Assessment of flexible pipes with damaged tensile armor wires using finite element models and symbolic regression

Anderson C. dos Santos¹, José Renato M. de Sousa ¹

¹Civil Engineering Dept - PEC, Federal University of Rio de Janeiro, COPPE/UFRJ, Centro de Tecnologia, Bloco I2000, Sala I116, Cidade Universitária, 21945-970, Rio de Janeiro, Brazil
anderson.santos@laceo.coppe.ufrj.br, jrenato@laceo.coppe.ufrj.br

Abstract. Flexible pipes are key components in several offshore oil and gas applications as production, gas lift, or gas and water injection lines. However, these structures may be damaged during their service lives, which may reduce their structural capacity and lead to costs associated with replacement, production loss, and oil spill. Among the possible damages in flexible pipes, the rupture of their tensile armors is critical, as these armors sustain the axial loads imposed on these pipes. Therefore, this paper deals with the structural analysis of flexible pipes with broken tensile armors subjected to tension loads. A previously proposed finite element model is employed to model the damaged pipes. Forty-seven flexible pipes are analyzed and the Stress Concentration Factors (SCF’s) of the tensile armor wires are collected to form a database, which is then employed by a symbolic regression package to fit analytical expressions for the obtained SCF’s. The goal of this work is therefore to find a closed formed analytical expression for the SCF’s, which is accurate and thus able to replace a time-consuming assessment with a finite element model.

Keywords: flexible pipes, damage assessment, finite element models, symbolic regression.

1 Introduction

Unbonded flexible pipes (or, simply, flexible pipes), such as the one presented in Fig.1, are largely employed in offshore oil and gas exploitation. These multilayered composite structures combine high axial strength and stiffness with low bending stiffness, resulting in highly compliant pipes. Each layer of these pipes has a specific function and may be either metallic or polymeric. Whereas the metallic layers provide structural resistance, the polymeric layers are used to seal the pipe and/or to mitigate wear and friction between layers. In a typical flexible pipe, three different metallic layers are found [1]: inner carcass (IC), which is made from profiled steel strips wound at angles close to 90 degrees with respect to the pipe axis that mainly resists radial inward forces; pressure armors (PA), which are usually Z-shaped steel wires also wound at angles close to 90 degrees with the main function of supporting the system internal pressure and also radial inward forces; and tensile armors (TA), which are constituted of several approximately rectangular steel wires laid in two or four layers, cross-wound at angles between 20 degrees and 55 degrees, which resist tension, torque, and pressure end cap effects.

Flexible pipes are critical assets, as they have elevated costs and their failure may lead to consequences such as oil spill and production loss. One of the most critical failure modes is the rupture of the tensile armors wires which threatens the structural integrity, as the load capacity may be significantly reduced. In the last few years, several inspection and surface in-field monitoring techniques have been proposed and developed aiming at monitoring the tensile armor layers of flexible pipes [2].

Therefore, the comprehension of the structural response of damaged flexible pipes is required to reduce costs and to ensure a secure operation of these structures. The main problem in analyzing a flexible pipe with damaged tensile armors wires is the need to individually represent these wires in the chosen model. Analytical models available in the literature are suitable for some specific situations but cannot be generally applied in the analysis of damaged flexible pipes [2]. However, De Sousa et al. [2], proposed a three-dimensional nonlinear finite element model to analyze the mechanical response of flexible pipes under installation or in-service loads such as tension...
and torsion or axial compression. This multi-purpose FE model represents each tensile armor wire and, consequently, localized defects, including total rupture, may be adequately simulated.

![Figure 1. Typical Flexible Pipe [2]](image)

In this paper, the FE model proposed by De Sousa et al. [2] is applied to study the mechanical response of flexible pipes subjected to pure tension. Forty-seven flexible pipes are studied and up to 25% of total wires at each flexible pipe outer tensile armor (OTA) are broken. Maximum Stress Concentration Factors ($SCF_{\text{max}}$) will be defined since they are a practical measure of robustness check [3]. Finally, to achieve a closed-form expression to $SCF_{\text{max}}$, the Symbolic Regression (SR) technique is used.

Next, a description of the employed FE model is presented followed by a brief review of the Symbolic Regression aspects and usage. After that, the results obtained are presented and discussed.

### 2 Finite Element Model

The finite element model (FEM) used in this study is the same employed in De Sousa et al. [2]. This model uses equivalent properties to represent each layer of the pipe with thick to moderate thick-walled shells. The tensile armor wires, however, are modeled with its actual cross-sections and three-dimensional beam elements are employed to represent each wire. The interaction between layers is ensured with the presence of contact elements. For the sake of conciseness, no further details about the FEM will be given here, and the reader is advised to consult De Sousa [2] to obtain more details about the proposed FE model.

The FE meshes of this model are generated by the in-house program called RISERTOOLS, which generates FE meshes to be analyzed in ANSYS® [4] program.

### 3 Symbolic Regression

Symbolic regression (SR) is the process of determining symbolic functions that describe a dataset effectively by developing an analytic model, which summarizes the data, and is useful to predict the different responses as well as facilitating human insight and understanding [5].

Unlike traditional linear and nonlinear regression methods that fit parameters to an expression (equation/relation) of a given form, SR simultaneously searches for both the parameters as well as the form of the expression. Therefore, SR searches for a set of basic functions (building blocks) and coefficients (weights) to minimize the error between the proposed expression and the dataset. The standard basic functions are addition, subtraction, multiplication, division, sine, cosine, tangent, exponential, power and square root, (“+”, “-”, “x”, “/”, “^”, “exp”, “log”). To select the optimal set of basic functions, Koza [6] suggested the employment of genetic programming (GP).

GP is a biologically inspired machine learning method that evolves computer programs to perform a task. To carry out genetic programming, the individuals, i.e., competing functions, should be represented by a binary tree.
In standard GP, the leaves of the binary tree are called terminal nodes represented by variables and constants, while the other nodes, the so-called non-terminal nodes are represented by functions. Figure 1 shows the binary trees of several functions. GP randomly generates a population of individuals, i.e., models represented by tree structures aiming to find the best-performing ones. There are two important features of the function represented by a binary tree: complexity and fitness. Complexity is defined as the number of nodes in a binary tree needed to represent the function. The fitness qualifies how good a model is. In this work, the Mean Squared Error (MSE), Eq. (1) is used as the fitness function.

\[
f_k = \frac{1}{m} \sum_{i=1}^{m} \left( SCF_{\text{max}}^{SR}[i] - SCF_{\text{max}}^{FEM}[i] \right)^2, \quad \text{for } 1 \leq k \leq n.
\]

where \( n \) is the size of the population, \( f_k \) is the fitness associated to the k-th model, \( m \) is the size of data set to be modeled, i.e., the amount of \( SCF_{\text{max}} \)'s extracted from the FEM proposed in the previous section, \( SCF_{\text{max}}^{SR}[i] \) is the i-th \( SCF_{\text{max}} \) predicted value for k-th model and \( SCF_{\text{max}}^{FEM}[i] \) is the i-th \( SCF_{\text{max}} \) value retrieved from FEM.

GP tries to improve the fitness of the population consisting of individuals (competing models) from generation to generation by mutation and crossover procedure. Mutation is an eligible random change in the structure of the binary tree, which is applied to a randomly chosen sub-tree in the individual. This sub-tree is removed from the individual and replaced by a new randomly created subtree. This operation leads to a slightly (or even substantially) different basic function as depicted in Fig. 1a. The crossover operation, representing sexuality, can accelerate the improvement of the fitness of a function more effectively than mutation alone can do. It is a random combination of two different basic functions (parents), based on their fitness, to create a new generation of functions, fitter than the original functions. To carry out crossover, crossing points (non-terminal nodes) in the tree of both parents should be randomly selected, as can be seen in Fig. 1b. Then, subtrees belonging to these nodes will be exchanged creating offspring.

An important goal in symbolic regression is to get a solution, which is numerically robust and does not require high levels of complexity to give accurate output values for given input parameters. A useful expression is both predictive and parsimonious. However, SR algorithms, instead of producing a single result, produce a set of expressions ranging from the most accurate, but overfit the data, to those that are more parsimonious but oversimplify. The prediction error versus parsimony relation, known as Pareto front, represents the set of proposed solutions given by SR. The Pareto front tends to contain a cliff where the predictive ability jumps rapidly at some minimum complexity. The predictive ability then improves only marginally with more complex expressions. Since the Pareto front provides the set of optimal solutions, the user should decide which ones are preferable.

The commercial package Eureqa® [7] is used to perform the SR analyses in this work.

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Figure 1 – Binary Tree Representation of several functions  a) Mutation Process b) Crossover Process
4 Case Study

4.1 Samples

Table 1 shows the general proprieties of the 47 flexible pipes used in this study. This set consists of flexible pipes of several usages, i.e., production, water injection, gas injection, and service lines.

4.2 FEM features

Each FE mesh considered a length of 2 OTA pitches. Moreover, the number of circumferential divisions was the same as the number of wires in OTA to ensure that only one wire has been damaged at a time. The meshes have 35,724 up to 210,926 nodes, and 67,353 to 399,386 elements. Altogether, FE meshes with 209,810 to 1,253,882 degrees of freedom were generated. A tension load of 1,000 kN was applied. Regarding the boundary conditions, one end of the pipe was fixed, whilst the other was free to rotate and elongate. Loads and boundary conditions were imposed on nodes at the ends of the pipe [2].

The damage was imposed in the mid-length of each pipe sequentially, as depicted in Fig. 1(a). One wire is broken at a time until 10 wires are damaged. After that, two wires are broken at a time up until 25% of the OTA wires were damaged. To model the damage itself, an element, belonging to each damaged wire has its stiffness matrix multiplied by a small number, e.g. $10^{-6}$, as depicted in Fig. 2b by magenta elements. In other words, the element is “deactivated”, while the stiffness matrices of all other elements are kept unchanged [2]. No wire has been removed from the model.

Table 1. Flexible pipes proprieties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID</td>
<td>Inner Diameter</td>
<td>63.5 mm (2.5 in)</td>
<td>368.30 mm (14.5 in)</td>
</tr>
<tr>
<td>$N_{ia}$</td>
<td>Total wire number at ITA$^1$</td>
<td>28</td>
<td>105</td>
</tr>
<tr>
<td>$N_{oa}$</td>
<td>Total wire number at OTA$^2$</td>
<td>30</td>
<td>108</td>
</tr>
<tr>
<td>$t_{ic}$</td>
<td>Inner Carcass thickness</td>
<td>3.5 mm</td>
<td>12.5 mm</td>
</tr>
<tr>
<td>$t_{pa}$</td>
<td>Pressure Armor thickness</td>
<td>6.2 mm</td>
<td>12 mm</td>
</tr>
<tr>
<td>$t_{ta}$</td>
<td>Tensile Armors wire thickness$^3$</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>$w_{ta}$</td>
<td>Tensile Armors wire width</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>$R_{ic}$</td>
<td>Inner carcass mean radius</td>
<td>33.25 mm</td>
<td>189.65 mm</td>
</tr>
<tr>
<td>$R_{pa}$</td>
<td>Pressure Armor mean radius</td>
<td>42.85 mm</td>
<td>212.65 mm</td>
</tr>
<tr>
<td>$R_{ita}$</td>
<td>ITA mean radius</td>
<td>46.95 mm</td>
<td>221.8 mm</td>
</tr>
<tr>
<td>$R_{ota}$</td>
<td>OTA mean radius</td>
<td>48.95 mm</td>
<td>227.6 mm</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Tensile Armors lay angle</td>
<td>25º</td>
<td>39º</td>
</tr>
</tbody>
</table>

Note:1- ITA – Inner Tensile Armor, 2- OTA – Outer Tensile Armor, 3- both OTA and ITA uses the same wire type

4.3 Empirical-analytical model

In this work, the Stress Concentration Factor (SCF) is defined as the ratio between the axial stress in the $i$-th OTA wire, after the $j$-th wire has been removed, $\sigma_{n_{ij}}$, and, the axial stress in the $i$-th wire considering the intact pipe, $\sigma_{n_{i}}$, eq. (2).

$$SCF_{n_{ij}} = \frac{\sigma_{n_{ij}}}{\sigma_{n_{i}}}$$  (2)
At each wire removal, only the greater SCF found (SCF\textsubscript{max}) has been chosen to compound the dataset to be analyzed using SR.

It was assumed the Empirical-Analytical Model (EAM) for SCF\textsubscript{max} would be a function only of geometric characteristics of layers, thus, no material parameters were included, moreover, the contribution of plastic layers has been neglected [2].

Following the approach used by Gonzalez [8], dimensionless groups were used as the dataset in which Eureqa\textregistered{} would search an analytical expression for SCF. Table 2 lists the dimensionless group used in the search and Eq. (3) shows the proposed equation format informed to Eureqa\textregistered{}.

The \( t_i/R_j, w_i/R_i^2 \) parameters enclose a dimensionless radial stiffness contribution for each layer, whilst \( t_iw_i/R_i^2 \) parameters enclose a dimensionless filling contribution of each layer, and \( N_i/N_k \) parameters were suggested from results found in De Sousa et al. [2]. The “r” superscript means “amount of removed wire”.

Relating to SR setup, the data set was composed of 559 lines where each line contains the SCF\textsubscript{max} associated with the i-th wire removed for the j-th pipe and the 16 parameters given by Table 2. Sum, subtraction, plus, division, and power were the “building blocks” functions used in the search. The solution with the best fit was chosen despite being the least parsimonious one. The search took 79 hr using an Intel i7 4.0 GHz machine with 4 cores and 32GB of RAM.

\[
SCF = f(\Pi_1, \Pi_2, \ldots, \Pi_{16})
\]

\[
\Pi_1 = N^r_{oa}
\Pi_2 = \frac{N^r_{oa}}{N_{oa}}
\Pi_3 = \frac{N^r_{oa}}{N_{oa} - N^r_{oa}}
\Pi_4 = \frac{N^r_{oa}}{N_{oa}}
\]

\[
\Pi_5 = \frac{N_{ta}}{N_{oa} - N_{oa}}
\Pi_6 = \frac{t_{oa}}{w_{ta}}
\Pi_7 = \frac{t_{oa}}{R_{oa}}
\Pi_8 = \frac{t_{oa}}{R_{ta}}
\]

\[
\Pi_9 = \alpha
\Pi_{10} = \frac{w_{ta}}{R_{oa}}
\Pi_{11} = \frac{w_{ta}}{R_{ta}}
\Pi_{12} = \frac{R_{oa}}{R_{ta}}
\]

\[
\Pi_{13} = \frac{t_{oa}w_{ta}}{R_{oa}^2}
\Pi_{14} = \frac{t_{oa}w_{ta}}{R_{ta}^2}
\Pi_{15} = \frac{t_{ic}}{R_{ic}}
\Pi_{16} = \frac{t_{ap}}{R_{ap}}
\]
5 Results

Eq. 4 depicts the EAM chosen from Eureqa® where just 6 of 16 proposed parameters in eq. (4) have been used. Tensile armors and Pressure armors seem to play an important role as well as the ratios between total and removed wires.

$$\text{SCF}_{\text{max}} = c_1 + \frac{w_{ia}}{R_{ia}} + \frac{t_{oa}}{R_{oa}} \left\{ \begin{array}{l} N_{oa} \bigg| \frac{t_{oa}}{N_{oa}} \bigg| + c_2 + c_3 \left( \frac{t_{oa}}{R_{oa}} \right)^2 + \frac{t_{oa}}{R_{oa}} \right\} - c_4 \left( \frac{N_{oa}}{N_{oa} - N_{oa}^r} \right) \sin(2\alpha). \right.$$  

\begin{align*}
c_1 &= 0.836137243594079 \\
c_2 &= 0.568256582998187 \\
c_3 &= 29.5601109901204 \\
c_4 &= 2536.18159456284 \\
c_5 &= 104.982049284956
\end{align*}

Figure 3 shows both FEM and EAM predictions to SCF$_{\text{max}}$ found at each wire removal to each pipe. Figure 3(a) shows the whole dataset. Each ascending line is related to one pipe. Figure 3(b) gives a closer view of what happens to the pipe marked with an arrow in Fig. 3(a). From Fig. 3 it is noticed that the maximum SCF increases as the number of removed wires increases, $N_{oa}^r$.

The correlation coefficient between FEM and EAM data found is 0.99659. The minimum absolute error found is -10.7% while the mean absolute error is -0.0678%.

Table 2 retrieved partially from Eureqa® gives the relative impact that one parameter has on the target value $f$, i.e., the SCF$_{\text{max}}$ based on several metrics.

Sensitivity is defined as $\frac{\partial f}{\partial \Pi_i} \cdot \sigma(\Pi_i) / \sigma(f)$, where $\frac{\partial f}{\partial \Pi_i}$ is the mean of the partial derivative of $f$ with respect to $\Pi_i$; $\sigma(\Pi_i)$, $\sigma(f)$ are the standard deviation of a given i-th parameter and standard deviation of $f$ respectively.

Positive Magnitude is a measure of how big an increase in a given parameter leads to an increase in the target value. Defined as $\frac{\partial f}{\partial \Pi_i} \cdot \sigma(\Pi_i) / \sigma(f)$ at all points where $\frac{\partial f}{\partial \Pi_i} > 0$.

Table 2 shows that the number of removed wire as well its several percentages play a significant role in SCF$_{\text{max}}$ value. This agrees with the results found in De Sousa et al. [2]. The underlying intact metallic layers, ITA and PA, seems to have almost the same impact on the SCF$_{\text{max}}$ value, but in distinct fashions. An increase in $\Pi_8$ leads to an increase in SCF$_{\text{max}}$ while an increase in $\Pi_{16}$ leads to a decrease in SCF$_{\text{max}}$. Finally, OTA itself
contribution is the least significant to $\text{SCF}_{\text{max}}$ value.

Table 2 – Sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Pi_1$</th>
<th>$\Pi_2$</th>
<th>$\Pi_3$</th>
<th>$\Pi_5$</th>
<th>$\Pi_8$</th>
<th>$\Pi_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity</td>
<td>0.4156</td>
<td>0.4084</td>
<td>0.1817</td>
<td>0.1568</td>
<td>0.1162</td>
<td>0.0327</td>
</tr>
<tr>
<td>Positive Magnitude</td>
<td>0.4156</td>
<td>0.4084</td>
<td>0.1817</td>
<td>0.1568</td>
<td>0</td>
<td>0.0327</td>
</tr>
<tr>
<td>Negative Magnitude</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1162</td>
<td>0</td>
</tr>
</tbody>
</table>

6 Conclusions

A compact and accurate analytical expression for maximum stress concentration factor has been found when a flexible pipe has its outer tensile armor wires broken. The expression suggested that the number of damaged wire plays the most significant contribution to the maximum stress concentration factor value. Pressure armor and inner tensile wires also play a secondary role as long as the pipe is being damaged. The SR approach has been able to fulfill the goal of this study despite its high computer time.

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References