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Numerical strategies for obtaining strut-and-tie models via topological optimization

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Abstract. The strut-and-tie model is widely used for analysis and design of reinforced concrete structures under plane stresses state. To apply this model, it is necessary to define strut-and-tie systems that represent the flow of stresses generated in the analysed structure. In order to make the concept of the model less dependent on the designer's experience, in many situations, like regions or structures with geometric or static discontinuity, this strut-and-tie model is defined through an evolutionary structural optimization (ESO) considering linear isotropic material, which provides the automatic generation of strut-and-tie models. The evolution criterion adopted by the topological optimization method considers the elimination of less requested elements in terms of tension, based on the elastic-linear analysis. The focus of this work is on the development and implementation of different strategies to control the way these elements are removed, which lead to different final configurations. In this context, it is possible to obtain optimized solutions to complex problems involving reinforced concrete structures. Two practical applications are presented and two of them had their results compared with the results provided in the literature to validate the efficiency of the algorithm proposed.

Keywords: Strut-and-Tie Model; Finite Element Method; Topological Optimization; Reinforced Concrete; Plane stress state.

1 Introduction

The analytical methods of design reinforced concrete bar elements subjected to normal stresses are quite accurate; however, when the shear force must be considered and has significant value, the Bernoulli hypothesis does not adequately describe the structural behavior or stress distribution and these methods are not efficient. Another example of the difficulty in using analytical methods is the cases in which a two-dimensional analysis considering the plane stress state is more appropriate.

Even in structures with well-defined analytical methods such as beams and frames there are regions in which the simplifying hypotheses that guarantee the accuracy of the analytical methods are not met. In these regions, called 'D regions' in the literature, the shear stresses are significant and the distribution of strains is nonlinear. As an example, we can mention corbels, nodes in frame, holes in beams, and dapped-end beams. Structural elements such as deep beams, footing, piles caps, are also practical examples in which the available analytical methods are not efficient.

For the analysis of these special regions and structural elements, alternatives such as the Finite Element Method and strut-and-tie model (STM) can be used. The latter is a simple method in which a truss is idealized within the 'D region' or the structural element, in order to represent the flow of stresses, allowing a simplified analysis of the mechanical behavior of the structural element. However, the efficiency of this method depends on the correct definition of the compressed and tensioned elements of the substitute model. For this purpose, in this paper is used a topological optimization technique ESO (Evolutionary Structural Optimization) for automatic generation of the STM, defining the best configuration to be adopted for the analysis.

Pioneering studies involving the STM originated at the beginning of the 20th century, when Ritter and Mörsch proposed, based on experimental results, the analogy of the truss model for the design of reinforced concrete beams under shear action. Schlaich *et al*. [1] allowed the systematic applications STM and extended to other types of structural elements, such as deep beams, corbels, footing, piles caps, beams with holes, among others. From there, several researchers used the STM in the development of their work, stands out: Schäfer and Schläich [2]; Tan *et al.* [3]; Tjhin and Kuchma [4]; Souza [5]; Zhang and Tan [6]; Wang and Meng [7]; Zhang and Tan [8]; Farghaly and Benmokrane [9]; Chen et al. [10]; Dashlejeh and Arabzadeh [11]; Ladeira *et al*. [12].

To propose an effective tool for the generation of the STM, the ESO algorithm is adopted in this paper. Techniques are presented to reduce the occurrence of numerical instabilities inherent to the evolutionary process. To achieve the main objective, the following specific objectives can be highlighted: implementation of a three-node triangular finite element for structures submitted to plane stress state; implementation of a topological optimization routine within a Finite Element Program. For more details about FEM implementation, see Ladeira *et al*. [12]. The choice of the three-node triangular element for the numerical analysis of structures in a plane stress state is because this element requires a very refined discretization of the continuum, thus allowing the definition of the compression and tension regions of the STM with more refinement. Furthermore, since in some stages of the analysis, the evolution technique used in the topological optimization process consists of eliminating the finite element, hence the requirement for a well-refined mesh, consequently a poorest element in terms of shape functions.

2 Evolutionary Structural Optimization

The ESO technique appears as an alternative to the mathematical rigor of classical optimization methods. This procedure presents a simple theoretical basis that consists of inserting voids in the structure through the gradual elimination of the least requested elements of the domain during the evolution process. In this method, a rejection criterion is defined (for example, stress level in the element), a rejection ratio (percentage of elements removed by iteration), an evolution ratio (after each iteration if no element meets the rejection ratio, this is increased by evolution ratio), and a final volume (the iterative process ends when the domain volume reaches a specified value).

In this paper, the ESO is an evolutionary algorithm based on the concept of stress: the maximum stress level in the structure, obtained by linear analysis via FEM, is taken as an indicator of the level of efficiency of each element. Elements with a low-tension level are systematically removed from the structure. At each iteration, new inefficient elements are eliminated from the mesh and the procedure is repeated until the field stress across the domain is practically constant and very close to the allowable stress of the material or that the minimum volume restriction is reached.

The removal criterion is made by comparing the von Mises stress of each element with the maximum von Mises stress across the entire structure. Therefore, at the end of each iteration all the elements that satisfy the eq. (1) will be eliminated. The form of removal of the element occurs by assigning low values for its Young's Modulus ($E = 10^{-12}$). This way, the new mesh generation of the structure is avoided, which greatly simplifies the computational implementation of the method within a platform of a finite element program. However, the degrees of freedom of a node connected to elements that have been removed from the analysis continue to produce equations in the structure's overall stiffness matrix, which can lead to a bad conditioning of that matrix. This is the reason for using a small elasticity module instead of zero for the element after its removal from the mesh.

$$\sigma_e^{vM} < RR_i \,.\, \sigma_{máx}^{vM} \tag{1}$$

In eq. (1), σ_e^{vM} is von Mises stress in the analyzed element, RR_i is the rejection ratio in the i-th iteration ($0<RRi<1.0$) and σ_{Max}^{vM} is the maximun von Mises stress in the i-th iteration. The rejection ratio is used to slow the process of removing the elements. The removal cycle occurs until no more elements can be removed for a given

RR_i. When this occurs, a steady state is reached. The evolutionary process is redefined by adding an evolution ratio, ER, to RR_i, according to eq. (2).

$$RR_{i+1} = RR_i + ER \quad i = 0,\ 1,\ 2\ ... \tag{2}$$

The initial value of the rejection ratio (RR_0) is defined empirically by the user. However, according to Querin [13], to ensure better convergence, the RR_0 and ER values should be approximately 1%. In this work, the evolutionary process continues until the structure does not reaches the final volume, VF, according to eq. (3).

$$VR < (1 - VF)\ .\ VT \tag{3}$$

where VR is the the total volume removed up in the i^{-th} iteration, VF is the final volume desired final and VT is the initial total volume of the structures.

Briefly, the ESO algorithm can be presented in the sequence of steps given below, represented schematically by the flowchart of Figure 1.

Step 1: discretization of the domain and application of the boundary conditions and prescribed actions;

Step 2: structural linear analysis via FEM and evaluate the von Mises stress for each element and maximum von Mises stress at each iteration;

Step 3: remove the elements that satisfy the Equation 19 for that iteration within a predefined volume limit;

Step 4: return to step 2 until the steady stage is reached;

Step 5: update the rejection ratio using the evolution ratio (ER) and start a new elements removal.

3 Strategies to control numerical instabilities

Although it is conceptually simple, ESO is a discrete optimization algorithm which in general results in problems related to numerical instabilities, as detailed in Sigmund and Petersson [14].

The checkerboard instability, is a very common problem in the topological optimization of continuous structures. It is characterized by the formation in the optimal topology of regions containing voids (without material) and solids (with material), alternately, assuming an aspect similar to a checkerboard. According to Díaz and Sigmund [15], this phenomenon results from numerical problems in the convergence of the FEM caused by the bad conditioning of the equilibrium equations.

In the present paper, the reduction of this type of instability was possible through the implementation of a code that controls the removal of elements that are 'loose' in the mesh during the evolutionary process, that is, elements that are not connected to any other element by through its edges. Figure 1 shows a breakdown of the ESO algorithm, which description is given below.

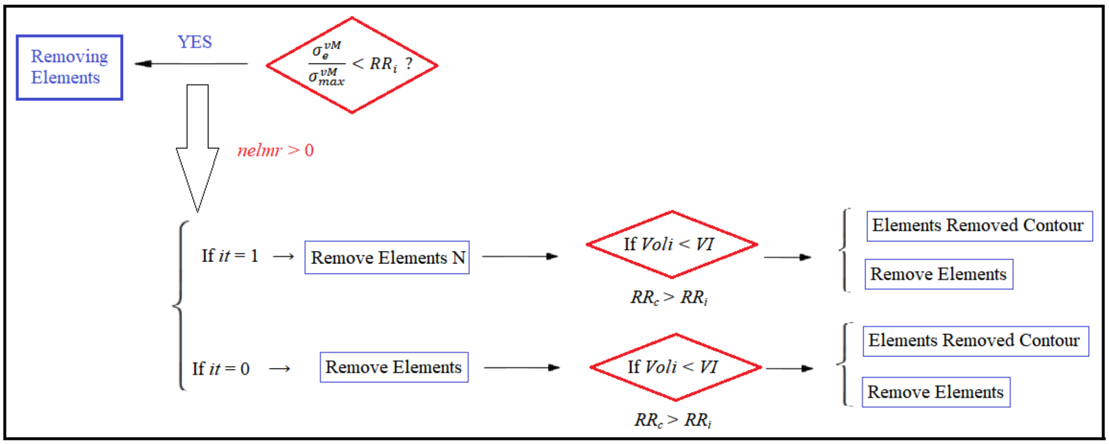

Figure 1. Flowchart for ESO algorithm when there is elements to be removed for a provided rejection ratio.

To make it clearer the explanation will be defined '*elmr*' as the list of elements to be removed, '*elmrc*' the list of contour elements of the updated domain to be removed, '*nelmr*' number of elements from '*elmr'* and '*nelmrc'* number of '*elmrc*' elements.

The algorithm creates a list (vector '*elmr*') with '*nelmr'* candidate elements to be removed in ascending order of rejection ratio, that is, elements that satisfy the eq. (1), regardless of whether is the contour or interior element. Here contour element is the element that has one or more of its edges not connected to any other element, otherwise the element is considered interior.

The removal of the elements from the '*elmr*' list occurs until the volume removed in a given iteration, '*Voli*', reaches the maximum volume allowed to be removed by iteration, defined by the parameter '*VI*'.

In order to control the way elements are removed, the '*it'* parameter was created, which allows the user to force (or not) eliminate elements of the mesh that are not connected to any other element through its edges, that is, they would be connected only by their nodes. Thus, when '*it'* is 1, these 'loose' elements are removed even if they do not satisfy eq. (1). This is done through the '*Remove Elements N'* function which excludes the first elements from the '*elmr'* list from the mesh until *Voli* = *VI* or until there is no elements to be removed from the '*elmr'* list, in this case *Voli* <*VI*. After that, the removal phase of the elements connected by their nodes is initiated. On the other hand, when '*it'* is 0, the algorithm doesn't remove 'loose' elements if they don't have low von Mises stress. In other words, the code then calls the function '*Remove Elements'*, which works as a function '*Remove Elements N'*, excluding the step of removing 'loose' elements that do not satisfy eq. (1).

Both for *it* = 1 and for *it* = 0, after removing all elements of the vector '*elmr'* or those allowed by the parameter '*VI'*, the algorithm looks for isolated elements or sections of isolated elements and these are removed elements. Unlike 'loose' elements, which are connected to other elements by their nodes, isolated elements are not connected to any other element either by their edges or by their nodes. The implemented algorithm can identify up to three elements connected to each other and isolated from the others.

Both for *it* = 1 and for *it* = 0, if the volume removed '*Voli'* has not reached the maximum volume allowed by iteration 'VI', there is the possibility of removing elements from the mesh that are exclusively at the contour until '*Voli* 'is equal to' *VI* '. For that, the RR_c parameter was defined, the rejection ratio of contour elements, and its corresponding evolution rate, ER_c. However, this is only valid for RR_c values greater than RR. Thus, the function 'Elements Removed Contour' creates a list with the end elements to be removed from the mesh, that is, elements such that $\sigma_e^{vM} < RRc.\ \sigma_{Max}^{vM}$. After that, the algorithm calls the function '*Remove Elements N'*, if *it* = 1, or '*Remove Elements*', if *it* = 0, to remove elements from the mesh, as described in the previous paragraphs.

If there is no element that satisfy eq. (1) for a given rejection ratio, that is, when the list '*elmr*' is empty ('*nelmr*' = 0) and the accumulated volume removed, *VR*, has not reached the stipulated value by the user, the removal rates elements, *RR* and *RRc*, must be updated and a new removal cycle started. However, before updating *RR* and RR_c, the algorithm checks if there are exclusively contour elements to be removed. The function '*Elements Removed Contour'* is called again to now create the '*elmrc*' list of elements exclusively at the edge of the updated domain. In Figure 2, '*nelmrc*' is the number of elements in the '*elmrc*' list. Again, this is only done if a contour element *RRc* rejection rate is set higher than the RR ratio for any elements. Figure 2 illustrates this step in the implemented code.

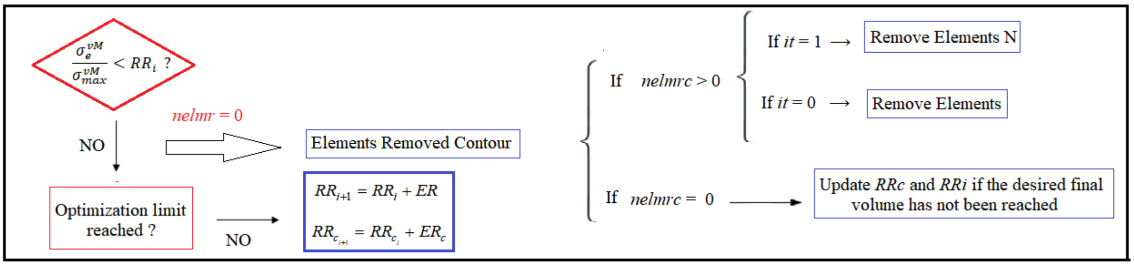

Figure 2. Flowchart for ESO algorithm when there is no elements to be removed for a provided rejection ratio

Another common problem in ESO is the mesh dependency and is related to the various final solutions obtained for different domain discretization's. Therefore, in topological optimization there is a change in the optimal topology each time the discretization is increased and, contrary to what would be expected, the use of a more refined mesh does not always result in better results.

Typical solutions adopted to minimize or solve problems of this type can be found in Jog and Haber [16] and Zhou et al. [17]. In this paper, the following strategy for calculating the stresses in the elements to representing the regions to be eliminated during the optimization process was adopted:

- Stresses are evaluated at the element's Gauss point (in the case of the implemented three-node triangular element, only one Gauss point is used for the numerical integration process);
- This stress is extrapolated to the element's nodes (for the three-node triangular element, as it is a constant deformation element, the stress at any point of the element assumes the same value as the stress at the Gauss points);
- Stress smoothing: the stresses in a node are calculated using a simple arithmetic average of the stresses of all the elements incident at that node;
- The stresses in the element are recalculated by averaging the stresses of the three element connectivity nodes.

In this way, the cavities are created automatically in internal points of low tension and it was observed that the application of this strategy reduces the occurrence of both the checkerboard and the mesh dependency during the iterative process.

4 Examples and Results

4.1 Deep beam with multiple load

The Figure 3(a) shows the initial geometry of the problem. It is a simply supported beam subjected to three concentered loads, where P1 = 40kN and P2 = 20kN. In this example, initially studied by Liang [18], the domain was discretized in a mesh of 3600 triangular elements with three nodes (see Figure 4(b)).

The Figure 3(c) shows the optimal configuration obtained by adopting the following parameters: *RR* = 1%, *ER* = 1%, *VF* = 39%, *VI* = 5%, *it* = 0, *RRc* = 0 and *ERc* = 0. This topology was obtained after 133 iterations of the topological optimization algorithm implemented in this work.

The optimal topology illustrated in Figure 3(d), on the other hand, was achieved after 121 iterations. The evolutionary process started with a rejection ratio (*RR*) equal to 1% and an evolution ratio (*ER*) of 1%. The volume removed per iteration (*VI*) is equal to 1%. In addition, *it* = 1, *RRc* = 0.03 and *ERc* = 0.03 were adopted.

In both situations the final desired volume is equal to 39% of the initial volume, the material used has a longitudinal modulus of elasticity E = 200GPa, Poisson's ratio υ=0.30 and *t*=10 mm. Figure 3(b) shows the result obtained by Liang [18]. This example shows how the optimal configuration is sensitive to the variation of the initial parameters for the same final volume.

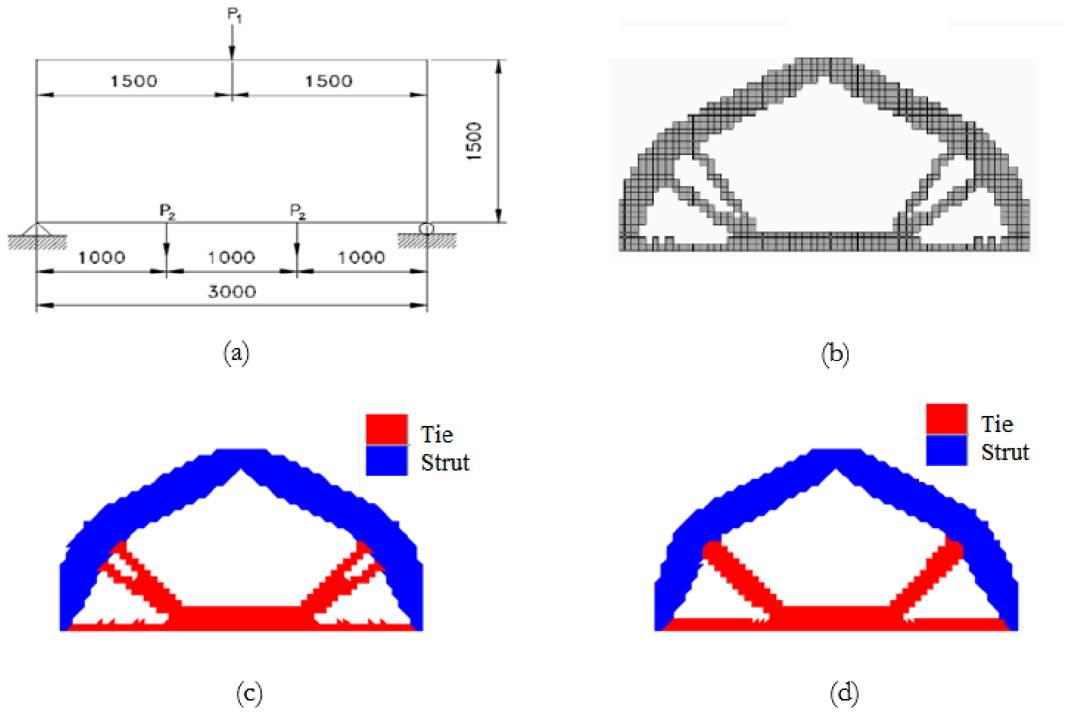

Figure 3. (a) Design domain and boudary conditions (b) Optimal Topology obtained by [33] (c) Iteration 133, RR=19%, VF=39% (d) Iteration 121, RR=6%, VF=39%

4.2 Bridge Structure

The Figure 4(a) presents the problem proposed by Liang and Steven [19], whose optimal configuration obtained by the authors is illustrated in Figure 4(b). A method called PBO (Performance-Based Optimization) was used in which the structure was discretized in a mesh of 90x30 quadrilateral elements with four knots, which led to a well-known "tie-arc" commonly used in bridge engineering projects.

It is a bridge with a central deck subjected to a uniformly distributed load modeled by concentrated loads of 500kN in all nodes on the upper face of the deck. In the present analysis, a material with elastic modulus E = 200GPa, Poisson's ratio υ=0.30 and *t*=30cm was considered. A mesh with 5400 three-node triangular elements was adopted.

The implemented algorithm prevents the removal of elements in which the external loading is applied as well as elements where the boundary conditions are imposed.

The Figure 4(c) shows the final configuration reached after 192 iterations of the algorithm. The following parameters were adopted: *RR* = 1%, *ER* = 1.5%, final volume *VF* = 35%, volume removed by iteration *VI* = 1.2% and *it* = 0 (therefore, *RRc* = 0 and *ERc* = 0).

Redefining the initial data, it was possible to obtain the optimal topology illustrated in Figure 4(d). In this case, *it* = 1, *RRc* = 2% and *ERc* = 3% and the other values were maintained, that is: *RR* = 1%, *ER* = 1.5%, final volume *VF* = 35% and volume removed by iteration *VI* = 1.2%. In this case, the evolutionary process ended after 139 iterations. The compression fields (struts) are represented in blue and the traction fields (tie) in red. Once again it is possible to notice that the optimal topology is sensitive to the way the elements are removed, interfering in the position, quantity and thickness of the ties, for example.

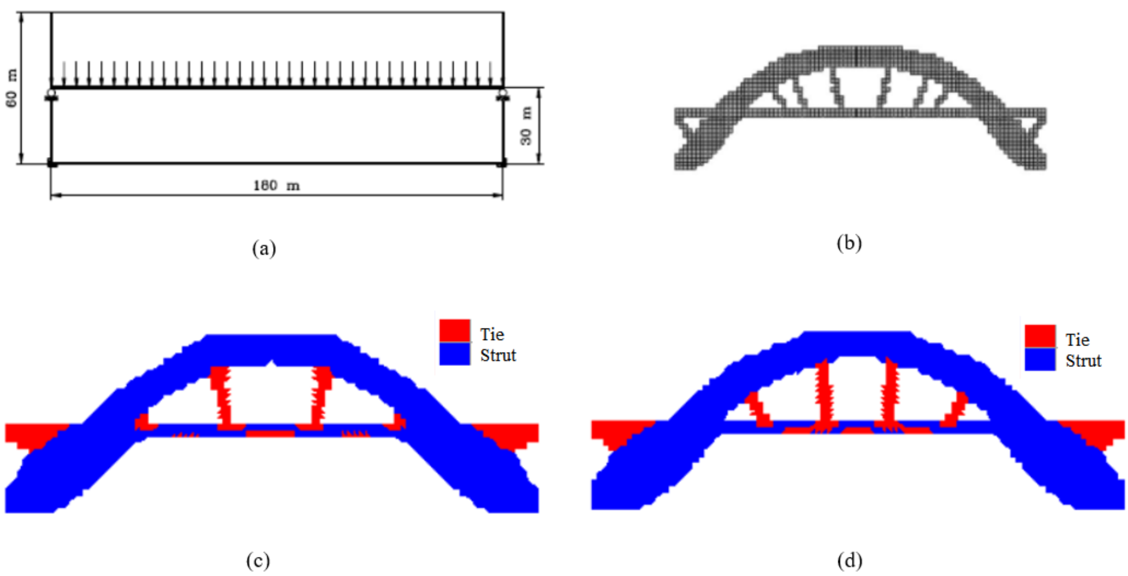

Figure 4. (a) Initial design domain (b) Optimal Topology obtained by [23] (c) Iteration 192, *RR*=19%, *VF*=35% (d) Iteratiom 139, *RR*=10%, *VF*=35%

5 Conclusions

The objective of the article is the presentation of a numerical techniques and formulation to generate the STM's in reinforced concrete structures under plane stresses state. The topological optimization algorithm ESO was used for this purpose.

The design domain is discretized in a refined finite element mesh and, iteratively, the algorithm converges to an optimal topological configuration, in which the STM are "automatically" defined. It was evidenced through the presented examples, that the different strategies of removal of the elements lead to different optimal topologies, depending on the calibration of the input parameters defined by the user. In addition, the present formulation contributes to reducing the numerical instabilities inherent to ESO. The three examples showed good accuracy with the results described by other authors. For the example of the continuous beam, the number and arrangement of the reinforcements were also proposed.

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