Modeling of conduction heat transfer between rice grains stored in a grain silo prototype

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Abstract. Grain storage is a vital operation to maintain the quality of the grains, for this you need to have control over the stored temperature and humidity. Therefore, the objective of this work is through mathematical modeling to simulate the internal temperature of the rice grains inside the silo. For the resolution of the model, the transfer of heat by conduction between grains was taken into account, taking into account the type of grain and the dimensions of the silo (height and radius), using classic methods of resolution, such as separation of variables, Laplace transform and GILTT. To validate the models, experimental data generated are used for the case study. A solution using only the dimension of the height of the silo T(z,t) showed a good correlation with the experimental data. While the model, with the temperature depending on the radius of the silo, T (r, t), had results that were more distant from the experimental data and the model T(r,z,t) had an accuracy higher than the one-dimensional ones in the first moments of time. Therefore, the results presented show that the proposed methods are great alternative tools to predict the temperature inside the silos.

Keywords: Separation of variables, Laplace transform, GILTT, Rice grain, Prototype silo

1 Introduction

Brazil is one of the largest producers and consumers of rice in the world. However, a large part of the rice produced is lost in each of the processing steps, among them, it was estimated by [1] that 10% of all the grain produced in Brazil annually is lost during its storage, indicating a huge deficit of good practices in this production process. Grain losses in storage can be quantitative due to weight or volume reduction due to breathing, or qualitative, which is characterized by changes in product quality due to loss of nutrients [2].

In the storage process, the objective is to maintain the same quality of the grain at the moment it was harvested, with temperature and humidity being its most important characteristics. In the case of rice, its ideal humidity is close to 13% and at low temperatures, thus reducing the chance of proliferation of insects and microorganisms [3]. According to [4], grains stored with humidity above 14.5% may result in the development of mold if the temperature of the grains exceeds the range of 22-24°C over a long period.

Because of this, to avoid grain losses, constant monitoring of these parameters is necessary. Monitoring the temperature in the grain mass is called thermometry [5]. According to [6], thermometry is the process of obtaining temperature values of a grain mass stored in silos or bulk warehouses, equipped with devices based on thermoelectric pairs. Among the various techniques to decrease the temperature gradient in the grain mass and minimize the migration of moisture, there is aeration [7].

Aeration consists of the forced movement of air through the grain mass under ambient or controlled conditions, with the objective of decreasing and standardizing the temperature and humidity of the stored grains, avoiding their loss of quality [8].

For better use of aeration, it is necessary to understand the phenomenon that characterizes the heat exchange between two substances between themselves, or between the system and the external environment due to their temperature difference, also called heat transfer or heat flow. This process occurs when a body or medium with a higher temperature transfers its heat to the lower temperature, which can be through conduction, convection or radiation [9].

With this, there are several studies on mathematical models to approach the storage characteristics, based on the thermophysical properties of the grains, to predict their conditions, be it temperature and/or humidity [10].

Thus, the main objective of this work is the use of a mathematical model, considering only the heat transfer for conduction between the grains, to predict the temperature distribution in a silo with stored rice grains.

2 Model and Experiment

The heat transfer occurs in the grain mass mainly by conduction, convection, and radiation. During conduction, heat is transferred at the points of contact between grains and of the grain to the silo wall. While through convection it is transferred from one session to another of the mass due to the flow of intergranular air created by the temperature difference in different points of the grain mass. Radiation is caused by an external heat source such as sinks or exposure to the sun [11].

The methods used for the resolution of the models are presented in [12] [13]. On models, it is assumed that the material is isotropic and its thermal conductivity is constant, and the problem of moisture migration in stored grains must be taken into account. Thus, the equation that represents the heat transfer in a silo is:

$$\left(\rho_m C_m\right)\frac{\partial T}{\partial t} + \left(\rho_a C_a\right)\left[V_r\frac{\partial T}{\partial r} + V_z\frac{\partial T}{\partial z} + V_\theta\frac{\partial T}{\partial \theta}\right] =$$

$$K_m\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + \rho_m H_{fg}\frac{\partial M}{\partial t} + q_h; \quad (1)$$

where, ρ_m - density of the grain mass, C_m - specific heat of the grain mass, T - internal grain temperature variation, t - time, ρ_a - air density, C_a - specific heat of the air, $\vec{V}$ - velocity vector, z - height of the grain mass, r - positioning of the grain mass around the length of the silo radius, θ - vertical angle, K_m - thermal conductivity of the grain mass, H_{fg} - evaporation heat, M - grain moisture and q_h - is the sum of heat flows caused by external or internal sources and sinks, such as solar radiation.

For the problem proposed in this work, the following simplifying hypotheses were used:

- The grains are considered to be compacted inside the silo, so there is no air passage between the grain masses, that is: $\left(\rho_a C_a\right)\left[V_r\frac{\partial T}{\partial r} + V_z\frac{\partial T}{\partial z} + V_\theta\frac{\partial T}{\partial \theta}\right] \approx 0$;
- Inside the silo there are no heat sources or sinks, the experimental silo is closed and no fans will be used: ($q_h \approx 0$);
- The change in humidity over time is close to zero and within the ideal storage values according [3], therefore: $\rho_m H_{fg}\frac{\partial M}{\partial t} \approx 0$.

Using the simplifying assumptions, heat transfer is considered only by conducting the grain to grain, so eq.(1) can be rewritten as:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}; \quad (2)$$

where α is the coefficient of thermal diffusivity of the grain. Separating the one-dimensional and two-dimensional models, we have:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t}; \quad (3)$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}; \quad (4)$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}. \quad (5)$$

In order to solve the one dimensional models, $T(z,t)$ and $T(r,t)$ and the two dimensional model $T(r,z,t)$ the following boundary and initial conditions in z and r were used:

- Condition in z

$$\left.\frac{\partial T}{\partial z}\right|_{z=0, z=H_s} = 0; \tag{6}$$

$$T(z,0) = F(z); \tag{7}$$

- Condition in r

$$T(r \to 0, t) \leq M, \quad \text{where} \quad M \in R; \tag{8}$$

$$T(R_s, 0) = T_{amb}; \tag{9}$$

$$T(r,0) = F(r); \tag{10}$$

where H_s is the maximum height of the silo, R_s is the radius of the silo and $F(z)$ and $F(r)$ is the profile function of the initial temperature in z and r. For $T(r,z,t)$ is used both boundary condictions, and for the initial condiction, is used the same of $T(z,t)$. This initial condition was chosen for the two-dimensional as the first test, due to the large variation in temperature along the grain column at the initial instant, compared with the variation in the radial direction.

The initial temperature profile function $F(z)$ and $F(r)$ was defined by heating the grains for a period of time of 30 minutes before the experiment, to have different temperatures in each layer. Table 1 contains the initial temperature values for each of the sensors used to perform the procedure.

$$F(z) = C_0 e^{C_1 z + C_2} + C_3. \tag{11}$$

Table 1. The initial temperature on each sensor and cable

Identification	Temperature (ºC)		
	Sensor 1	Sensor 2	Sensor 3
Cable 1	31.3	24.2	22.8
Cable 2	30.3	24.3	23.3
Cable 3	31.1	23.7	23.8

For the initial temperature in $F(r)$, the temperature was used after heating the beans in this first study, due to the small variations that we have in the sensors of each layer, we chose to consider $F(r)$ constant. To obtain the coefficients of this profile function, $F(z)$, represented by eq.(11), the temperatures obtained by the sensors after heating were used and an exponential adjustment of the curve, was carried out using the Least Squares Method (MMQ), represented in Table 2.

Table 2. $F(z)$ function constants on each cable.

Identification	C_0	C_1	C_2	C_3
Cable 1	9.31188	-7.23951	0	22.6384
Cable 2	30	-6.16646	-1,40319	23.0666
Cable 3	10	-15.2809	-0.161737	23.7988

In addition, for the experiment, styrofoam plates were installed at the base and top of the experimental silo, to isolate it (conditions of zero heat flow at these points). To solve the mathematical models proposed, analytically, the methods of Variable Separation (SV), Laplace Transform (TL) [14], and Generalized Integral Laplace Transform Technique (GILTT)[15] were used.

The solutions obtained using the SV method are presented below. The SV method consists to transform the partial differential equation in a set of ordinary differential equations, which must be solved using the initial or boundary conditions. The accounts and solutions using the TL and GILTT methods can be found at M.V. Henriques[1], G.J. Weymar[2], D. Buske[1,2][16]. The results of SV method were:

- One dimensional model $T(z,t)$

$$T(z,t) = \frac{1}{H_s}\left(\frac{C_0 e^{C_2}(e^{C_1 H_s} - 1)}{C_1} + C_3 H_s\right)\cos\left(\frac{n\pi}{H_s}z\right)e^{-\left(\frac{n\pi}{H_s}\right)^2 \alpha t} + \tag{12}$$

$$\sum_{n=1}^{\infty}\frac{2}{H_s}\left(\frac{C_0 C_1 e^{C_2} H_s^2[(-1)^n e^{C_1 H_s} - 1]}{C_1^2 H_s^2 + n^2\pi^2}\right)\cos\left(\frac{n\pi}{H_s}z\right)e^{-\left(\frac{n\pi}{H_s}\right)^2 \alpha t}.$$

- One dimensional model $T(r,t)$

$$T(r,t) = T_{amb} + \sum_{n=1}^{\infty}\frac{2(F(r) - T_{amb})J_1(R_{0,n})}{R_s\beta_n[J_0(R_{0,n})^2 + J_1(R_{0,n})^2]}J_{0,n}\left(r\frac{R_{0,n}}{R_s}\right)e^{-\left(\frac{R_{0,n}}{R_s}\right)^2 \alpha t}. \tag{13}$$

- Two dimensional model $T(r,z,t)$

$$T(r,z,t) = \sum_{m=1}^{\infty}F_{0,m}J_{0,m}\big(r\frac{R_{0,m}}{R_s}\big)\cos\left(\frac{n\pi}{H_s}z\right)e^{\left(-\frac{R_{0,m}}{R_s}^2 - \frac{n\pi}{H_s}^2\right)\alpha t} +$$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}F_{n,m}J_{0,m}\big(r\frac{R_{0,m}}{R_s}\big)\cos\left(\frac{n\pi}{H_s}z\right)e^{\left(-\frac{R_{0,m}}{R_s}^2 - \frac{n\pi}{H_s}^2\right)\alpha t} + T_{amb}. \tag{14}$$

3 Results and discussion

In this work, temperatures were measured experimentally inside a silo prototype containing rice, to compare the results of three analytical solutions, two one-dimensional (in r and z) and one two-dimensional (in r and z). The prototype is shaped like a cylinder with a height of $0.7m$, a radius of $0.5m$, vertical plates of styrofoam at the base and time of the silo and an initial temperature distribution produced by heating the rice that was at room temperature of 22.5ºC by a jet of hot air at the base of the silo. Nine temperature sensors were used, which were placed in tree cables (triplicate) all with the same radial distance of $0.28m$ from the cylinder axis. Each cable had three sensors positioned at the heights: $0.01m$, $0.23m$ and 0.56 m along the cable. The prototype was filled with rice husks with a thermal diffusivity of $3.27 \times 10^{-6} m^2 s^{-1}$[17], and the temperature were analyzed every 30 minutes over a total period of 24 hours (1440 minutes).

Among all the models tested, the SV method was the one with the best precision, this is because the SV method is an entirely analytical procedure where no approximation is carried out beyond of the series truncation, while by the TL and GILTT method, in addition to the truncation, there is still the numerical inversion in the Laplace Inverse Transform, using the Gaussian square method.

Because the results of the triplicate (Cable 1, 2 and 3) are very close to each other, for simplicity it was chosen to demonstrate the results of Cable 1 only.

Several simulations were made to analyze the convergence of the results of the eq.(12), (13) and (14). The truncation of the solutions was estimated using cable 1 and sensor 1 as a reference, represented in Figure 1.

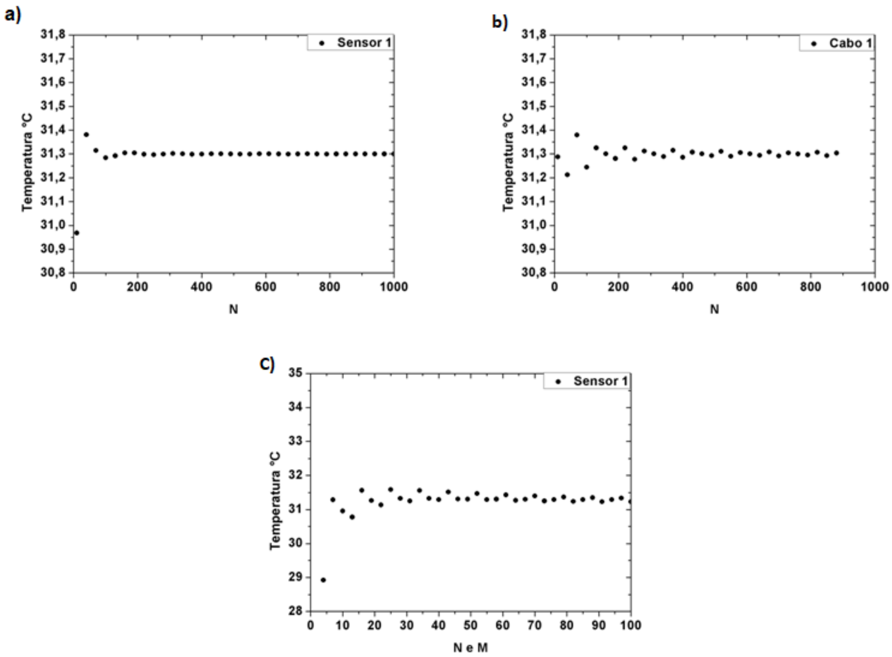

Figure 1. Truncation of the solution by the SV method on Cable 1, sensor 1 on model a) $T(z,t)$ b) $T(r,t)$ c) $T(r,z,t)$

Although the convergence occurs at $N = 400$ for $T(z,t)$, $N = 500$ in $T(z,t)$ and N e $M = 40$ in $T(r,z,t)$ for a better approximation of the data $N = 1000$, $N = 900$ and N e $M = 100$ respectively were used.

To facilitate the visualization of the distribution of the internal temperature over time in each of the solutions of the one dimensional and two dimensional models, the graph is shown in Figure 2.

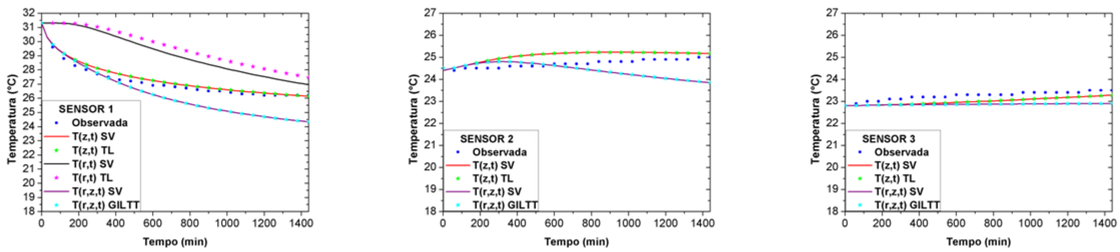

Figure 2. Comparison between observed and predicted temperatures in Cable 1 for each sensor, model and method

Analyzing the observed and the predicted temperature, it is observed that the temperature profile obtained with the SV and TL method follow the physics of the problem (temperature distribution along the grain column), in model $T(z,t)$, because the variations between the predicted (model) and observed temperatures are small (less than 1°C).

When comparing the results of the $T(z,t)$ model with the one dimensional heat transfer model around the radius of the silo, $T(r,t)$, a large difference was observed between the observed and predicted temperatures, in which there was a high frequency of errors above 2°C, to the one-dimensional model $T(r,t)$, while the other was less than 1°C. Thus, it is concluded that there is a greater portion of vertical heat transfer. Model $T(r,t)$ was used only sensors 1 because that sensor is closest to the base of the silo was used for less interference from radial heat transfer.

While the two-dimensional model showed excellent results in the first moments, having temperatures closer to the simulated ones, however, over time the two-dimensional model worsens its prediction. Comparing with the one dimensional models, its result was inferior to the $T(z,t)$ model. This is because the $T(z,t)$ model considers the transfer only at the height of the grain and for the real problem we have a great influence of the convection transfer in z.

Then, to verify the agreement of T_p (predicted temperature) with the T_o (observed temperature), a statistical analysis is made on the model that presented the best result $T(z,t)$ [18], in order to evaluate its performance. The statistical indexes used are:

- **Normalized mean square error (NMSE):** it is a dimensionless measure that informs about the temperature deviations observed in the experiment with those obtained by the methods, where the lower the value, the better the model.
- **Correlation coefficient (COR):** it is a dimensionless measure that describes the degree of association or agreement between the variables, the closer to 1 the better their performance.
- **Fractional bias (FB):** it is a dimensionless measure in which it indicates the tendency of the data to overestimate or underestimate when compared to the real ones, with its optimal value equal to zero.
- **Standard fractional deviation (FS):** is a dimensionless measure in which it indicates how much the model can simulate the dispersion of the observed data, with its ideal value equal to zero.

In Table 3, the statistical indices NMSE, FB, FS and COR are required for model $T(z,t)$ in each sensor for cable 1.

Table 3. Statistical analysis of the one-dimensional model $T(z,t)$ on each sensor in cable 1.

Statistical indexes			
indexes	Method SV cable 1		
	Sensor 1	Sensor 2	Sensor 3
NMSE	0,0001	0,0002	-0,0001
COR	0,9739	0,7610	0,9023
FB	-0,0087	-0,0145	0,0108
FS	-0,0460	0,2840	0,1525

Through the results of the statistical indices, it is observed that NMSE, FB, and FS are very close to zero (ideal value). Thus, it is concluded that the deviations between the observed and predicted temperatures are almost zero and that the model do not tend to underestimate or overestimate the values found.

Regarding the COR index, the results are close to 1 (ideal value), which shows a high degree of agreement between the variables (T_o and T_p), indicating good results.

To illustrate the temperature variation over time, the internal temperature of the grain mass is calculated, varying the height of the grain mass from $z = 0.01m$ to $0.7m$ for model $T(z,t)$ and the radius of the silo $r = 0.01m$ to $0.5m$ for model $T(r,t)$. Figure 3 showing this variation at certain points in time ($t = 0$ to $20h$).

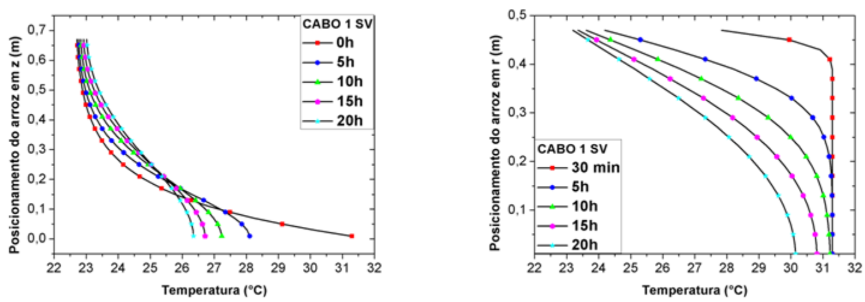

Figure 3. Evolution of temperature over time (from 0 to 20h), temperature distribution along the grain column and the radius of the silo in Cable 1 by the SV method

It is observed that the model $T(z,t)$ is coherent with the physical phenomenon, that is, the temperature of the grains inside the silo tends to come into balance since the contours are isolated (boundary conditions imposed in the problem).

The $T(r, t)$ model shows that in the center, the temperature changed approximately 1.5°C from the initial moment until the end (20h). This is because as you approach the walls of the silo, the temperatures are balanced with the environment over time and the line with red dots has high temperatures for all positions of the beam because in 30 min the heating had just been turned off, before the beginning of the experiment.

4 Conclusions

Among all the applied methodologies that showed the best results was the SV method. The one-dimensional solution $T(z, t)$ discussed in this work showed a good correlation with the experimental data, since the statistical indices are very close to the ideal values, with a maximum error of 0.57°C in cable 1 between all sensors. When comparing with the model, $T(r, t)$, there was a big difference between the observed and predicted temperatures, in which there was a high frequency of errors above 1°C, reaching close to 3°C difference in a given case. Thus, it is concluded that there is a greater portion of vertical heat transfer.

Then, in order to analyze the influence of the spatial temperature distribution inside the silo, the solution of the two-dimensional model,$T(r, z, t)$, was developed, where in the first moments (first 10 hours) it presented temperatures close to those observed, showing to be as effective or better than the $T(z, t)$ model during this period. However, over time, its accuracy tends to decrease.

As in the experiment, the grain volume was heated for 30 min, with the air inlet at the base of the silo, the temperatures in sensors 1 (located 1 cm from the base) on all cables are higher from the base to the top and, over time, from the center to the silo wall, where we have room temperature in the boundary conditions. After a few hours, as the external temperature is lower than the internal one and the hot air is lighter and tends to rise, it is believed that considering the natural convection the precision of the solutions found will be better.

Therefore, the implemented methods proved to be effective as a low-cost tool where it demonstrated good accuracy without needing expensive equipment to simulate.

In this first study, we tried to explain the problem considering the boundary and initial conditions proposed in this work. In future works, other boundary conditions will be addressed, and the experiment will be replicated trying to have different positions.

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