Nonlinear Thermal Analysis of Steel Sections with Boundary Element Method

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Abstract. The fire safety is a topic that has been under discussion lately, mainly due to the last fires that have befallen humanity and to which countless losses are linked. In this context, the role of fire resistance of structural elements is essential for the security of buildings. Therefore, a non-linear thermal analysis of steel sections is proposed with the Boundary Element Method. The partial differential equation of non-linear heat conduction is linearized using the Kirchhof Transformation; the non-homogeneous term of the mentioned equation is approximated by a series of radial basis functions variable, in order to control the conditioning of the interpolation matrix and using the Dual Reciprocity Method. In addition, boundary conditions for heat exchange by convection and radiation are introduced in the model, in order to simulate heat exchange when the section is subjected to standard fire. Finally, in order to validate and compare, the temperature field obtained for two different steel sections is compared with the results of the simplified method presented in NBR 14323; obtaining a good identity of the solutions. The results reveal the validation of the constructed numerical model and the precision of the simplified method for thermal analysis of sections of steel.

Keywords: Boundary Element Method, Dual Reciprocity Method, Thermal analysis, Steel Sections.

1 Introduction

The fire safety is a topic whose discussion has surfaced lately, mainly due to the last fires that have befallen humanity and to which countless losses are linked; as a recent example, we mention the fire of Notre-Dame Cathedral, which occurred in April 2019 and which resulted in considerable wear and tear on that historic structure. In this regard, the British government released, on September 12, 2019, a report, which shows that, from April 2018 to March 2019, 73214 primary fires were attended by the Fire and Rescue Services (FRSs) - that is, fires whose effects are potentially more serious and that cause damage to people or damage to property - of which 44575 occurred in homes or other buildings, which represents more than 60% of primary fires (GOV.UK [1]).

In this context, the role of fire resistance of structural elements is essential to the security of buildings. To determine this resistance, it is common, due to the complexity and non-linearity of the behavior of structures in fire situations, to adopt approximate solutions in Finite Element Method (FEM); examples of these applications are found in Drury et al. [2], Kucukler [3], Vitorino et al. [4] and Zhang et al. [5]. For the solution of this problem, another numerical alternative is the Boundary Element Method (BEM), which, in relation to the FEM, has the advantage of discretization only in the contour of the domain, resulting in a smaller amount of input data (coordinates of points and mesh), matrices with smaller dimensions and shorter processing times.

Therefore, in this article, a non-linear thermal analysis of steel sections with BEM is proposed, as an alternative to FEM. There is naturally literature on BEM with applications in problems in various fields, more recently Bin et al. [6] and Zhou et al. [7]. BEM is an efficient and precise numerical technical for solving partial differential equations (PDE). In this method, the PDE is converted into an integral equation of equivalent boundary; this being its main advantage over other classic methods, such as FEM and the Finite Differences

Method; resulting in the need to discretize only the boundary and a high convergence rate. In order to deal with the non-homogeneous term of PDE, keeping discretization only in the boundary and without the need for domain integrals, a tool is the Dual Reciprocity Method (DRM); in this approach, such non-homogeneous term is approximated by a series of simple functions and transformed into a boundary equation (PARTRIDGE et al. [8]).

When dealing with the non-linearity of the heat conduction equation, the Kirchhof transformation is used to linearize it (WROBEL; BREBIA [9]). Once linearized, the heat equation is solved using BEM. The methodology is then applied to the section of steel elements; the results obtained here are compared with the results from the methodology presented in NBR 14323.

2 Formulation

2.1 Initial considerations of the problem

Consider a transient temperature field $u(\boldsymbol{x},t)$ in an isotropic medium of two-dimensional domain Ω with boundary $\Gamma = \Gamma_u \cup \Gamma_q \cup \Gamma_h$, without internal heat source and whose representation is shown in Fig. 1. For this case, the equation that governs heat conduction is

$$\frac{\partial}{\partial x}\left(K\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial u}{\partial y}\right)=\rho c\frac{\partial u}{\partial t} \tag{1}$$

with the initial condition

$$u(\boldsymbol{x},0)=u_0(\boldsymbol{x}) \tag{2}$$

and with boundary conditions

$$u(\boldsymbol{x},t)=\bar{u}(\boldsymbol{x},t), \qquad \boldsymbol{x}\in\Gamma_u \tag{2}$$

$$q(\boldsymbol{x},t)=\bar{q}(\boldsymbol{x},t), \qquad \boldsymbol{x}\in\Gamma_q \tag{3}$$

$$q(\boldsymbol{x},t)=h(u-u_\infty)+\sigma\epsilon(u^4-u_\infty^4), \qquad \boldsymbol{x}\in\Gamma_h \tag{4}$$

where $\boldsymbol{x}$ is the position vector; the parameter t denotes time; K is the thermal conductivity of the material; c is called specific heat; ρ is the specific mass of the material; $q(\boldsymbol{x},t)$ is the heat flow defined as $q(x,t)=-K\partial u/\partial n$ where n is the vector normal to the boundary Γ outside the domain Ω; $u_0(\boldsymbol{x})$ represents the initial temperature distribution; $\bar{u}(\boldsymbol{x},t)$ and $\bar{q}(\boldsymbol{x},t)$ are the prescribed temperature and flow respectively. The h parameter is the convection heat transfer coefficient; σ is the Stefan-Boltzmann constant and ϵ is the radiative factor between the surface and the outside at a temperature u_∞.

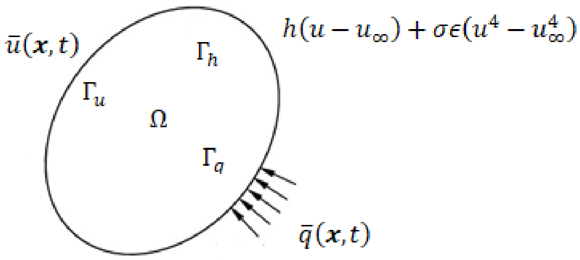


Figure 1. Domain configuration.

Assuming that K, ρ and c are all temperature dependent, eq. (1) is notoriously non-linear, because it takes the form

$$K\nabla^2 u-\rho c\frac{\partial u}{\partial t}=-\frac{dK}{du}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{5}$$

where non-linear terms appear explicitly on the right side. In addition, another non-linearity comes from the radiation boundary condition of eq. (5). The purpose of this work is to solve eq. (1) with the said non-linearities and with application to the problem mentioned in the introduction; to this end, the following considerations are preliminarily made.

2.2 Boundary Element Method

For didactic purposes, initially consider the Laplace equation

$$\nabla^2 u = 0 \tag{6}$$

in a two-dimensional domain similar to Fig. 1, but only with essential boundary conditions (Dirichlet boundary condition) and natural - Neumann boundary condition -, that is, $\Gamma = \Gamma_u \cup \Gamma_q$. Therefore, the convective and radioactive boundary conditions are excluded (eq. (5)).

According to Brebbia and Dominguez [10], the integral equation on which BEM is based is

$$c_i u_i + \int_\Gamma u q^* d\Gamma = \int_\Gamma q u^* d\Gamma \tag{8}$$

where u^* is the weighting function; q^* its derivative with respect to the vector normal to the boundary, that is, $q^* = \partial u^*/\partial n$ and c_i is a function of the internal angle at point i of the boundary. Discretizing the boundary into N elements, the above equation can be rewritten as follows

$$c_i u_i + \sum_{j=1}^{N} \int_{\Gamma_j} u q^* d\Gamma = \sum_{j=1}^{N} \int_{\Gamma_j} q u^* d\Gamma \tag{9}$$

Alternatively, in matrix form, we have

$$\boldsymbol{Hu} = \boldsymbol{Gq} \tag{10}$$

It should be noted that eq. (10) is written without the introduction of boundary conditions; in doing so, a system of equation with N unknows is obtained.

2.3 Dual Reciprocity Method for the linear case

In order to deal with the non-homogeneous term of PDE within the scope of BEM, there is the Dual Reciprocity Method (DRM). Consider the subsecutive equation

$$\nabla^2 u = b(x, y, t, u) \tag{11}$$

in which b is any arbitrary function. The objective is to solve it without the need for internal domain discretization. The solution of eq. (11) is expressed as the sum of the homogeneous solution with a particular solution indicated by $\hat{u}$, that is,

$$\nabla^2 \hat{u} = b \tag{12}$$

A series of particular equations $\hat{u}_j$ is proposed instead of a single $\hat{u}$, such that b is approximated by

$$b \cong \sum_{j=1}^{N+L} \alpha_j f_j \tag{13}$$

where N and L are respectively the number of boundary points and internal to the domain; α_j is the set of coefficients initially unknow and f_j is the set of interpolation functions. Interpolations functions are required to relate to particular solutions $\hat{u}_j$ as follows

$$\nabla^2 \hat{u}_j = f_j \tag{14}$$

With such equations and applying the discretization procedure described in the previous sections, we have

$$\boldsymbol{Hu} - \boldsymbol{Gq} = \left(\boldsymbol{H\hat{U}} - \boldsymbol{G\hat{Q}}\right)\boldsymbol{F^{-1}b} \tag{15}$$

where $\boldsymbol{\hat{U}}$ and $\boldsymbol{\hat{Q}}$ are constituted by the columns $\boldsymbol{\hat{u}_j}$ and $\boldsymbol{\hat{q}_j} = \boldsymbol{\partial \hat{u}_j / \partial n}$ respectively. To solve linear transient thermal conduction problem, $b = \eta \dot{u}$ is made in eq. (15), so that

$$\boldsymbol{Hu} - \boldsymbol{Gq} = \eta\left(\boldsymbol{H\hat{U}} - \boldsymbol{G\hat{Q}}\right)\boldsymbol{F^{-1}\dot{u}} \tag{16}$$

Making $\boldsymbol{C} = -\eta\left(\boldsymbol{H\hat{U}} - \boldsymbol{G\hat{Q}}\right)\boldsymbol{F^{-1}}$, we have

$$\boldsymbol{C\dot{u}} + \boldsymbol{Hu} = \boldsymbol{Gq} \tag{17}$$

Such equation is similar to that obtained by the FEM and can be solved by a direct integration technique. Such procedure is adopted in this work.

2.4 Dual Reciprocity Method for nonlinear case

In order to deal with the nonlinear case, the Kirchhof transform is used, which consists in the construction of a new variable $U = U(u)$ such that eq. (1) becomes linear in this new variable. The derivatives of U with respect to x and y are respectively

$$\frac{\partial U}{\partial x} = \frac{dU}{du}\frac{\partial u}{\partial x}$$
$$\frac{\partial U}{\partial y} = \frac{dU}{du}\frac{\partial u}{\partial y} \tag{18}$$

When comparing the above equation with eq. (1), it appears that the appropriate choice of U is such that

$$\frac{dU}{du} = K(u) \tag{19}$$

or in integral form

$$U = T[u] = \int_{u_a}^{u} K(\varphi)d\varphi \tag{20}$$

where u_a is an arbitrary reference value. Consequently, eq. (1) becomes

$$\nabla^2 U = \frac{1}{k}\frac{\partial}{\partial t} \tag{21}$$

where $k = k(u) = K/\rho c$. It can be noted that eq. (21) contains a residual nonlinear term, namely k. Therefore, another transformation is necessary. According to Partridge et al. [8], it is possible to write $k = k(x, y, t)$, as long as u is a continuous field. A new variable τ is proposed, defined as

$$\tau = \int_0^t k(x, y, t)dt \tag{22}$$

From the partial derivative of τ with respect to t

$$\frac{\partial \tau}{\partial t} = k \tag{23}$$

Therefore, replacing the above equation in eq. (21), finally, we have

$$\nabla^2 U = \frac{\partial U}{\partial \tau} \tag{24}$$

This equation can be solved as described in the previous section. However, since the new variable τ depends on the position, an iterative solutions process must be employed. Therefore, DRM is used in eq. (24) together with the Newton-Raphson method (see Wrobel and Brebbia [9] for more details), resulting

$$(2\boldsymbol{\overline{C}} + \boldsymbol{H})U^{m+1} - 2\boldsymbol{G}Q^{m+1} = (2\boldsymbol{\overline{C}} - \boldsymbol{H})U^m \tag{25}$$

where U and Q represent values in the transformed space and matrix $\boldsymbol{\overline{C}}$ contains the step value of the modified time variable at each node, that is, $\Delta\tau_j = k_j \Delta t$. For more details on inverse transformation and boundary conditions, see Wrobel e Brebbia [9] and Azevedo [11].

3 Results and discussions

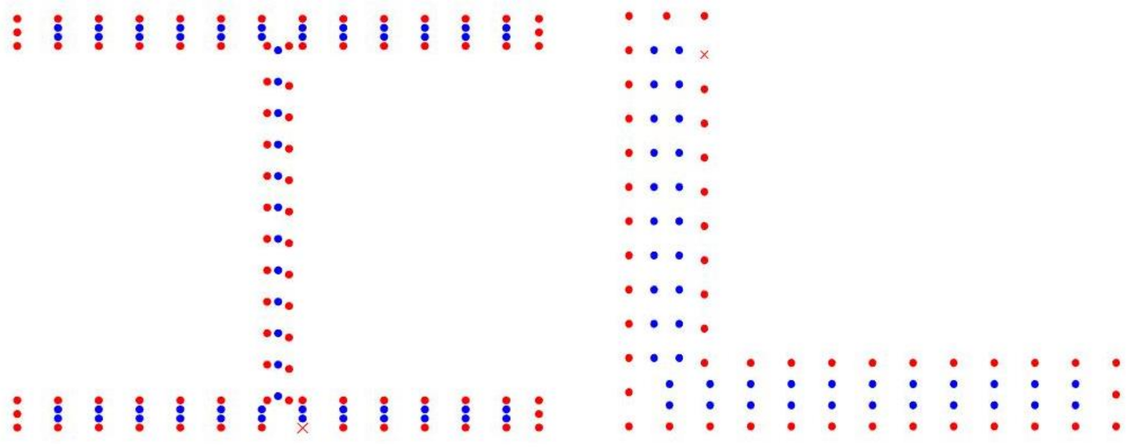
Based on the formulation described in section 2, a computer program is developed in MATLAB to solve the aforementioned 2D heat conduction equation with the different types of boundary conditions also mentioned. The use of the program is illustrated by the application to two steel sections, comparing the results with that proposed by the respective norm, in order to provide evidence of the efficiency and accuracy of the analysis.

The thermal properties of steel used in the simulations are the same as those presented in NBR 14323. It should also be noted that the said norm allows the use of a fixed value for the specific mass of the steel, regardless of the temperature. Table 1 shows the geometric and numerical characteristics of the tested sections. When observing the specific heat and thermal conductivity curves in the aforementioned norm, one can notice the variation of properties as a function of temperature, especially the specific heat, which in itself already leads to the non-linearity of the heat conduction equation in the section. However, NBR 14323 simply allows the use of constant values for these properties, which definitely does not generate much difference from the point of view of the thermal capacity of the section, because - as can be seen in the figures below - the temperature difference between using the value constant or variable value is no more than a hundred degrees; this is mainly due to the fact that the steel sections are slim, so that conduction is not a predominant phenomenon.

Table 1. Steel sections tested.

Section	1	2
Type	W 250 x 89,00	L 51 x 7.9
$A\ (cm^2)$	113.9	7.42
$d\ (mm)$	260	-
$b_f\ (mm)$	256	50.8
$t_w\ (mm)$	10.7	7.9
$t_f\ (mm)$	17.3	-
Perimeter(mm)	1522.6	203.2
Element type	Linear	Linear
Nº of boundary points	82	48
Nº of internal points	60	42

The following figures show the meshes of the tested sections respectively; the red dots are the boundary points and the blue dots are internal points. In addition, the point plotted with the x is the one for which the temperature is analyzed in the graphs.



(a) Section 1.

(b) Section 2.

Figure 2. Mesh of steel sections.

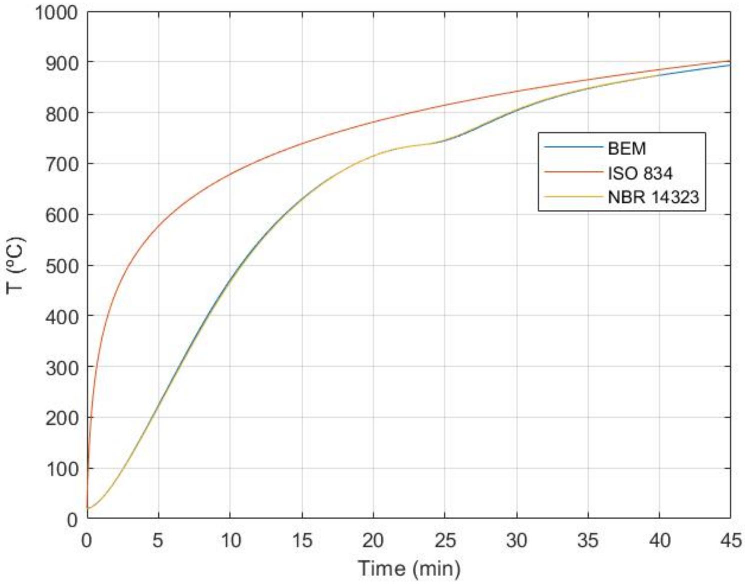


Figure 3. Section 1 with variable specific heat.

All results presented in the figures refer to the temperature at the point marked on the mesh in Fig. 2 and are compared with the simplified procedure of NBR 14323. In all tested sections, a result was obtained by BEM close to that obtained by the simplified standard methodology. When analyzing the Fig. 3 and Fig. 4, it appears that the main difference is the waviness of the curve close to 750 ºC; this is due to the growth of the specific heat close to this temperature. On the other hand, this reveals the method's ability to capture the non-linearity of the problem.

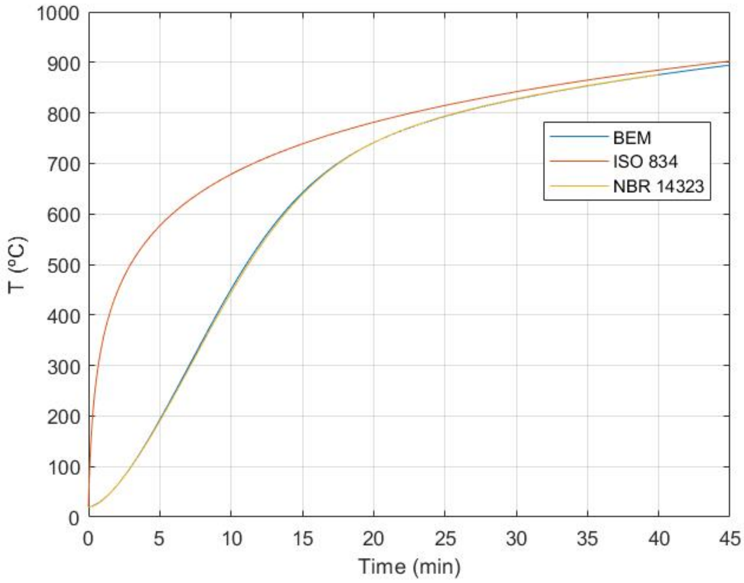


Figure 4. Section 1 with constant specific heat.

In the case of section 2, we see a faster increase in temperature compared to section 1 (see Tab. 2), mainly because it is a smaller section. Finally, the results reveal, on the one hand, the validation of the method (BEM) and, on the other hand, the precision of the simplified method presented by NBR 14323 for slim steel sections.

Table 2. Temperature results for the two sections.

	Section 1			Section 2		
t (min)	BEM	NBR	e (%)	BEM	NBR	e (%)
5	223.38	220.68	1.22%	353.55	353.76	0.06%
10	470.38	465.85	0.97%	602.28	603.77	0.25%
15	629.50	627.73	0.28%	703.27	704.07	0.11%
20	714.54	714.78	0.03%	740.09	740.56	0.06%
30	803.94	805.71	0.22%	833.57	833.81	0.03%
40	873.50	874.06	0.06%	880.03	880.14	0.01%

The table above shows the temperature results for the two sections tested at different time points, comparing the results with those obtained by the simplified method of the standard. It should be noted that the k_{sh} for thermal analysis of sections mediating NBR 14323 was taken equal to 1.

4 Conclusions

The presented methodology proved to be capable and precise to solve the linear and non-linear 2D thermal problem in steel sections. In addition, the simplified method for steel sections presented by NBR 14323 proves to be quite accurate when it comes to slim sections. In particular, BEM coupled with DRM is an efficient way to solve non-homogeneous partial differential equations.

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Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors.

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