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Computation of mooring systems' forces of vessels berthed at dolphins

Arina L. B. Carvalho[1], Kalil J. Skaf[1], Eduardo M. B. Campello[1]

[1]*Dept. of Structural and Geotechnical Engineering, Polytechnic School, University of São Paulo*
Av. Prof. Luciano Gualberto, 380, 05508-010, São Paulo - SP, Brasil
arina.vlc@usp.br, kalil.skaf@egtengenharia.com.br, campello@usp.br

Abstract. The operational safety of a port terminal is assured by the adequate restriction of the ship's movements by the mooring system, composed of lines and fenders. In the current practice of structural design, the mooring system's forces are determined by simplified linear analyzes enhanced by dynamic amplification coefficients. However, the system's stiffness has physical and geometric nonlinearities. Besides, the main forces acting on a ship at berth come from environmental sources and have random and oscillatory behavior in nature, which may induce dynamic responses that exceed the static estimated. This work presents partial results of an on-going research on the dynamic behavior of a vessel berthed at dolphins and the current design practice simplification's consequences. Different hierarchical models will be analyzed (linear static, non-linear static and linear dynamic) at a case study of a Floating Storage and Regasification Unit (FSRU) permanently berthed at dolphins at Barcarena's Liquefied Natural Gas (LNG) terminal, in Pará, Brazil. The results in terms of the mooring system's forces will be discussed and dynamic amplification coefficients will be estimated.

Keywords: mooring lines, marine fenders, dolphins, nonlinear static analysis, linear dynamic analysis.

1 Introduction

The operational safety of a port terminal is ensured by the appropriate limitation of movements of berthed vessels during their loading and unloading operations. In such operations, the main forces acting on a vessel come from environmental sources. The displacement restrictions on the horizontal plane are given through mooring lines and fenders, which comprise the vessel´s mooring system. The current common practice on structural design of such systems is based on the relevant international standards, which prescribe the computation of the mooring forces (lines' tension and fenders' compression) through simplified methods. These methods invariably consist of a linear static analysis that decouples the vessel's movements and does not consider the relative stiffness of its elements. The purpose of this work is to investigate the forces developed in mooring systems through the development of more elaborate (hierarchically superior) models, namely, a nonlinear static model and a linear dynamics model, and compare the results against those of the simplified methods. The models are applied to a case study of a natural gas' regasification terminal at the Port of Barcarena, Pará, Brazil. The case consists of a Floating Storage Regasification Unit (FSRU) berthed at dolphins and subjected to environmental wind and current loads. For the static model, the nonlinearities considered are both physical (such as the fenders´ stiffness and buckling-column-type behavior) and geometrical (caused by the slacking of the mooring lines and the occasional loss of contact between the vessel and the fenders). The corresponding system of nonlinear equations is iteratively solved through a fully consistent Newton-Raphson scheme. For the dynamics model we formulate the problem in the frequency domain taking into account the oscillatory nature of the environmental loadings. To this aim, the system´s stiffness is linearized around the equilibrium configuration as determined by the previous (nonlinear static) model. For comparison purposes, the same input parameters as used in both models were adopted to compute the mooring system´s forces through the simplified, linear static methods prescribed by the international standards. A thorough assessment of the results obtained with each modeling approach is performed.

This paper is organized into six sections, the first being the introduction. Section 2 presents the linear and non-linear static models. Section 3 explains the linear dynamic model's formulation. Section 4 describes the case study that is considered here. Section 5 presents and discusses the results obtained in both analyzes. Finally, in section 6, the conclusions and final considerations are presented. Throughout the text, italic letters ($a, b, \ldots, A, B, \ldots$) represent scalar quantities, while bold-italic letters ($\boldsymbol{a}, \boldsymbol{b}, \ldots, \boldsymbol{A}, \boldsymbol{B}, \ldots$) represent vectors and matrices.

2 Linear and Nonlinear Static Models

The linear static model is taken from the Spanish standard *Recomendaciones de Obras Marítimas* - ROM 2.0-11 [1]. The system is balanced on the horizontal plane by the active environmental forces that move the ship and the reaction forces caused by the mooring lines' stretching and the fenders' compression. In this approach, we have a statically indeterminate system, with multiple unknowns, given by the number of lines and fenders, and only three balance equations. To solve this problem and facilitate the design practice, the Spanish standard defines some simplifications. When the resulting forces act to move the vessel away from the structure (towards the sea), the simplification decouples the vessel's movements and does not consider the relative stiffness of its elements. In the case when the resulting forces move the vessel towards the structure, the simplifications also decouple the movements but consider the relative stiffness of its elements. However, in the last case, an iterative calculation is necessary as the equilibrium configuration is not known a priori. The detailed methodology can be found in ROM 2.0-11 [1] itself or at a previous work by the authors [2].

The non-linear static model, in turn, was based on the work of Barros [3], supplemented here with some additional features. Accordingly, the ship is modelled as a rigid body and the mooring lines and fenders as oriented vectors. The lines' tension and fenders' compression are written in terms of the vessel's center of gravity´s displacements and rotation. The model considers geometrical nonlinearities given by the slacking of the mooring lines and the loss of contact between the vessel and the fenders. Differing from Barros [3], the physical nonlinearity of the fenders´ stiffness and buckling-column-type behavior are also considered through the formulation proposed by Antolloni *et al* [4]. The horizontal plane balance is given by the sum of forces in the vessel's center of gravity. In this case, the balance leads to a nonlinear system composed of three equations: sum of longitudinal and transversal forces and in-plane moments; and three unknowns: longitudinal and transversal displacements and in-plane rotation of the vessel´s CG. Its numerical solution is formulated and implemented in the software *Mathematica* by the authors through a Newton-Raphson's procedure. To ensure the solution's convergence, the external loads are incrementally applied from the vessel´s initial position. The detailed methodology can also be found in the authors' paper [2].

In both analyzes, environmental actions are considered as static loadings and are calculated following the procedure stated by ROM 2.0-11 [1]. Hence, it is necessary to multiply the forces by a dynamic amplification coefficient (γ_D) to acknowledge their oscillatory behavior. In this work, we adopt $\gamma_D = 1.5$ (based on the work of Barros [3]) and this value will be verified by the dynamic analysis.

3 Linear Dynamic Model

The oscillatory behavior of the environmental actions (winds, waves and currents) can induce a dynamic response of the mooring system composed of the ship, mooring lines, fenders and dolphins. In such a response, the ship oscillates around an equilibrium configuration. Depending on the excitation frequency range, the displacements may overcome those estimated in the static analysis and exceed the mooring lines' tension and fenders' compression limits.

A free-floating (rigid) vessel has six degrees of freedom: three displacements and three rotations about its center of gravity. The displacements in the longitudinal, transversal and vertical axes are named surge, sway and heave, respectively. The rotations around these same axes are named roll, pitch and yaw, respectively. In this context, the motion equation that describes the ship's behavior may, according to Gaythwaite [5], be written as:

$$(\boldsymbol{m} + \boldsymbol{a}(\boldsymbol{\omega}))\, \ddot{\boldsymbol{x}} + \boldsymbol{b}(\boldsymbol{\omega})\, \dot{\boldsymbol{x}} + \boldsymbol{c}\, \boldsymbol{x} = \boldsymbol{F}(\boldsymbol{t})\ , \tag{1}$$

where $\ddot{\boldsymbol{x}}$, $\dot{\boldsymbol{x}}$, and $\boldsymbol{x}$ are the acceleration, velocity, and displacement vectors of the ship's six degrees of freedom, respectively; $\boldsymbol{m}$ and $\boldsymbol{a}(\boldsymbol{\omega})$ are the mass and the hydrodynamic added mass matrices; $\boldsymbol{b}(\boldsymbol{\omega})$ is the damping matrix; $\boldsymbol{c}$ is the system's restorative forces stiffness matrix and $\boldsymbol{F}(t)$ is the time-varying acting forces. Still in (1), $\boldsymbol{\omega}$ is the vector of oscillation frequencies of the system.

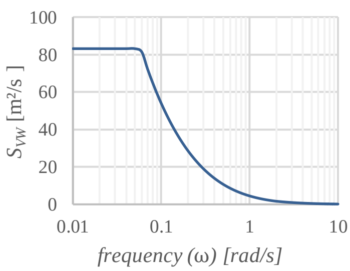
The added mass and damping are usually called hydrodynamic coefficients. Both depend on the oscillation frequency (ω) of the respective vessel's degree of freedom and are different for each one. The restorative forces stiffness ($\boldsymbol{c}$) also depend on the analyzed degree of freedom. For the displacements of surge, sway and yaw, they are related to the mooring lines' and fenders' stiffnesses, which, as explained previously, have geometrical and physical nonlinearities. For the heave, pitch and roll degrees-of-freedom, in turn, they are related to the hydrostatic forces. Finally, the acting forces $\boldsymbol{F}(t)$ may come from environmental sources, such as winds, waves and currents, or else result from the port's operations.

In this study, the analysis will be restricted to the horizontal plane, considering only the degrees of freedom of surge, sway and yaw. After defining the motion equation, its solution will be made in the frequency domain, which requires the linearization of the former so that classical methods of structural dynamics may be applied. If the mooring system's response is (approximately) locally linear around its equilibrium configuration, the system's stiffness' linearization and the frequency's domain analysis are good approximations to the problem's real behavior.

3.1 Motion's Equation Definition

According to Barros [3], the surge and sway inertial forces correspond to the vessel's displacement in the analyzed load situation. For the yaw moment of inertia, the distribution of the ship's mass in the analyzed load situation must be taken into account. The effect of the added mass of each degree of freedom is also considered. Therefore, the total inertia (named "virtual mass") is given by the sum of both components. Concerning the stiffness matrix, it is assumed that the dynamic problem can be interpreted as small disturbances around the static equilibrium configuration. Therefore, the system's stiffness will be defined based on the static equilibrium calculated through the previously defined nonlinear model. Then, small displacements will be imposed on each degree of freedom at this equilibrium configuration. The stiffness can then be calculated through standard arguments, i.e., by the ratio of forces' variation in the direction of interest by the imposed displacement. Determination of the hydrodynamic coefficients of added mass and damping was done according to the work developed by Oortmerssen [6]. Therein, the author created parameterized curves that relate dimensionless hydrodynamic coefficients to the excitation frequency for the vessel's six degrees of freedom. The hydrodynamic coefficients are then obtained in terms of the analyzed vessel's dimensions and geometry. Regarding the external loads, only the wind's dynamic effect will be taken into account in this work. Its oscillatory behavior is related to the wind's fluctuation component (turbulence). The latter can be described through a power spectral density of the wind speed $S_{Vw}(\omega)$. This study will use the Ochi-Shin's spectrum, presented by Feikema and Wichers [7]. From the speed spectrum, it is possible to determine the forces' spectrum $S_{Fw}(\omega)$ for any direction through the numeric formulation detailed by Barros [3] and presented on eq. (2). In this equation, V_w is the mean wind's velocity, F_{ws} is the wind's static force calculated in the static analysis, ω and $\Delta\omega$ are the oscillation's frequency and its discretization interval, respectively, and $S_{F,unit}(\omega)$ summarizes the oscillatory component. Figure 1 shows the Ochi-Shin spectrum, its formulation and an example of the wind's force spectrum.

$$S_{Fw}(\omega_i) = F_{ws}{}^2 \left[\left(\frac{2}{V_w}\right)^2 S_{Vw}(\omega_i) + 8\left(\frac{1}{V_v{}^2}\right)^2 \sum_{j=0}^{N} S_{Vw}(\omega_j)\, S_{Vw}(\omega_{i+j})\, \Delta\omega \right] = F_{ws}{}^2\, S_{F,unit}(\omega_i) \tag{2}$$



$$S_{Vw}(\omega) = C\, V_v{}^2\, F_g / \omega \;,$$

$$0 \le \omega \le 0{,}001885\, V_v \rightarrow F_g = 583\, x \;,$$

$$0{,}001885\, V_v \le \omega \le 0{,}0628\, V_v \rightarrow F_g = \frac{420\, x^{0{,}7}}{[1+x^{0{,}35}]^{11{,}5}} \;,$$

$$0{,}0628\; V_v \le \omega \rightarrow F_g = \frac{838\, x}{[1+x^{0{,}35}]^{11{,}5}} \;,$$

$$x = {}^{1{,}592\, \omega}/_{V_v} \,.$$

$$C = (750 + 69\; V_v) 10^{-6} \,.$$

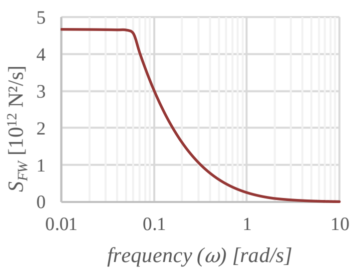


Figure 1. Ochi-Shin's spectrum (left), its formulation (center) and wind's force spectrum (right).

As the force spectrum is proportional to the static wind force F_{ws} , knowing the wind is a random, Gaussian and ergodic process, and based on the Fourier's transform, Barros [3] defines the wind force's components $F_{wd}(\omega)$ in the frequency domain as a function of the excitation's frequency ω and its discretization $\Delta\omega$:

$$F_{wd}(\omega) = F_{ws}\sqrt{2\, S_{F,unit}(\omega)\, \Delta\omega} \tag{3}$$

3.2 Frequency Domain Analysis

The frequency domain analysis is made according to Barros' [3] procedure and through the modal's superposition method. The frequency range of interest (0.01 to 10 rad / s) was analyzed in intervals of 0.01 rad/s. The procedure consists of calculating the respective hydrodynamic coefficient, virtual mass, and damping matrix for each frequency ω_i. The stiffness matrix is calculated for the vessel's static equilibrium position considering wind and current loads. The system's natural frequencies of modal matrices are then defined for the excitation frequency ω_i.Therefore, the virtual mass, stiffness, damping matrices, and wind dynamic force vector are calculated in the modal coordinates. It is important to note that the damping matrix is non-proportional. Similar to the work of Barros [3], the modal damping matrix is approximated by a linear combination of the mass and stiffness matrices to maintain the orthogonality of the vibration modes. Finally, for each excitation frequency ω_i, the problem is solved according classical dynamics theory and the decoupled displacements are obtained in modal coordinates. It is then possible to transform these displacements into global coordinates and estimate its maximum amplitudes ($x_{i,k}$) for each degree of freedom k. The results are presented in the form of *displacement x frequency* curves. According to Barros [3], the total displacements (X_k) for each degree of freedom k can be estimated by eq. (4), wherein $\overline{x_k}$ are the displacements obtained by the static nonlinear model of the ship under the wind and current static loads and $x_{i,k}$ are the dynamic displacements' maximum amplitude in the analyzed frequency range of 0.01 to 10 rad / s.

$$X_k = \overline{x_k} \pm \sqrt{\sum x_{i,k}}. \tag{4}$$

4 Case Study

The analyzes were carried out into a case study of a Liquefied Natural Gas (LNG) terminal located in the Port of Barcarena, Pará, Brazil. In this study, a Liquefied Natural Gas Carrier (LNGC) vessel is permanently berthed at dolphins and is responsible for store and re-gasify the LNG from the incoming loaded ships. This vessel is named Floating Storage and Regasification Unit (FSRU) and its representative characteristics were taken from the ROM 2.0-11 [1] considering a membrane type vessel with a capacity of 140,000 m³. The terminal structure is composed of four breasting dolphins, six mooring dolphins located 40m backward (towards land) and two mooring dolphins aligned to the main platform. The coordinate axis' origin O is located on the structure's symmetry axis, aligned with the vessels' center of gravity. The layout of the structure is illustrated in Figure 2.

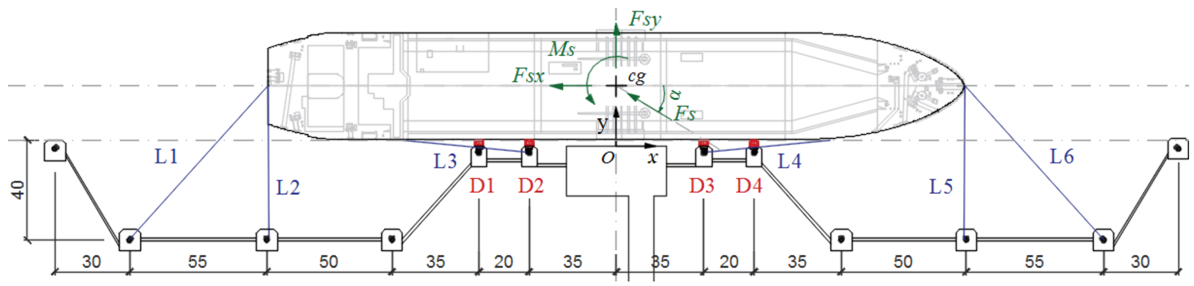


Figure 2. Layout of the case study

The mooring lines' arrangement was adopted following the recommendations of the ROM 2.0-11 [1]: a simplified symmetrical configuration with vertical angles lesser than 25º and the minimum number of cables, with two stern/bow lines, two breasts lines, and two springs. Although according to Gaythwaite [5] large ships usually use 16 to 18 lines, we adopted the minimum number of cables as possible, even if it results in larger lines' diameter with the only purpose of investigating the above-described models. The mooring lines used in this analysis are made of synthetic fiber of HMPE (*High-Modulus Polyethylene*). As it is a material with a high elasticity modulus, it is recommended to use a more flexible material at its end as to provide some flexibility. Hence, synthetic fiber tails (polyolefin and polyester) are adopted. In this analysis, the stiffness of the lines and tails will be considered linear with the elongation. The fender used, in turn, is the SCK Cell Fender type. As previously explained, its stiffness has a nonlinear behavior and it is described by the formulation developed by Antolloni *et al* [4].

The environmental loads were estimated according to the Oil Companies International Marine Forum's (OCIMF) [7] recommendation. Concerning the wind, an average speed of 31 m/s (60 knots) is adopted at any incidence angle (α_V) direction. This load must be added to the current load. OCIMF [7] recommends current's speeds of 1.54 m/s (3 knots) for incidence angles (α_C) of 0º or 180º about the ship's longitudinal axis; 1.03 m/s (2 knots), for angles of 10º or 170º; and 0.5 m/s (0.75 knots) at a 90º angle. The study was carried out considering forces acting towards the sea (0º-180º) or the land (180º-360º). The static forces were calculated according to the ROM 2.0-11's [1] formulation and the dynamic wind, in accordance with Section 3 of this study.

5 Results and Discussion

Tables 1-2 below show the results of the linear and nonlinear static analyzes for the maximum mooring lines' tension (Table 1) and fenders' compression (Table 2). Table 3 shows the vessel's center of gravity (cg) equilibrium's displacements of surge, sway and yaw (u_{cg}, v_{cg}, θ) as resulted from the nonlinear analysis. As previously explained, the dynamic amplification coefficient of γd= 1.5 was adopted for both analyses.

Table 1: Static Analysis – Mooring line's tension – γd= 1.5

α_V [º]	α_C [º]	Analysis	Stern L1	Aft Breast L2	Spring L3	Spring L4	Fwd Breast L5	Bow L6
90	90	Linear	**3178**	**3178**	0	0	2167	2167
90	90	Non-Linear	2400	**3513**	1314	867	2356	1750

Table 2: Static Analysis – Fender's compression – γd= 1.5

α_V [º]	α_C [º]	Analysis	Stern D1	Stern D2	Bow D3	Bow D4
240	190	Linear	**3226**	2636	567	0
240	270	Non-Linear	**3145**	2995	2143	1788.7

Table 3: Static Analysis – Vessel's cg displacements (nonlinear analysis) – γd= 1.5

α_V [º]	α_C [º]	u_{cg} [m]	v_{cg} [m]	θ [10^{-3} rad]
90	90	-0.018	0.686	-1.278
240	270	0.073	-0.238	2.272

For the linear dynamic analysis, firstly it is calculated the static nonlinear equilibrium for wind and current forces (following the same procedure as used in the static analysis) with no dynamic amplification, i.e., with γd = 1.0. The linearized stiffness matrix is then calculated and the dynamic's displacements under the excitation of the wind turbulence are estimated. The results of the vessel's cg displacements, maximum mooring lines' tension, and fenders' compression are shown in Table 4, Table 5, and Table 6, respectively. When analyzing Table 5, it is noted that the system's stiffness for the maximum fender's compression changes with the maximum dynamic's displacements, Dynamic (1), because spring L3 slackens. Therefore, the analysis is repeated now considering the new stiffness with two lines slackened (Aft Breast L2 and Spring L3), Dynamic (2).

Table 4: Dynamic Analysis – Vessel's cg displacements

α_V [º]	α_C [º]	Analysis	u_{cg} [m]	v_{cg} [m]	θ [10^{-3} rad]
90	90	Static γd= 1.0	-0.012	0.366	-0.856
		Dynamic	± 0.028	± 0.183	± 0.454
		Total	-0.041	0.549	-1.310
240	270	Static γd= 1.0	0.053	-0.119	0.988
		Dynamic (1)	± 0.080	± 0.054	± 0.921
		Total (1)	0.133	-0.173	1.909
240	270	Static γd= 1.0	0.053	-0.119	0.988
		Dynamic (2)	± 0.185	± 0.080	± 1.309
		Total (2)	0.238	-0.199	2.297

Table 5: Dynamic Analysis – Mooring line's tension

α_V [°]	α_C [°]	Analysis	Stern L1	Aft Breast L2	Spring L3	Spring L4	Fwd Breast L5	Bow L6
90	90	Static γd= 1.0	1653	**2284**	1058	758	1509	1220
		Dynamic	2093	**3083**	1320	702	1897	1509
240	270	Static γd= 1.0	294	0	335	955	768	649
		Dynamic (1)	76	0	0	1325	1007	653
		Dynamic (2)	109	0	0	1734	1098	522

Table 6: Dynamic Analysis – Fender's compression

α_V [°]	α_C [°]	Analysis	Stern D1	Stern D2	Bow D3	Bow D4
90	90	Static γd= 1.0.	0	0	0	0
		Dynamic	0	0	0	0
240	270	Static γd= 1.0	**2246**	2107	1539	1352
		Dynamic (1)	**2836**	2651	1737	1391
		Dynamic (2)	**3025**	2841	1839	1433

The figures below illustrate the *displacement x frequency* curves of the surge, sway and yaw for the load cases mentioned before.

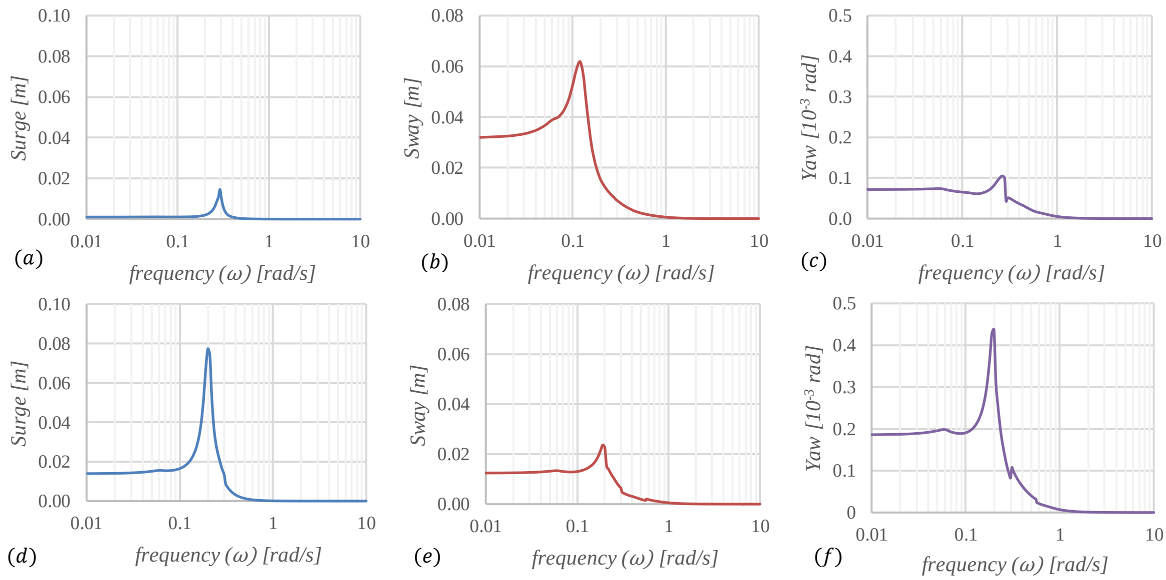


Figure 3. (*a*), (*b*), and (*c*) shows the *displacement x frequency* curves of the surge, sway and yaw, respectively, for $\alpha_V = \alpha_C = 90°$ and (*d*), (*e*), and (*f*) for $\alpha_V = 240°$, $\alpha_C = 270°$.

In respect to the static analysis, the maximum mooring's line tension obtained in the nonlinear analysis (3513 kN) is 10.6% greater than that of the linear analysis (3178kN). This difference is due to the proposed Spanish standard's simplification, which does not consider the lines' relative stiffness caused by their different lengths. From Table 1, the breasts lines receive about 50% more load than the stern/bow lines. The maximum fenders' compression of the linear analysis (3226.3kN) is close to the nonlinear result (3114.5kN), with a 2.6% discrepancy. In the dynamic analysis, regarding wind and current at 90º, the displacements of the surge, sway, and yaw show amplifications of 3.42, 1.50, and 1.53, respectively, in comparison to the nonlinear static analysis with γd = 1.0. In relation to the maximum line's tension, the dynamic amplification coefficient is 1.35. Interestingly, although the displacement in the longitudinal direction presents great dynamic amplification, it has a small influence on the final result. In a general way, the dynamic amplification coefficient γd = 1.5 adopted in the nonlinear static analysis proved to be adequate, as the maximum line's tension obtained in this (3513kN) is greater than the dynamic result (3083kN). Concerning the wind at 240º and current at 270º, the dynamic's analysis (Dynamic 2) displacements of the surge, sway, and yaw show amplifications of 4.49, 1.67, and 2.32, respectively, in comparison to the nonlinear static analysis with γd = 1.0. However, when analyzing the maximum fender's compression, the dynamic amplification coefficient is 1.35. This fact can be attributed to the highly nonlinear system's behavior around the static equilibrium under these loading conditions. The system's stiffness changes with the dynamic displacements because the spring (L3) slackens from the nonlinear static equilibrium. Therefore, the adopted stiffness in the dynamic analysis is slightly lower than the real stiffness, leading to greater displacements and fender's compression. One way to consider the real stiffness is through a nonlinear dynamic analysis in the time domain. Nevertheless, at least for the mooring system´s forces, the dynamic amplification coefficient $\gamma d = 1.5$ adopted in the nonlinear static analysis proved to be adequate, as the maximum fender's compression obtained in this (3145kN) is greater than the dynamic result (3025kN). Finally, when analyzing the *displacement x frequency* curves, we can observe the frequency range in which the system's response is approximately static, that is, the displacement is proportional to the force spectrum applied and the frequency range that approaches the system's natural frequency and causes a resonant response, with large displacements' amplifications.

6 Conclusions

A simplified linear static analysis of a ship berthed at dolphins is a quick and practical way to estimate the maximum mooring lines' tension and fenders' compression. On the other hand, the nonlinear analysis indicates that the lines' relative stiffness causes uneven forces' distribution between them. Regarding the fender's compression, as both static methods take into account the relative stiffness of its elements, and as the resulting displacements are small in comparison to the ship´s dimensions, the results are very similar. However, the simplified linear method is iterative and much more laborious. Such observed results are compatible with the considered literature. In general, the linear static analysis presents a good estimate of maximum mooring systems' forces. However, the nonlinear static analysis shows results closer to the problem's physical reality, as long as it is calibrated with an adequate dynamic amplification coefficient. The definition of this coefficient is based on dynamic analysis. In this context, linear dynamic analysis shows excellent results for locally linear equilibrium configurations. However, when dynamic displacements alter the system's stiffness, it is necessary to make approximations that can increase the response obtained, being on the safe side. A dynamic approach in the time domain is necessary to validate these situations' results. Generally, the dynamic amplification coefficient of $\gamma d = 1.5$ adopted in the static analyzes proved to be adequate (at least for the purpose of determining the mooring system´s forces). In any case, we remark that the present work presented partial results of a research that is under development. A dynamic nonlinear formulation of the mooring problem shall be presented by the authors when appropriate.

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