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CONTACT PROBLEMS IN ORTHOTROPIC MATERIALS USING THE BOUNDARY ELEMENT METHOD

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Abstract. The main objective of this work is the development of a boundary element formulation for the evaluation of frictional contact problems in bodies of anisotropic materials under centrifugal loads. The fundamental solution is obtained using the Lekhnitskii formulation. Centrifugal loads are treated as body forces and are not included in the computation of the fundamental solution. Because of this, domain integrals arises in the formulation. In order to avoid domain discretization, these integrals are transformed into boundary integrals using an exact transformation. As a result, an elegant formulation is obtained where only boundaries need to be discretized. Quadratic continuous boundary elements are used in order to discretize boundaries of contact bodies. As the contact area is unknown, there is a non-linear problem in which the Newton's method is used in order to compute tractions and displacements. Problems with known analytical solutions are used in order to assess the accuracy of the proposed formulation. A numerical analysis of a contact problem in a high pressure compressor of the turbojet engine is carried out. This problem comprises of anisotropic monocristal blades in contact with an isotropic rotor. Due to centrifugal and random loads, blades are subjected to stresses of fretting. They are attached at the rotor by dovetail joints where occur severe contact. Fatigue due to fretting is the main cause of mechanical failure in turbojet engines, what shows the relevance of this work. If compared to other traditional numerical methods, as the finite element method, there are many advantages in the boundary element formulation, considering accuracy in the computation of stress field and an easier mesh manipulation.

Keywords: Boundary Element Method, Fretting, Anisotropic Material.

1 Introduction

Fatigue failure occurs in components subject to dynamic and oscillatory loads, normally at stress levels much lower than the necessary level to fail at static loads. In general, fretting occurs on firmly attached surfaces who suffered small displacements among themselves due to oscillatory loads. In the aeronautical sector, this phenomenon is present in the contact between the blades and the turbine rotor. These parts are generally manufactured separately and mounted by connection on a dovetail joint.

The objective of this article is to present an alternative method that can be as precise as the FEM and computationally cheaper for the purpose of computation of stress and strain on anisotropic bodies in contact and under centrifugal loads. The main application is the rotor-blade dove tail fretting analysis of turbo-jet engines.

Another important application of anisotropic materials in aeronautical structure is the use of composite material components. Composite materials have aroused the interest of several researchers in decades, due to the low weight and variation of mechanical properties, which can be designed for specific uses. In structures made of composite material, the union between structural members using screwed or riveted joints is associated with great difficulties, due to the fact that they are laminated components that may eventually present couplings between the stresses and high stress concentration factors as shown in Daniel and Ishai [1].

It is necessary to highlight some of the advantages of using the Boundary Element Method (BEM) to evaluate contact systems, in relation to other numerical techniques, such as the Finite Element Method (FEM). The main advantages that should be highlighted is the decrease in an order the dimension of the proposed problem, being possible to notice that there is a decrease in the amount of input data, in the processing time and in the storage of processed information, guaranteeing less computational effort.

2 Exact Solutions

2.1 Rotating disc

Consider an orthotropic rotating disc of radius *R*, rotating with angular velocity *w*, around its center, and a system of origin coordinates in the center of the disc and whose axis directions coincide with the directions of the axes of greater and lesser rigidity of the material Fig. 1.

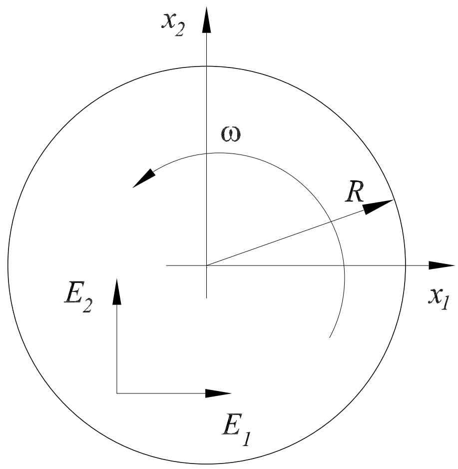

Figure 1. Orthotropic rotating disc.

$$
\begin{aligned}
u_1 &= \frac{a_{11}\rho\omega^2 x_1}{2}\left\{K\left(\frac{x_1^2}{3}+3x_2^2-R^2\right)+\left[R^2-\left(\frac{x_1^2}{3}+x_2^2\right)\right]\right\}+\\
&\quad \frac{a_{12}\rho\omega^2 x_1}{2}\left\{K\left(x_1^2+x_2^2-R^2\right)+\left[R^2-\left(\frac{x_1^2}{3}+x_2^2\right)\right]\right\}\\
u_2 &= \frac{a_{12}\rho\omega^2 x_2}{2}\left\{K\left(x_1^2+x_2^2-R^2\right)+\left[R^2-\left(x_1^2+\frac{x_2^2}{3}\right)\right]\right\}+\\
&\quad \frac{a_{22}\rho\omega^2 x_2}{2}\left\{K\left(3x_1^2+\frac{x_2^2}{3}-R^2\right)+\left[R^2-\left(x_1^2+\frac{x_2^2}{3}\right)\right]\right\}
\end{aligned}
\tag{1}
$$

where a_{ij} are material constants and

$$
K=\frac{a_{11}+2a_{12}+a_{22}}{3a_{11}+2a_{12}+a_{66}+3a_{22}}
\tag{2}
$$

2.2 Hertzian contact problem

A classic problem with an analytical solution deduced by Hertz [2] is presented in this section. A cylindrical punch with a flat profile is pressed against a semi-spacial elastic by a normal force P. Considering that the punch is rigid and in full contact, the analytical solution to the problem is given by:

$$
p(x)=-\frac{P}{\pi\sqrt{a^2-x^2}}
\tag{3}
$$

this type of contact pressure distribution is unique at the tips, with the analytical solution tending to infinity when $|x|=a$.

3 Boundary Integral Formulation

Considering an infinitesimal element inside an orthotropic rotating disc of domain Ω and boundary Γ, the equilibrium equation can be written as

$$C_{ijkl}u_{k,jl} - \rho\omega^2 x_i = 0 \tag{4}$$

where C_{ijkl} is the elastic constant tensor, u_i is the displacement vector, and ρ is the density, ω is the angular velocity of the disc.

The integral equation is obtained by integrating the equilibrium Eq. (4), using Maxwell-Betti reciprocal theorem and Hooke's law. Following the procedure presented by Albuquerque [3], the displacement integral equation is given by:

$$c_{ij}u_j + \int_\Gamma T_{ij}u_j d\Gamma = \int_\Gamma U_{ij}t_j d\Gamma + \rho\omega^2 \int_\Omega U_{ij}x_i d\Omega \tag{5}$$

where c_{ij} is a constant which depends on the position on the boundary; U_{ij} and T_{ij} are the displacement and traction Lekhnitskii fundamental solutions, respectively. The last integral, that is a domain integral, is transformed into boundary integral using the radial integration method. Details of Lekhnitskii fundamental solutions can be found in Albuquerque [3]

The contact mode is checked at each load step to decide which nodes are in contact and which are not. Considering the contact pair a and b, with surface forces and displacements known in step $m-1$, if in step m no contact is detected, the following relationship is true:

$$\begin{aligned}(\Delta u_n^a + \Delta u_n^b)^m &< g_0^{m-1},\\ t_n^{m-1} + \Delta t_n^m &\geq 0.\end{aligned} \tag{6}$$

if the opposite happens,

$$\begin{aligned}(\Delta u_n^a + \Delta u_n^b)^m &\geq g_0^{m-1},\\ t_n^{m-1} + \Delta t_n^m &< 0,\end{aligned} \tag{7}$$

nodes a and b are not in contact. The g_0 function represents the gap between nodes while u_n and t_n are normal displacements and normal surface forces, respectively.

In the event of contact, a second check is made regarding the tangential movement of the contact nodes. Peer evaluation for the mode of adherence is given by,

$$|t_t^m| = |t_t^{m-1} + \Delta t_t^m| \leq \mu|t_n^{m-1} + \Delta t_n^m| = \mu\,|t_n^m| \tag{8}$$

if this condition is valid, the pair is in a state of adhesion. Oppositely:

$$|t_t^{m-1} + \Delta t_t^m| \geq \mu|t_n^{m-1} + \Delta t_n^m| \tag{9}$$

the slip condition is achieved. Coulomb's law of friction defines two friction coefficients, one static and one dynamic. According Nowell [4], in problems where there is sliding and sticking simultaneously, f can be approximated by an average value.

4 Numerical Results

4.1 Orthotropic rotating disc

In order to assess the ability of the code to model centrifugal load problems, consider an orthotropic disc in plane stress with Young's moduli E_1 = 17.24 Pa and $E_2 = 48.26$ Pa, shear modulus $G_{12} = 6.89$ Pa, Poisson's ratio $\nu = 0.29$, radius $R = 1$ m rotating with angular speed $\omega = 20$ rad/s. This problem is analysed using the developed formulation. Due to the symmetry, only one quarter of the disc was discretized with boundary conditions given by Fig. 2.

The quarter of the disc is discretized with a boundary mesh with 18 boundary elements per segment.

The numerical results obtained for displacement in x direction is shown in Fig. 3 and in the y direction is shown in Fig. 4. They are compared to analytical solutions given by equations (1), showing good agreement.

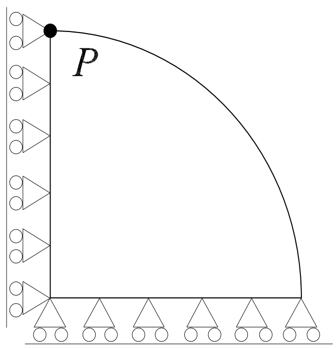

Figure 2. Boundary conditions for the one quarter of the orthotropic rotating disc.

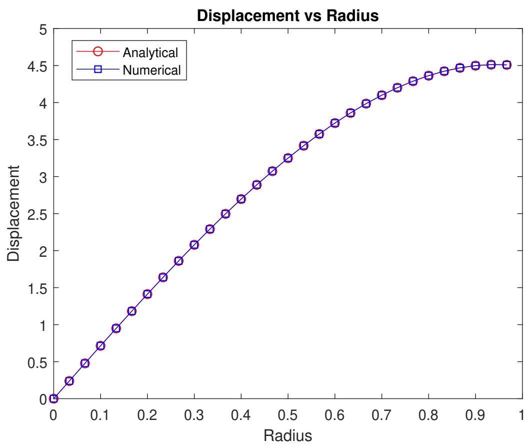

Figure 3. Boundary stress along x direction.

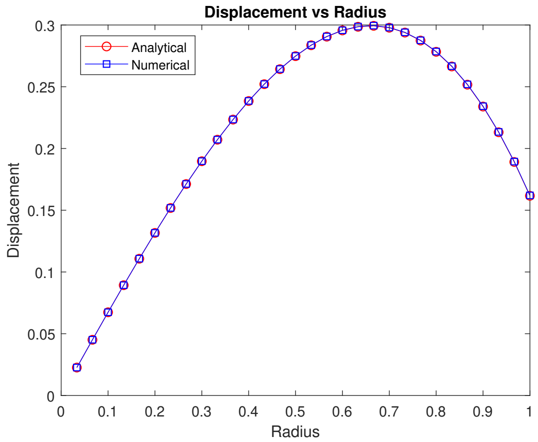

Figure 4. Boundary stress along segment along y direction.

4.2 Hertz contact problem

A Hertzian contact was simulated to validate the code ability to assess contact problems. It consists of a cylindrical punch in contact with a rigid half-plane. For this case, we have an analytical solution, using Eq. (3).

The geometry, shown in Figure 5, and material properties of the cylindrical punch with Young's moduli E = 73.4 GPa pressure modulus $P = 90$ N/mm, Poisson's ratio $\nu = 0.33$, radius $R = 70$ mm, lenght $w = 6.5$ mm, height $h = 6.5$ mm.

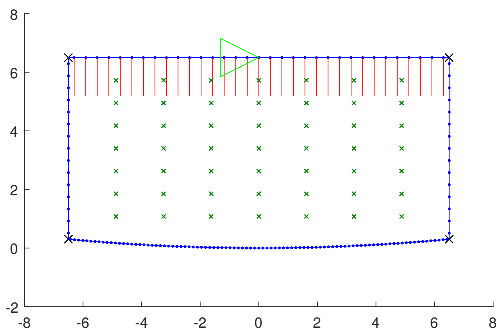

Figure 5. Boundary conditions and mesh.

By observing Fig. 6 it can be notice a good agreement between the numerical results and analytical solutions given by equation (3).

4.3 Dovetail contact problem

In the present work, Papanikos [5] was used as a reference for the numerical simulation of the blade dovetail joint analysis using the BEM. The representation shown in Figure 7 is the disc and dovetail of an airplane turbine.

Using numerical modeling presented in this work, it is possible to compute the normal traction in the contact region using the boundary element method.

The boundary conditions applied in Fig. 8, illustrates the movement restrictions, in which, the green triangles is the known displacement, the red segment is the known traction. In Fig. 8, it is shown the normal tracion in the contact region. It presents the expected behaviour predicted in literature. However, in order to validate the code used, the simulation will be performed using the FEM and compared to the BEM results.

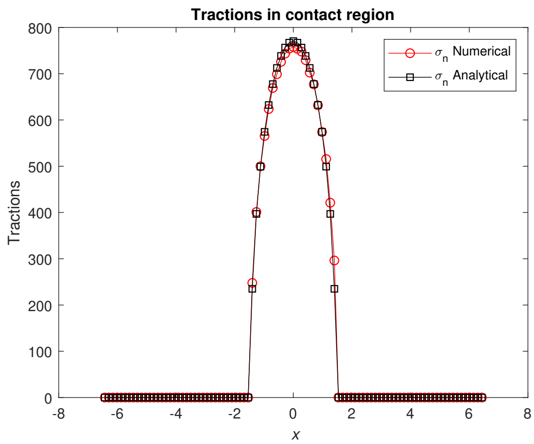

Figure 6. Tractions in the x and y direction for hertzian contact

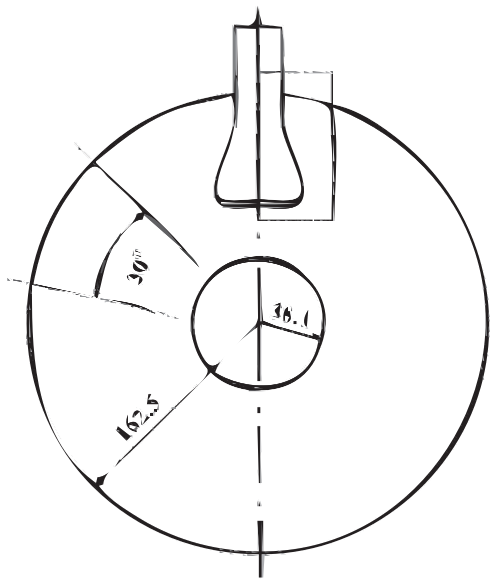

Figure 7. Disc and dovetail, Papanikos [5].

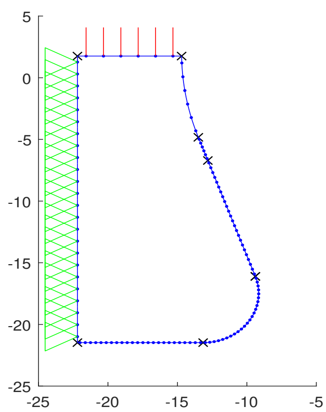

Figure 8. Detailed representation with boundary conditions.

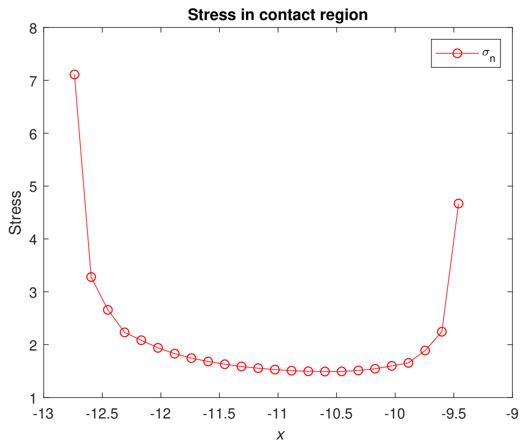

Figure 9. Stress normal in contact region.

5 Conclusions

This work presents a formulation based on the boundary element method for the solution of contact of bodies under centrifugal forces . It was able to evaluate problems with mode to node contact, with different boundary conditions and contact between bodies made up of different materials. For that, each body was modeled and the contact conditions were applied using constraint conditions and force equilibrium, forming a non-linear system of equations that was solved by Newton's method.

The results is in good agreement with literature. The formulation is extended for anisotropic allowing the analysis of materials such as single crystal alloys or ceramic matrix composites used in turbine blades.

The implementation of domain forces to reproduce centrifugal was carried out without the generation of domain integrals. Thus, there is no need discretize the domain, keeping the main characteristics of the boundary element method that is the necessity to discretize only the boundary.

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References

[1] Daniel, I. & Ishai, O., 1994. *Engineering Mechanics of Composite Materials*. Oxford University Press.
[2] Hertz, H., 1882. über die berührung fester elastischer körper and über die harte(on the contatc of rigid elastic solids and on hardness). *J. reine und angewandte Mathematik*.
[3] Albuquerque, M. H. Aliabadi, P. S., 1999. The dual boundary element formulation applied to dynamic fracture mechanics in anisotropic materials. *Boundary Element Techniques*, pp. 23–29.
[4] Nowell, D., 1988. *An analysis of fretting fatigue*. PhD thesis, University of Oxford.
[5] Papanikos, S.A. Meguid, Z. S., 1998. Three-dimensional nonlinear finite element analysis of dovetail joints in aeroengine discs. *Finite Elements in Analysis and Design*, vol. 29, n. 3–4, pp. 173–186.