XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

A Elasto-Plastic Constitutive Model Based on Chaboche Kinematic Hardening of Aluminum Alloy 7050-T7451

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Abstract. The elasto-plastic behavior of the aluminum alloy 7050-T7451 subjected to cyclic loading was investigated and modeled. The objective of this work is to improve the constitutive model of Chaboche with a special emphasis in the compression. In the proposed model, the part under tension of the stress-strain curve in the stable hysteresis cycle is modeled separately from the compressed one, where it is considered that they have different Modulus of Elasticity. The work includes both simulation and experimental data, and they are compared to each other. The values were collected experimentally by the application of symmetric cyclic strain-driven loading in the strain range between 0.75% and 2.25%. The experimental results were used to calculate the parameters of the constitutive model in a MATLAB® software. The responses obtained with the proposed model have greater adherence to the experimental results than those obtained with the Chaboche model.

Keywords: Elasto-plastic Strain, Constitutive Model, Stable Hysteresis Cycle.

1 Introduction

The aluminum alloy 7050-T7451 contains several advantageous properties such as high strength, good toughness, low density and good corrosion resistance and this makes it widely used in the aerospace industries and in various components of modern aircraft (Hao *et al*. [1]; Xu *et al*. [2]). These and many other applications cause their structural components to be frequently subjected to complex cyclical loads (Zhu *et al*. [3]).

During the last decades, cyclic constitutive models have been widely developed for metallic materials and are being used successfully in academic researches and industrial applications (Zhou *et al*. [4]). In cyclical loading, the cyclic hardening or softening of materials plays a very important role in the safety assessments and life prediction related to the fatigue of these structural components and this topic has attracted a lot of attention (Chaboche *et al*. [5]). It is important to understand the variables responsible for the cyclic mechanical response of the material. Be able to accurately describe this behavior have great importance in life prediction and project optimization.

Thus, the objective of this study is to modeling the elasto-plastic behavior of the aluminum alloy 7050-T7451 through an adaptation of the non-linear constitutive model of Chaboche kinematic hardening. The proposed model is compared with the data collected experimentally and this error is compared with the error between the Chaboche model and the experimental data. This study is done using the experimental data together with simulation in MATLAB®.

2 Chaboche constitutive theories

Chaboche nonlinear model is based on research by Frederick and Armstrong [6], which consists of a superposition of several non-linear models defined by eq. (1):

$$d\alpha_{ij} = \frac{2}{3} C d\varepsilon_{ij}^{p} - \gamma \alpha_{ij} d\bar{\varepsilon}_{p} \tag{1}$$

Where C and γ are parameters of the material, $d\varepsilon_{ij}^{p}$ is the plastic strain increment, $d\bar{\varepsilon}_p$ is the equivalent plastic strain increment, while α_{ij} is the total translation of the center of the initial yield surface, known as back stress (Fig. 1).

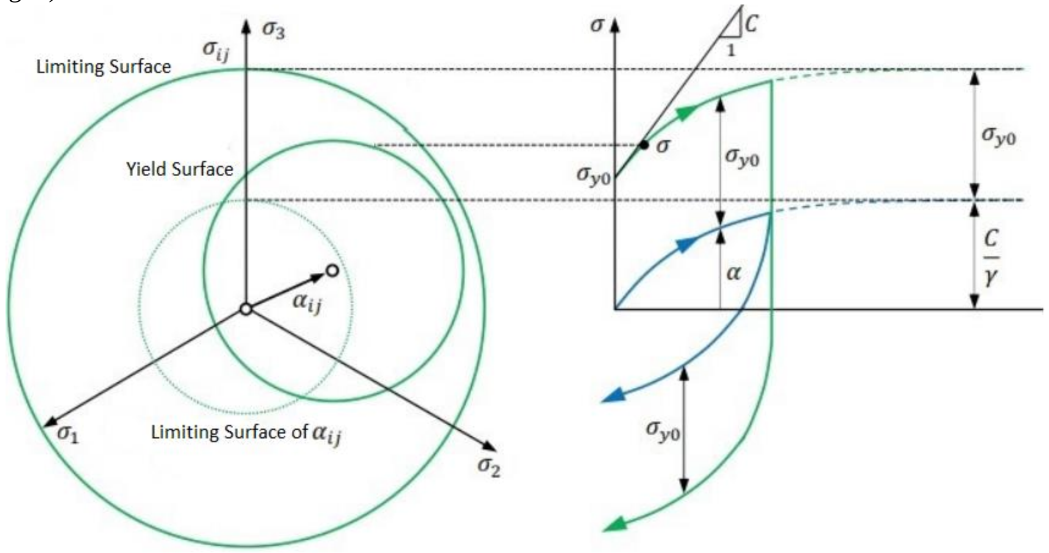

Figure 1. Graphical representation of Frederick-Armstrong's nonlinear kinematic hardening model
Adapted from Souza Neto *et al*. [7]

Where σ_{y0} is the yield stress. The Chaboche model is used for this paper and its back stress is given by:

$$\alpha_{ij} = \sum_{i=1}^{n} d\alpha_{ij}^{(i)} \tag{2}$$

$$d\alpha_{ij}^{(i)} = \frac{2}{3} C_i d\varepsilon_{ij}^{p} - \gamma_i \alpha_{ij}^{(i)} d\bar{\varepsilon}_p \tag{3}$$

Where n is the number of superposition models. C_i and γ_i are parameters of the material. For a stable hysteresis cycle of a strain-driven loading, the solution of the back stress in the uniaxial loading is given by:

$$\alpha_i = \frac{C_i}{\gamma_i}\left(1 - 2e^{\left[-\gamma_i\left(\varepsilon_x^p - (\varepsilon_L^p)\right)\right]}\right) \tag{4}$$

Where ε_x^p is the plastic strain when applied the loading and ε_L^p is the limit of plastic strain of the hysteresis loop. The graphical representation of the superposition of 3 independent back-stresses is given in the Fig. 2.

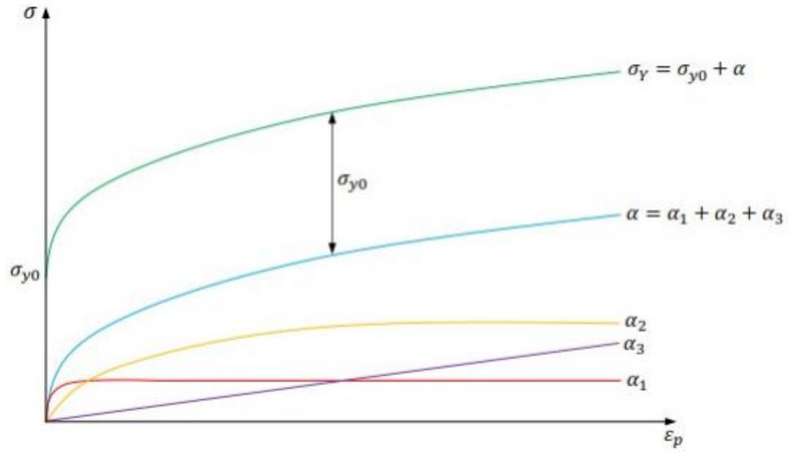

Figure 2. Representation of Chaboche nonlinear kinematic hardening model
Adapted from Hashiguchi [8]

3 Materials and methods

3.1 Experimental details

The experimental tests were performed on an MTS® LANDMARK servo-hydraulic machine model 370.10 with 100 kN of maximum load capacity and served by an MTS® FlexTestGT controller. Strain was measured using an MTS® extensometer model 632.26F-20.

Three specimens were used for each maximum strain value in the range. Figure 3 contains the dimensions of specimen.

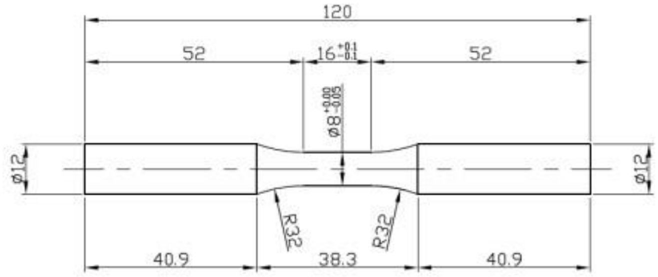

Figure 3. Dimensions of specimen (mm)

After the preparation of the specimens, they were subjected to strain-driven cyclic mechanical tests, following the information in Tab. 1. As recommended by MIL-HDBK-5H (Handbook [9]), the strain control was performed by means of triangular wave in the frequencies between 0.111 Hz and 0.333 Hz, in order to maintain constant the strain rate at 0.3 1/min.

Table 1. Strain amplitudes and frequency for each specimen

Specimen	R_ε (-)	Strain (%)	$\Delta\varepsilon$ (%)	Frequency (Hz)
C1, C2 and C3	-1	-0.75 to 0.75	1.5	0.333
C13, C14 and C15	-1	-1.25 to 1.25	2.5	0.200
C31, C32 and C33	-1	-1.75 to 1.75	3.5	0.143
C49, C50 and C51	-1	-2.25 to 2.25	4.5	0.111

Where R_ε is the ratio between the minimum and maximum strain and $\Delta\varepsilon$ is the strain variation. The number of specimens as well as the number of strain variations is based on the ASTM E739-10 standard (E739-10 [10]), which classifies the data as permissible for design.

3.2 Proposed constitutive model

First, some considerations were made:

• The material is considered isotropic: The aluminum alloy is an anisotropic material (Schubbe [11]), but if the specimens are made in the same direction as the lamination of the plate and the loads are applied in the same direction, this assumption can be made (Lima, Massaroppi Junior and Bose Filho [12]);

• There is a distinction between tensioned ($d\varepsilon_x^p \geq 0$) and compressed ($d\varepsilon_x^p < 0$) parts. These two parts will be modeled separately, considering the respective Modulus of Elasticity;

• The true stress and true strain are used, and these are based on the hypothesis of constant volume during deformation;

• As the distance between the extensometer knives is 8 mm, this is the initial size (l_0) considered;

• Although there are three specimens for each strain amplitude, only one result is used, as these are very similar;

• The analyzed results are taken from the stable hysteresis loops.

With that, the model can be defined. The Chaboche model for the stable hysteresis loop, with $n = 3$, can be defined as:

$$\sigma_x = \sigma_0 + \sum_{i=1}^{3} \alpha_i \ , d\varepsilon_x^p \geq 0 \tag{5}$$

$$\sigma_x = -\sigma_0 + \sum_{i=1}^{3} \alpha_i \ , d\varepsilon_x^p < 0 \tag{6}$$

$$\alpha_i = \frac{C_i}{\gamma_i}\left(1 - 2e^{\left[-\gamma_i\left(\varepsilon_x^p - \left(\varepsilon_L^p\right)\right)\right]}\right), d\varepsilon_x^p \geq 0 \text{ for } i\text{=1 and 2} \tag{7}$$

$$\alpha_i = -\frac{C_i}{\gamma_i}\left(1 - 2e^{\left[\gamma_i\left(\varepsilon_x^p - \left(\varepsilon_L^p\right)\right)\right]}\right), d\varepsilon_x^p < 0 \text{ for } i\text{=1 and 2} \tag{8}$$

$$\alpha_3 = C_3\varepsilon_x^p \ , d\varepsilon_x^p >= 0 \text{ for } \ i\text{=3} \tag{9}$$

$$\alpha_3 = C_3\varepsilon_x^p \ , d\varepsilon_x^p < 0 \text{ for } i\text{=3} \tag{10}$$

In Chaboche constitutive model, the Modulus of Elasticity of the tensioned part is the same as the compressed one and the modification proposed by this work is to separate these parts. Separating these parts, the calculation of the plastic strain is different for each one.

Chaboche only considers the Modulus of Elasticity (E) in the tension to calculate the entire plastic strain. In this research it was found that there is a difference between the Modulus of Elasticity in tension (E_t) and the Modulus of Elasticity in compression (E_c).

Dividing the hysteresis loop into two parts, the function ε_x^p is given by:

$$\varepsilon_{x\,t}^p = \varepsilon_{x\,t} - ln\left(1 + \frac{\sigma_{x\,t}}{E_t}\right) \tag{11}$$

$$\varepsilon_{x\,c}^p = \varepsilon_{x\,c} - ln\left(1 + \frac{\sigma_{x\,c}}{E_c}\right) \tag{12}$$

Where the sub-indexes c and t represent the compressed and tensioned parts respectively. With this, one can separate the Chaboche model into tension and compression, with $n = 3$, which are:

- Tension

$$\sigma_{x\,t} = \sigma_{0\,t} + \sum_{i=1}^{3} \alpha_{i\,t} \tag{13}$$

$$\alpha_{i\,t} = \frac{C_{i\,t}}{\gamma_{i\,t}}\left(1 - 2e^{\left[-\gamma_{i\,t}\left(\varepsilon_{x\,t}^p - \left(\varepsilon_{L\,t}^p\right)\right)\right]}\right) \text{ for } i\text{=1 and 2} \tag{14}$$

$$\alpha_{3\,t} = C_{3\,t}\varepsilon_{x\,t}^p \tag{15}$$

- Compression

$$\sigma_{x\,c} = \sigma_{0\,c} + \sum_{i=1}^{3} \alpha_{i\,c} \tag{16}$$

$$\alpha_{i\,c} = -\frac{C_{i\,c}}{\gamma_{i\,c}}\left(1 - 2e^{\left[\gamma_{i\,c}\left(\varepsilon_{x\,c}^p - \left(\varepsilon_{L\,c}^p\right)\right)\right]}\right) \text{ for } i\text{=1 and 2} \tag{17}$$

$$\alpha_{3\,c} = C_{3\,c}\varepsilon_{x\,c}^p \tag{18}$$

With the values obtained experimentally and using these equations, the parameters C_i and γ_i are calculated through an algorithm in the MATLAB® software.

4 Results

The data collected experimentally are force (F), but for the model it is necessary to find the related tensions (σ). The initial radius of the specimen cross section is r_0=4mm and the initial length l_0=8mm. For that and considering deformation at constant volume, the instant length of the specimen (l), the true stresses (σ) and true strains (ε) are given by the relationships:

$$S = \frac{F}{\pi r_0^2} \tag{19}$$

$$e = \frac{l}{l_0} \tag{20}$$

$$\sigma = \frac{F}{\pi r_0^2}\frac{l}{l_0} \tag{21}$$

$$\sigma = S(1+e) \tag{22}$$

$$\varepsilon = ln\,(1+e) \tag{23}$$

Where S, F and e are the engineering stress, applied force and engineering strain respectively.

With the parameters found, the true stress curves as a function of the true strain are presented. In the left column of the Fig. 4 to Fig. 7 is shown the comparison between the proposed model and the experimental results and the one on the right between Chaboche and the experimental.

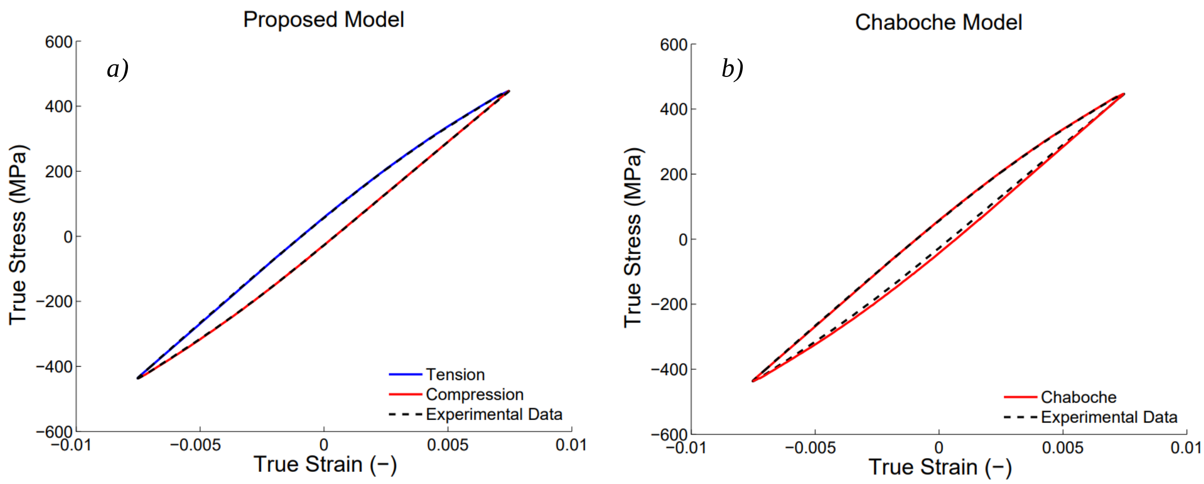

Figure 4. True stress-strain curve. Comparison between: *a)* proposed model and *b)* Chaboche model both with experimental results for $\Delta\varepsilon$ = 1.5% of the cyclic loading on Aluminum Alloy 7050-T7451.

Figure 4, Fig. 5, Fig. 6 and Fig. 7 shows that there is a difference in the parts under compression, in which it appears that the proposed model has more adherence to the experimental results than the Chaboche model. In order to know which model is more compatible with the experimental results, the error is calculated by means of the maximum difference between the stress found in the respective model and the experimental stress for the same strain in the entire cycle. The results are given in the Tab. 2:

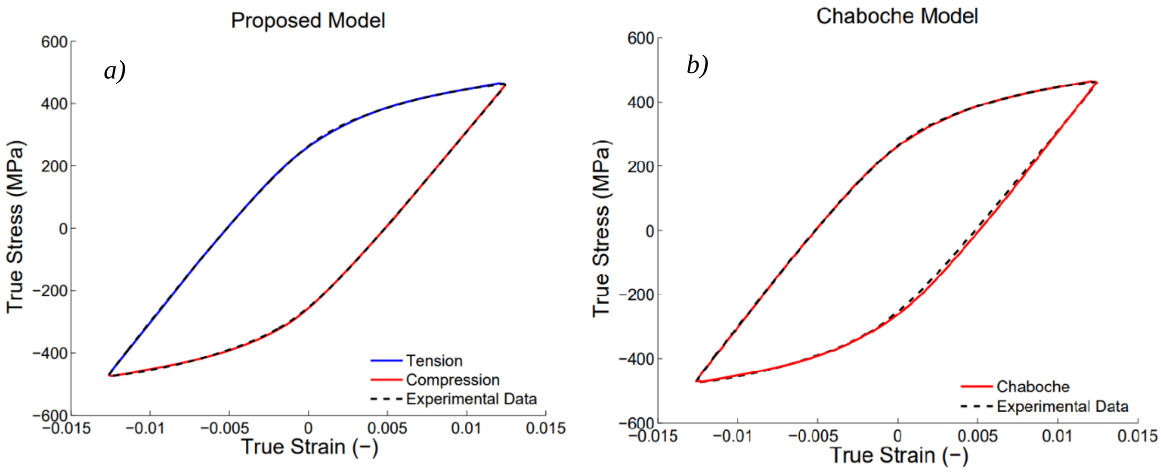

Figure 5. True stress-strain curve. Comparison between: *a)* proposed model and *b)* Chaboche model both with experimental results for $\Delta\varepsilon$ = 2.5% of the cyclic loading on Aluminum Alloy 7050-T7451.

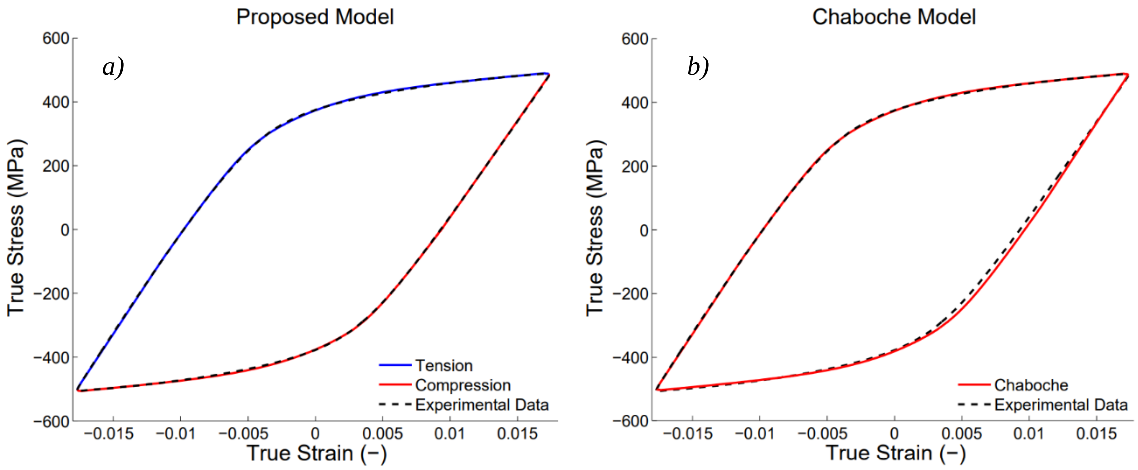

Figure 6. True stress-strain curve. Comparison between: *a)* proposed model and *b)* Chaboche model both with experimental results for $\Delta\varepsilon$ = 3.5% of the cyclic loading on Aluminum Alloy 7050-T7451.

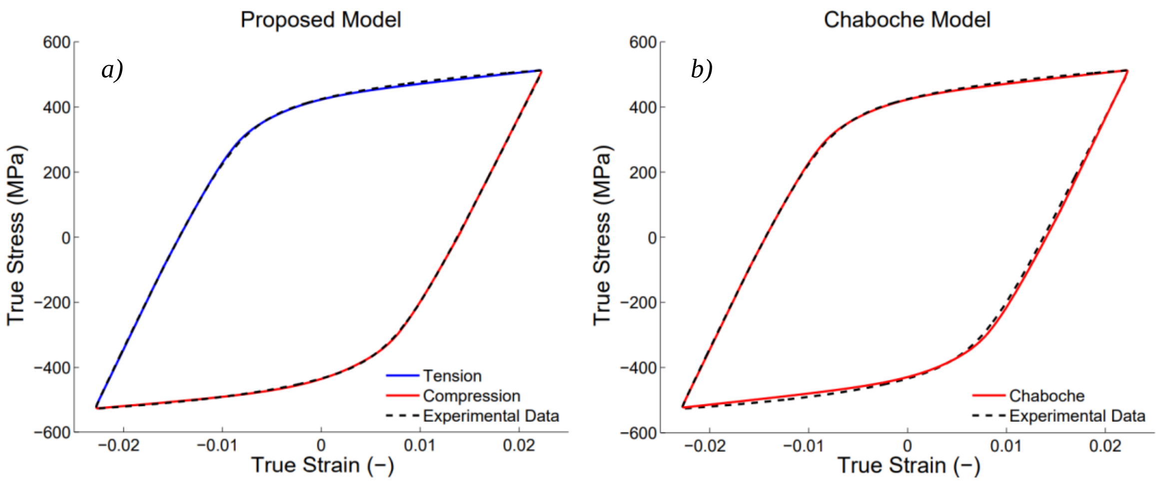

Figure 7. True stress-strain curve. Comparison between: *a)* proposed model and *b)* Chaboche model both with experimental results for $\Delta\varepsilon$ = 4.5% of the cyclic loading on Aluminum Alloy 7050-T7451.

Table 2. Maximum differences between the tensions found in the models and the experimental results.

CP	$\Delta\varepsilon$ (%)	$\Delta\sigma_{chab}$ (MPa)	$\Delta\sigma_t$ (MPa)	$\Delta\sigma_c$ (MPa)	E_t (GPa)	E_c (GPa)
C1	1.5	16.324	4.2340	4.1740	66.491	63.374
C13	2.5	15.2846	11.5778	13.5928	63.985	60.581
C31	3.5	20.3824	8.1301	5.4907	64.344	60.291
C49	4.5	15.8373	12.4194	11.8516	63.038	59.762

Where $\Delta\sigma_{chab}$ is the maximum difference of the experimental stress and the calculated by the Chaboche model in the entire stable hysteresis cycle. $\Delta\sigma_t$ is the difference in parts under tension, while $\Delta\sigma_c$ is the difference between the compressive stress of the proposed model and the experimental compressive stress. Note that the proposed model has the error always smaller than the Chaboche model.

5 Conclusions

Based on the results obtained with the proposed model, it can be conclude that:

• Separating the hysteresis loop that are under tension and under compression improves the prediction of the material cyclic behavior;

• The ratio between $\Delta\sigma_{chab}$ to $\Delta\sigma_c$ or $\Delta\sigma_t$ tensions varied by almost 400% in some cases;

The proposed model proved to be a valid tool, since the results found are closer to the collected data if compared to the Chaboche model.

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