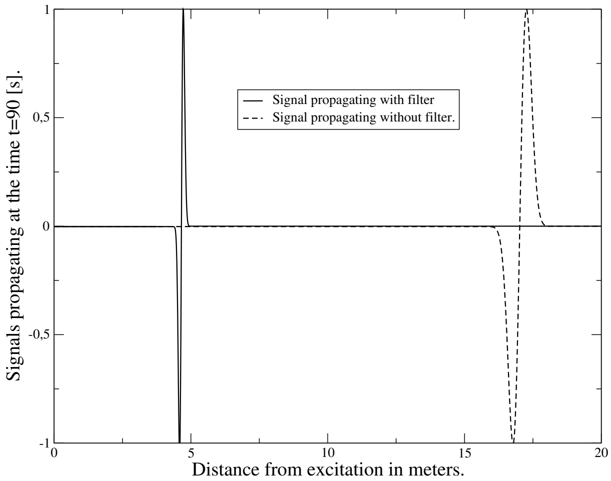

Figure 4. Signals propagating at 90 [s], $c = 0,2$[m/s], $c_m = 0,054675$[m/s], $c_f = 0,054109$[m/s], Error=$1,03384\%$. The signal on the left side is propagating with the presence of filter and the one on the right side is the signal propagating without the presence of a filter.

and R. L. Taylor [3], the filtering in the time domain, although the Butterworth filter transfer function module in the frequency domain has unitary value over the entire frequency range of the original signal, became the method unstable in all attempts. It was not possible to obtain satisfactory results, even if a mesh was used with the edge length around 100 (one hundred) times smaller than the shortest wavelength contained in the signal.

Acknowledgements. We appreciate the financial support of Petróleo Brasileiro S.A.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] S. Haykin, B. V. Veen, 1957. Signals and Systems. *Bookman*,
[2] Allen Tavlove Susan C. Hagness, 2000. Computational Electrodynamics. *Artech House Publishers*,
[3] O. C. Zienkiewicz and R. L. Taylor, 1989. *The Finite Element Method. McGraw-Hill*,
[4] Davis, Timothy, 2011. UMFPACK User Guide
[5] C. Geuzaine and J.-F. Remacle, 2009. Gmsh: a three-dimensional finite element mesh generator with built-in pre and post-processing facilities. In *International Journal for Numerical Methods in Engineering 79(11).*