XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

PREPROCESSED CONSTRAINT GUIDED QUADRILATERAL MESH GENERATION

Daniel Masarin[1], Ernesto Massaroppi Junior[1]

[1]*Laboratory of Applied and Computational Mechanics of the Department of Mechanical Engineering of the São Carlos School of Engineering (EESC), University of São Paulo (USP)*
Av. Trabalhador são-carlense, 400, 13566-590, São Carlos/SP, Brazil
daniel.masarin@gmail.com, massarop@sc.usp.br

Abstract. Numerical methods, such as the finite element method, have been recognized as a great tool for product design in the present market, helping engineers to increase product quality in less development time. However, the quality of the element mesh affects the numerical results, so it is necessary to seek a method that generates good elements. Quadrilateral (two-dimensional) and hexahedral (three-dimensional) mesh is recognized to produce good numerical results, although the automatic generation techniques are still complex and face some restrictions in complex geometries to inhibit distorted elements. In general, methods that generate meshes with fewer singularities require a more restricted geometric shape to generate elements, which demands a greater time in its construction. The algorithm proposed aims to produce good quality quadrilateral meshes automatically over arbitrary geometry, for that it preprocesses the current geometry correcting common features that difficult quadrilateral mesh generation. In sequence, it generates a quadrilateral mesh, first adjusting in a topological coordinate system a mesh that considers constraints coming from geometry, like element density and boundary line angles. Thus, the singularities are arranged in a way that reduces distorted elements. Hence, it maps the resulting mesh to the Euclidean coordinate system using the transfinite mapping method for all regions. As a result, it is possible to generate good quality quadrilateral mesh even on distorted geometries in comparison with methods available in actual commercial software.

Keywords: Boundary Lines Angle Constraint, Element Density Constraint, Geometry Preprocessing, Quadrilateral Mesh Generation

1 Introduction

The use of the finite element method (FEM) for simulations of mechanical behaviors in solid mechanics has become a necessary step in the design of new projects. However, to obtain an accurate result, care must be taken at the mesh generation. As showed by Du et al. [1], proper elements shape and distribution lead to better numerical results and different types of meshes have their pros and cons in processing time or numerical efficiency.

According to Armstrong et al. [2], triangular or tetrahedral meshes have the most robust algorithms, capable to produce elements in almost any arbitrary geometry with good shape and respecting variable element density. On the other hand, quadrilateral and hexahedral subdivisions are harder to obtain. As reported by Yu et al. [3] actual meshing algorithms cannot guarantee to generate a satisfactory mesh for complex geometries. Moreover, in this type of mesh is problematic to do local topological modifications or refinements because it forms chains of elements that are difficult to rearrange. Their generation algorithms generally apply only to a portion of geometries that fits a certain criterion. Nevertheless, that technique presents some advantages over the triangular or tetrahedral meshes due to several reasons: Natural tensor-product representation, higher computational accuracy, and easier feature alignment.

Another common classification in mesh generation is structured and unstructured meshes. Structured meshes are characterized by equal numbers of connections in each node of the mesh. In unstructured meshes, the number of connections can vary freely. As stated by Ito [4] structured meshes generate more distributed elements and display a better convergence in numerical methods, although it is harder to achieve.

Like showed by Owen [5] the generation of quadrilateral unstructured elements in surfaces is commonly subdivided into two categories: the direct and indirect methods. The direct methods generate the elements in its final form, without a further transformation after the mesh generation. The most used techniques are the advancing front method and the block subdivision method. In the indirect methods, the geometry first is subdivided in triangles and then these are joined together to generate quadrilaterals. Generally, the adoption of this approach produces distorted elements and/or mixed mesh with triangles and quadrilaterals.

The generation of structured meshes with quadrilateral elements is generally made using mapping methods. That sort of mesh is recognized to produce good elements, although presents several difficulties in its generation process, usually involving some sort of block decomposition as showed by Baker [6].

The algorithm presented in this paper is part of a larger FEM mesh generation computational package. This software aims to produce good quality quadrilateral and hexahedral meshes automatically over arbitrary geometry. For that it preprocesses the current geometry correcting common features that difficult quadrilateral and hexahedral mesh generation. It includes the identification of lines contact, overlap and intersection and also lines over surfaces. Besides that, surfaces with three, five or more edges as well as with internal boundaries are manipulated, joined and/or subdivided, until quadrilateral surfaces are obtained. Similarly, volumes with any number of lateral surfaces are also modified until hexahedral volumes with quadrilateral surfaces are obtained. The part of the complete software shown in this paper is responsible for the generation of quadrilateral elements in an area also quadrilateral that respect conditions imposed by the geometry, achieving the best possible subdivision.

2 Quadrilateral surface mesh generation

Frey and George [7] subdivided the mesh generation into five methods: i) manual and semi-automatic, ii) parametric mapping, iii) decomposition of domains in subdomains, iv) insertion points and/or elements creation and v) constructive methods. When dealing with quadrilateral mesh generation, it is common to use constructive methods. These techniques use a combination of the other methods to generate the mesh. That is because the techniques intended to produce quadrilateral elements generally require some requisites conditions on the geometry. The decomposition of domains in subdomains is one of the solutions used to transform a complex model in a set of simple blocks capable to meet these restrictions.

In the insertion points and/or elements creation methods, the advancing front method is the most applied for quadrilateral mesh generation. This method proceeds by creating one element at a time from a moving front. Then the unmeshed area is filled recursively until de front fills all the space. As reported by Staten et al. [8] this technique generally produces good elements in the boundary where the front started, and bad ones in internal regions, where it finishes.

The parametric mapping methods are known to generate a balanced element quality in the model, seeing that a mesh is generated in a topological space and then mapped to the Euclidean space. The function used to do the transformation can adjust the mesh to the boundary, thus it becomes uniformly distributed inside the geometry. Problems with the elements shape can be caused because the original geometry is distorted.

For the use of this method, two solutions are possible: the finite element interpolation technique [9] or the transfinite mapping [10]. The first method uses the Lagrange bilinear elements linear or quadratic to map the topological mesh to the Euclidean space. It is less used since it is limited to surfaces with boundary composed by linear or quadratic functions. The second method, instead, is capable to deal with every type of curve in the boundary. The only requirement is that the parametric equations of the curve are known. Thus, due to the fact of the quality of the mesh and the facility to work with a lot of geometries this method offers a great solution for the generation of quadrilateral elements.

Although the transfinite mapping is a good technique that produces efficient quadrilateral meshes, it faces some difficulties when used in certain conditions. Specific care must be taken with the size of the elements and with its internal angles since elements with small element aspect ratios or inner angles near 0º or 180º generate problems during the numerical calculus and must be avoided. As stated by Beatty and Mukherjee [11], internal angles near 90º are considered the ideal value.

The problem with the angle can have three causes: a bad distribution of elements in the topological space; a great curvature of the surface, distorting the mapped mesh in a way that the quadrilaterals turn distorted; or the angles between the curves of the boundary are too obtuse or too acute, forcing the elements in the region to be the same way. To resolve the third problem, the algorithm proposed in this paper corrects the elements near such occurrences, imposing some restrictions on the topological mesh. One example of such an occurrence is shown in figure 1.

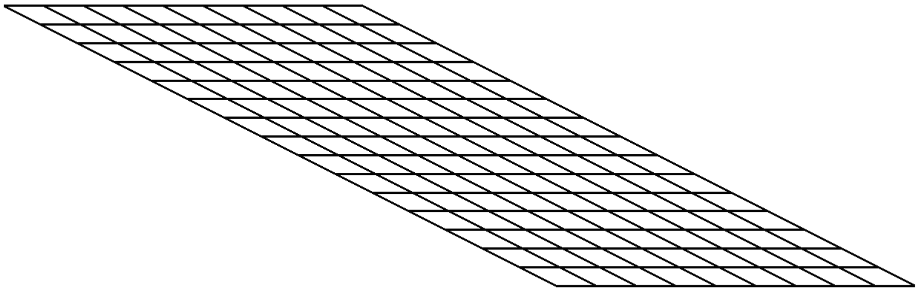

Figure 1. Mesh with obtuse and acute angle in their boundary vertices

3 Proposed algorithm

The algorithm proposed in this paper aims to generate quadrilateral elements that respect conditions imposed by the geometry, trying to achieve the best possible subdivision using mapping methods on deformed surfaces. For that, it first distributes the elements in a topological system, using the information read from the model to predict the best elements arrangement. Then it uses the transfinite mapping method to project the mesh to the Euclidean space.

In a structured mesh of quadrilaterals, two types of problems influence the numerical result in the application of the FEM: the distortion of the elements and the number of singularities in the mesh. The problem with the elements shape is for the reason that in a finite precision floating point arithmetic realized by the computer, the multiplication of great numbers with low numbers generates larger approximation errors than multiplying numbers with the same magnitude.

As showed by Armstrong et al. [2] singularities in a mesh represent the nodes that have a different number of connections in comparison with their neighbors. In a quadrilateral subdivision, four connections are the natural number. The presence of singularities influences the mesh quality and therefore the convergence of the FEM calculus. Thus an efficient mesh should combine elements with good shape and a minimum number of singularities.

Two types of elements arrangement can be used in a corner vertex of the geometry. In this paper, it is called as an orthogonal or polar arrangement. To apply any of them, a reference point is first created. That point is oblique to the corner vertex and its distance of each boundary is determined by both element edge lengths in the respective boundary lines. In an orthogonal arrangement, one element is formed at the corner, composed by the connection between the reference point and the end of the first element edge in each boundary line of the corner. In the polar arrangement, the reference point is connected with the corner vertex, forming two elements at the position, both divided by the bisector formed. An example of the arrangement types could be seen in figure 2.

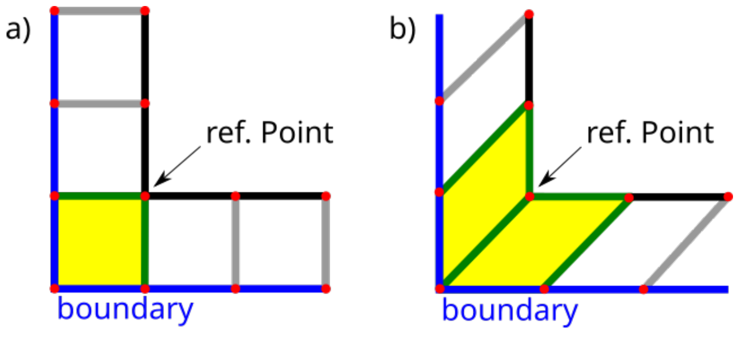

Figure 2. Elements arrangement: a) orthogonal; b) polar.

Both arrangements can be used for two reasons: to correct some different density between opposite lines of the boundary and/or to minimize the impact of the angle between the boundary lines on the elements shape. In the first case, the lines of an area can have different densities requirements. It is either because of some restriction imposed by the user or in case of a great difference between the lengths of its lines, supposing a defined element size is sought. In this case, four possible dispositions can be applied to correct the different densities. Like showed in figure 3 the a) situation corrects one element difference between both two opposed lines and can be used, by means of rotation, in the four vertices. The same analogy is valid for other cases.

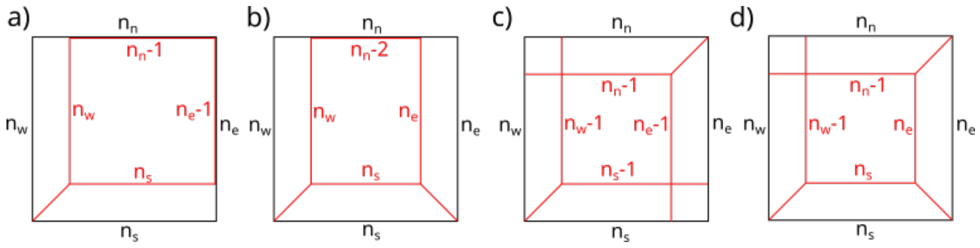

Figure 3. Elements arrangement to correct different densities: a) one polar vertex; b) two consecutives polar vertices; c) two opposite polar vertices; d) three polar vertices.

Moreover, these two types of arrangements generate an effect on the inner angles of the elements, depending on the angle between the boundary lines of the area. The orthogonal arrangement will maintain the angle of the boundary lines in the internal element angle, but the polar arrangement divides the angle between the two elements in the corner. Only doing the correct treatment of the angle between the boundary lines, it is possible to generate elements with better internal angles in the corners of the Euclidean area.

Therefore, to do that correction, the algorithm imposes a polar arrangement when the angle is greater than 120º and an orthogonal arrangement when it is less than 60º. Any corner that does not fill inside this angle range the algorithm interprets that is free, and both types of arrangement could be used. As a consequence, if it is necessary to perform a correction since there are problems of density, any correction will be allowed. In the current algorithm only the first layer of elements receives the angle restriction, although depending on the element size and on the angle magnitude, more inner layers could be corrected for better mesh quality.

There are sixteen possible angle arrangements as shown in figure 4. In each type of imposition, the mesh density is changed, hence the corrections of density presented before should be applied together, correcting the densities when the angle restrictions are not valid anymore.

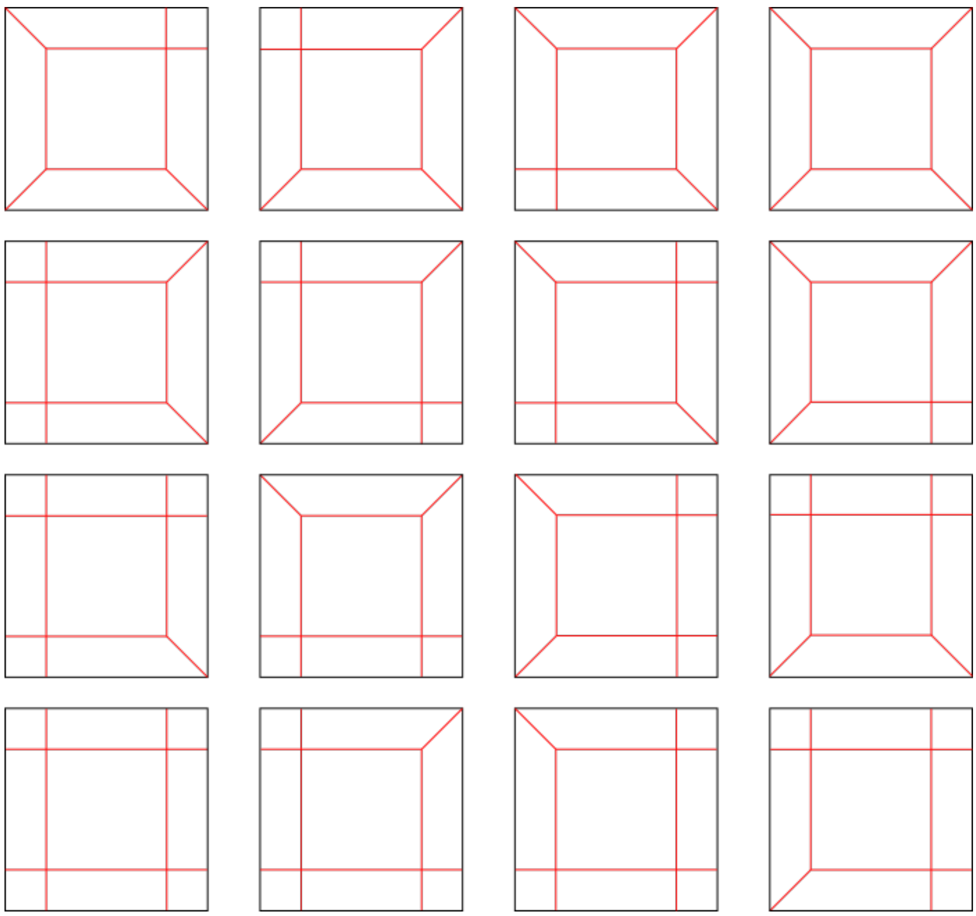

Figure 4. Angle imposition possibilities using polar and orthogonal arrangements

The algorithm then, using the restrictions and density information, creates a disposition list. This list represents the sequence of the arrangements that together fill the area of the geometry. This list considers the order of the arrangement that presents the minimal number of elements and, consequently, nodes. This is an important constraint that increases quality of the mesh and decreases the number of degrees of freedom. For example, in a sequence of a) and b) cases of figure 3, the minimum number of elements is achieved when a) cases are first used.

Initially, this list considered only the arrangements shown in figure 4, but as more examples were considered some additional provisions should be included as an orthogonal L disposition, present in the first case of figure 5. In addition, some adjustments had to be made in the existing dispositions. One of them is the calculation of reference points in order to generate the most uniform mesh and maintain the continuity of the elements chain.

4 Results

Using the algorithm proposed in the paper, a topological mesh was generated for each possible angle imposition in a quadrilateral geometry. Furthermore, each side of the geometry used a specific element density for all cases, $n_e = 8$, $n_n = 5$, $n_w = 7$ and $n_s = 8$. That way the algorithm was tested for its capability to deal with angle impositions and with different mesh densities corrections. The resulting mesh is showed in figure 5.

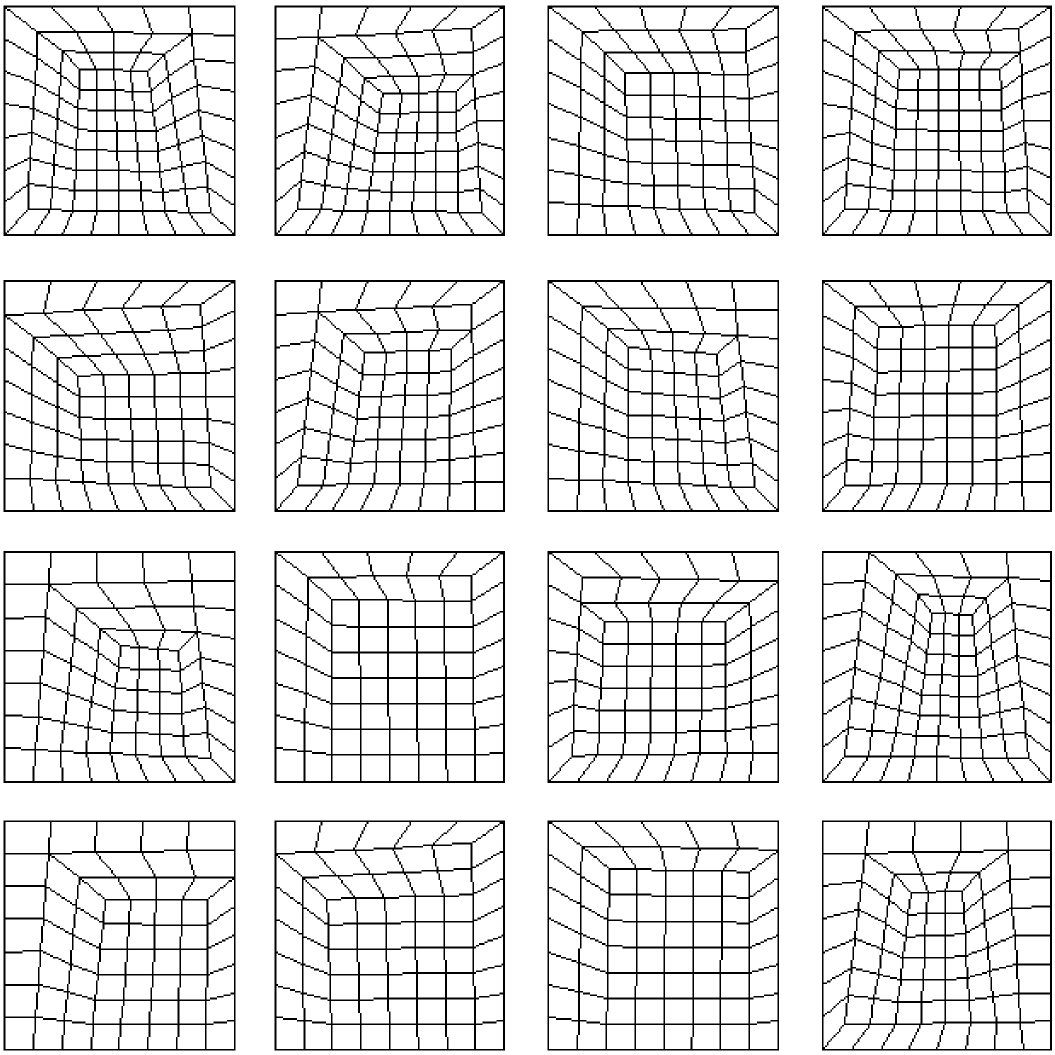

Figure 5. Topological mesh for all angle imposition possibilities

In all the cases presented, the mesh generated was capable to correct different densities in each boundary line of the topological area of the model and perform the correction due to the angle imposition using the two different corner elements arrangements presented in this paper. All the elements generated presents a good shape in the topological coordinate system and exhibit internal angles near 90º and a satisfactory aspect ratio.

Besides that, the number of singularities was minimal possible, when using the type of correction proposed in the paper, for the most of the examples. These singularities were mostly present near the boundary of the models, revealing a structured mesh in its internal parts.

5 Conclusions

Observing the results, the proposed algorithm can resolve a problem generally found in mapping methods for the generation of quadrilateral mesh. For that it uses the angle analysis of the boundary lines, adjusting the topological mesh to compensate the angle using different elements arrangements on the respective corners.

Moreover, the algorithm also was capable to correct density differences between opposite boundary lines using the same technique. The elements generated in the topological coordinate system presents good internal angles and a satisfactory aspect ratio.

Acknowledgements. The authors are grateful to LAMAC (Laboratory of Applied and Computational Mechanics) for the license of the ANSYS® computational package used for the generation of the images used in the present work. The first author would also like to thank EESC (Escola de Engenharia de São Carlos) for its support in research and for the availability of its lecturers in helping with problems faced during the preparation of the work. The second author would like to thank Dr.-Ing. Andreas Heckmann, who at the time of working together in which he completed his doctoral thesis entitled ”Zerlegungs- und Vernetzungsverfahren für die automatische Finite-Elemente-Modellierung”, inspired the realization of this work.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is property and authorship of the authors.

References

[1] Du, Q., Wang, D., & Zhu, L., 2009. On Mesh Geometry and Stiffness Matrix Conditioning for General Finite Element Spaces. *SIAM Journal on Numerical Analysis*, vol. 47, n. 2, pp. 1421–1444.

[2] Armstrong, C. G., Fogg, H. J., Tierney, C. M., & Robinson, T. T., 2015. Common Themes in Multi-block Structured Quad/Hex Mesh Generation. *Procedia Engineering*, vol. 124, pp. 70–82.

[3] Yu, W., Zhang, K., & Li, X., 2015. Recent algorithms on automatic hexahedral mesh generation. In *2015 10th International Conference on Computer Science & Education (ICCSE)*, pp. 697–702. IEEE.

[4] Ito, Y., 2013. Challenges in unstructured mesh generation for practical and efficient computational fluid dynamics simulations. *Computers & Fluids*, vol. 85, pp. 47–52.

[5] Owen, S. J., 1998. A Survey of Unstructured Mesh Generation Technology. In *7th International Meshing Roundtable*, pp. 239–267, Dearborn, Michigan, USA.

[6] Baker, T. J., 2005. Mesh generation: Art or science? *Progress in Aerospace Sciences*, vol. 41, pp. 29–63.

[7] Frey, P. J. & George, P. L., 2000. *Mesh Generation: Application to Finite Elements*. Hermès Science, illustrate edition.

[8] Staten, M. L., Kerr, R. A., Owen, S. J., Blacker, T. D., Stupazzini, M., & Shimada, K., 2009. Unconstrained plastering-Hexahedral mesh generation via advancing-front geometry decomposition. *International Journal for Numerical Methods in Engineering*, vol. 81, n. 2, pp. 135–171.

[9] Lo, D. S. H., 2017. *Finite element mesh generation*. CRC Press, 1st edition.

[10] Gordon, W. J. & Hall, C. A., 1973. Transfinite element methods: Blending-function interpolation over arbitrary curved element domains. *Numerische Mathematik*, vol. 21, n. 2, pp. 109–129.

[11] Beatty, K. & Mukherjee, N., 2000. A Transfinite Meshing Approach for Body-In-White Analyses. Technical report, SIEMENS, Ohio.