XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Thermomechanical analysis of spherical domes using Geckeler and Finite Element approximations

João M. F. dos Santos[1], Anne C. A. de Oliveira[1], João P. L. Lessa[1], Paulo C. do Carmo[1]

[1]*Technology Center, Federal University of Alagoas*
Maceió, 57072-900, Alagoas, Brazil
eng.jmarcosf@gmail.com, annecaroline.alb@gmail.com

Abstract. Structural analysis consists of obtaining the structure's response to the various actions to which it will be subjected throughout its useful life. In this work, it is proposed to develop the analytical calculation for the resolution of a concrete spherical dome, supported on its base, subjected to own and accidental loads. At the end, the effort diagrams, the stresses and displacements from these approximate analytical methods are obtained, where these results are compared with the Finite Element Method (FEM). The chosen example is the semi-spherical dome, where the influence of the different types of loading on these structures is studied, and the variability of their importance at the expense of changing the element's geometry. To validate the analytically determined results, a refined shell-type mesh (S4R8) was initially used in the ABAQUS program. For the convergence of results, the type of the element was varied in triangular and quadrilateral using the options of elements S4R8, S3, STRI65 and S4R. In the end, it was concluded that the adherence of the element type is important for approximations with the least number of elements possible, however, for this type of element, the triangular types with reduced integration yielded better adherence.

Keywords: Concrete domes, thermomechanical actions, FEM.

1 Introduction

Several times the structural engineer is faced with special structural elements in concrete or other type of material, these elements differ from the usual ones in geometry, loading and often bring an analysis challenge. As an example, we have surface structures, especially thin ones. Such structures are widely used in several areas of engineering due to their versatility of application, whether in isolated elements, such as domes; even structural elements formed by the union of one or more surface structures, such as reservoirs.

According to Billington [1], the domes are defined as thin-walled shells in the form of a surface of revolution, which is obtained by rotating a flat curve on an axis located in the plane of the curve. The domes, in their majority, are structures that present a great slenderness, a characteristic that is possible due to the fact that the domes are subjected, predominantly, to normal efforts. For domes supported on the surface of the terrain, Silva [2] states that the interaction of the structure with the foundation soil is not usually considered in the practice of structural projects. In most cases, this effect is neglected as a means of simplifying design procedures.

Because of this extensive use, it is necessary to know how the structural behavior of these elements works. Classical theories provide solutions, at first analytical, for patterns of geometries and loads through the solution of differential equations. Timoshenko and Woinowsky-Krieger [3] and Billington [1] present an approach, in principle analytical, that allows obtaining efforts and displacements of two-dimensional structures. Depending on the arrangement of the problem, the analytical models demand solutions of complex differential equations, making the analysis procedure exhaustive. With the intense use of computers and models and approximate methods, it is possible to have less complex solutions so that there is greater applicability in the study of these structural elements. The finite element method (FEM) is the most widely used method between the numerical models. Assan [4], Soriano [5], Bathe [6] confirm that the basic idea of the MEF is based on the premise of dividing the integration domain, continuous, into a finite number of regions called finite elements, forming a mesh.

This work proposes a calculation development to evaluate and obtain the displacements and efforts according to the variables of the spherical dome with the analytical formulas of Billington [1] and approximations of Geckeler, obtaining in the end the effort diagrams from the analytical methods and comparing with results from the Finite

Element Method (FEM) varying elements and approach configurations.

2 Methodology

2.1 General Assumptions

For the analyzes developed in this work regarding shells, the Kirchhoff-Love kinematic hypotheses are used, according to Marques [7], developed for the study of thin shells and assuming the following requirements: the material that constitutes the structure it is homogeneous, isotropic and obeys Hooke's law; the thickness is small in relation to the other dimensions of the plate and the radii of curvature of the medium surface; normal stresses on the medium surface are negligible in relation to other stresses; straight lines normal to the middle surface before the deformed state remain straight and normal to the medium surface after the deformation; the displacements are very small in relation to the thickness of the shell, being possible to neglect their influence in the study of the equilibrium conditions of the surface element.

The analyzed model of the spherical dome has the dimensions and characteristics expressed in Figure 1 below. It is necessary to point out that all analyzes will be carried out in a membrane regime, where only N_θ and N_ϕ efforts act, which is only possible by admitting a sliding foot, not preventing the horizontal variation of the dome, this is possible from the use of Neoprene as a connection on all edges of the dome. The dome is idealized in concrete, with an average thickness of $11\ cm$ and a radius of $10\ m$ in its projection measure. The inclination used was $22,62°$.

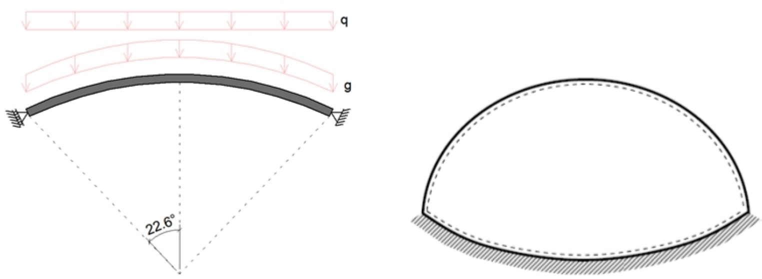

Figure 1. Dome representation scheme

2.2 Analytical procedures according to Geckelerr

In general, all types of stresses can act on a shell, such as normal stresses, shear stresses perpendicular to the middle surface, bending moments and also twisting moments. However, restricting the dome to work only in the membrane regime, there will be only normal efforts on the element, which can be seen in Figure 2.

Where N'_ϕ is the normal membrane effort in the direction of the angle ϕ , N'_θ is the normal effort in the direction θ, p_y is the loading component in the y direction, p_z is the loading component in the direction z, r_0 is the horizontal radius of the dome at any time, r_1 is the radius of curvature of the meridians, r_2 is the radius of curvature of the parallels, ϕ is the opening angle of the parallels and θ is the opening angle of the meridian.

As the problem analyzed in this work presents geometry and axisymmetric loading, the terms of effort and loading involving $d\theta$ are null due to symmetry. Therefore:

$$p_\theta \equiv 0,\ N'_{\phi\theta} \equiv 0,\ N'_{\theta\phi} \equiv 0\ and\ \frac{\partial}{\partial\theta} \equiv 0 \tag{1}$$

According to Marques [8], for the analysis of efforts and displacements, one must distinguish the type of influence that the loading of own weight imposes on the dome when compared to accidental loading. The following Table 1 highlights the formulations for the first case, the equations for accidental loads are similar.

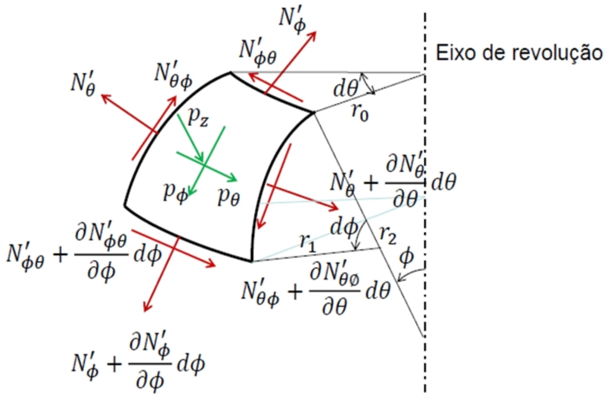

Figure 2. Differential element: membrane efforts

Fonte: Adapted from Marques, 2019.

Table 1. Analytical implementation equations

Description	Corresponding equation
Resulting from loading	$R = 2\pi a^2 g \int_0^{\phi} sin\phi d\phi = 2\pi a^2 g(1 - cos\phi)$
Normal steering effort ϕ	$N'_{\phi} = -\frac{ag}{1+cos\phi}$
Normal steering effort θ	$N'_{\theta} = ag\left(\frac{1}{1+cos\phi} - cos\phi\right)$
Horizontal displacement	$\Delta_H = \frac{a^2 g}{Eh}\left(\frac{1+v}{1+cos\phi_b} - cos\phi_b\right) sin\phi_b$
Rotation	$\Delta_{\phi} = -\frac{a^2 g}{Eh}(2+v)\, sin\phi_b$

2.3 Numerical FEM Procedures

For the initial modeling of the dome, a shell element was chosen, which allows the modeling of perfect and imperfect plates. ABAQUS includes three groups of shell elements: general purpose, thick shell and thin shell. The general purpose bark elements, as treated by Lustosa [9], are used for both thick and thin bark analysis and provide good quality solutions for most applications. These elements take into account shear deformation, which becomes very small as the thickness of the plate decreases. Thick shell elements are necessary when the consideration of transverse shear is important.

As a practical criterion Bazant and Cedolin [10]] consider that a shell is thick when its thickness is greater than $1/15$ of the smallest gap, otherwise it is considered thin. Thin-shell elements neglect the effects of transverse shear, as they are very small compared to bending and follow Kirchhoff's theory.

For this type of work, Billington [1] and Bezerra [11] recommend, for the discretization of the model, the use of the main element *S8R5*, which is a type of thin-house element that satisfies Kirchhoff's theory numerically. This type of element is quadrilateral, has eight nodes and uses reduced integration. The degrees of freedom considered are five: three displacement components and two rotation components in the plane, per node. Figure 3 highlights the characteristics of this element.

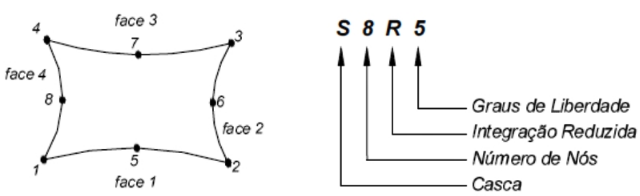

Figure 3. Element S8R5

Several shell elements in ABAQUS use reduced integration in the procedures associated with the calculation of the respective stiffness matrix. The reduced integration considerably reduces the execution time, especially in three-dimensional problems, and does not compromise the accuracy of the results obtained.

The boundary conditions assigned to the example are to lock displacements at the inner edge of the shell, allowing rotation in the direction of growth of the angle θ.

In order for the support and analysis to be reliable to the calculations performed analytically, it was necessary to create a spherical coordinate system. For this, the software requests that 3 points be defined for the spherical coordinates, where the first point concerns the radial coordinate, the second point defines the angle θ, and the third point defines the positive growth direction of θ, and consequently the direction of growth of the angle ϕ. After this stage, it can be seen that the dome supports are tilted at such an angle that they are tangent to the surface of the dome, where this fact is essential, since for analytical theory it is assumed that the supports are also tangent to the shell.

In relation to loading, it is necessary to distinguish between permanent loading and accidental loading, where the first is distributed along the dome profile as a surface load and the second defined by technical standards, is given as a projection of a horizontal load on the surface spherical.

3 Results and discussions

This section will be dedicated to the results of the spherical dome, initially expressing the comparison between analytical and numerical results with an intermediate mesh followed by the comparison of changing elements and integrations, bringing finally an analysis of mesh refinement.

To validate the results already analytically determined in Maple, we use a well-refined mesh of 2646 elements of the shell type (*S4R8*). Figure 4 expresses the shell in undisturbed condition and after applying the load with its deformed state.

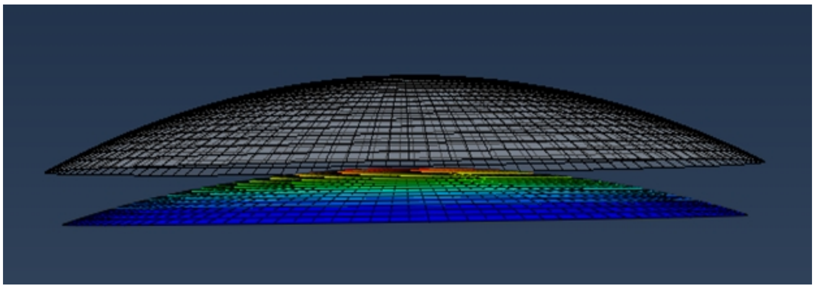

Figure 4. Dome in deformed and undeformed condition

3.1 Comparison of analytical and numerical results

Results of displacements and efforts were compared according to Table 2. For the edge of the dome, there was an error in the normal meridional effort N_θ of 0.01% between the analytical values and those calculated by the finite element method. For the center of the dome, the error was 1.29%, as can be analyzed by the values shown in Table 2. Such difference in normal effort N_θ in the center of the dome may have as one of its causes the influence of normal effort circumferential in effort N_θ, since analytical theory does not take such influence into account.

It is observed that, in order to determine the horizontal displacements in the dome, it was necessary to analyze such information through the Cartesian axis system, in such a way that the spherical coordinate system does not show displacement values in the horizontal plane. For the analysis of rotations in the dome, the spherical coordinate system was used.

The concept of adaptivity and the very essence of the finite element method are intrinsically linked to the definition of error, since the finite element method is a numerical method, that is, it solves problems in an approximate way and the idea of using adaptivity came to improve the characteristics of the meshes that had generated poor results in the analysis of finite elements.

Table 2. Results of displacements and efforts

Analysis Type	Unity	Analytical Result	Result FEM - S8R5	Error (%)
Displacement on the border	m	$9,08e^{-5}$	$9,19e^{-5}$	$1,20\%$
Normal effort N_ϕ	kN/m	$-56,94$	$-56,936$	$-0,01\%$
Normal effort $N_\theta - border$	kN/m	$-43,213$	$-43,142$	$-0,16\%$
Normal effort $N_\theta - center$	kN/m	$-55,25$	$-54,538$	$-1,29\%$
Rotation on border shell	rad	$-2,88e^{-5}$	$-2,46e^{-5}$	-14,54%

3.2 Element variation

For this analysis it was decided to vary the type and generate meshes with close amounts of finite elements comparing the displacement values obtained, thus seeing which one provides more convergent results to the real analytical value.

All elements used 3 Gauss points as a condition. The following types were used: *S4R8* - Quadrilateral shell element with 8 knots (quadratic) and reduced integration; *S3* - Triangular element with 3 knots. (Linear); *STRI65* - Quadratic triangular element with 6 knots and 5 degrees of freedom; *S4R* - Element with predominantly square 4-node Linear. Figure 5 expresses such variation.

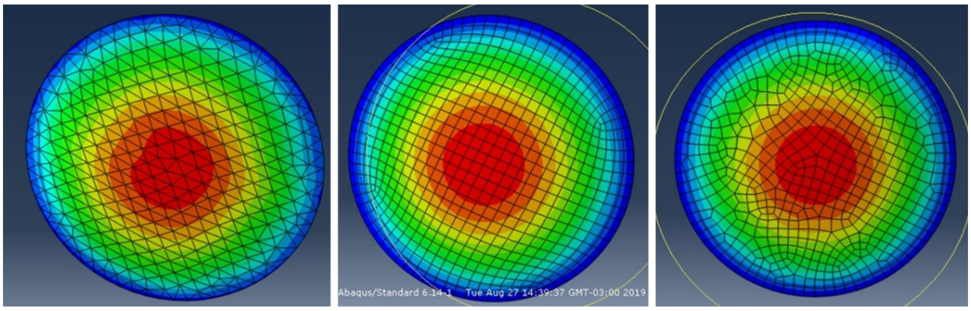

Figure 5. Mesh refinement stages

With the results described in Table 3 we realize that the dominant triangular and quadrilateral element, due to the circular shape of the piece, is able to capture a range of results with greater precision, attributed to its geometric adaptability.

Table 3. Comparative results changing element type

Element type	Displacement (m)	Rotation (Rad)	Error (displacement)	Error (rotation)
S4R8	$-9,195e^{-5}$	$-2,467e^{-5}$	$1,3\%$	$-14,39\%$
S3	$-9,146e^{-5}$	$-2,820e^{-5}$	$0,76\%$	$-2,14\%$
STRI65	$-9,188e^{-5}$	$-2,477e^{-5}$	$1,23\%$	$-14,04\%$
S4R	$-9,187e^{-5}$	$-2,481e^{-5}$	$1,22\%$	$-13,90\%$

3.3 Mesh refinement and convergence

As for the convergence study, only the opening of the mesh was varied to obtain a point indicating where the error variation was no longer noticeable or was no longer relevant. This can be seen in the Figure 6.

Table 4 summarizes the displacement data at the edge compared to the value of $-9,076e^{-5}$ *m* and the rotation of $-2,881e^{-5}$ *rad*, both calculated analytically, thus obtaining the average error in each quantity of elements.

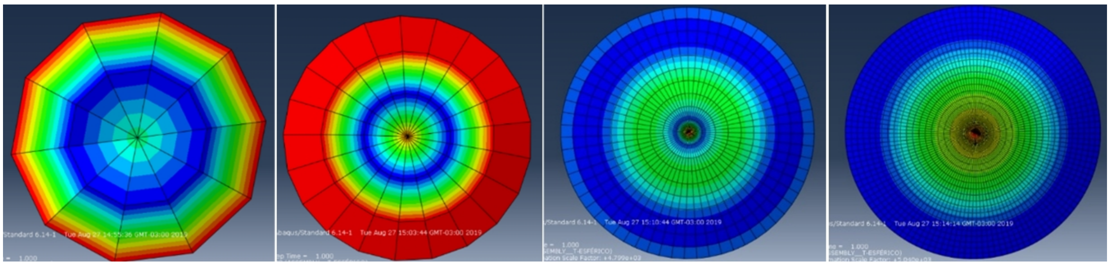

Figure 6. Mesh refinement stages

Table 4. Mesh refinement and convergence

Mesh opening	Numbers of elements	Displacement ΔH_m - (m)	Error ΔH_m	Rotation Δ_ϕ - (rad)	Error Δ_ϕ
6	20	$-0,000158$	$72,11\%$	$0,0002305$	$-1033,20\%$
3	63	$-0,0001163$	$26,69\%$	$0,00001336$	$-154,09\%$
1	630	$-0,00009442$	$2,85\%$	$-0,00002212$	$-10,45\%$
0,5	2646	$-0,00009255$	$0,82\%$	$-0,00002361$	$-4,41\%$
0,2	16014	$-0,00009194$	$0,15\%$	$-0,00002462$	$-0,32\%$
0,15	28492	$-0,0000918$	$0,00\%$	$-0,0000247$	$0,00\%$
0,1	64684	$-0,0000918$	$0,00\%$	$-0,0000247$	$0,00\%$

3.4 Analysis of reduced integration and nodal points

For this comparative analysis, two spherical dome shells were modeled with the same amount of finite elements and in one of them, the option of not using the reduced integration was marked. At the end, very approximate results were obtained, as shown in Table 5, however, it was noticed that the work processing time was slightly longer. This is justified because, according to Bathe [6], reduced integration reduces data processing without losing the quality of the results.

Table 5. Analysis of reduced integration: Element S4R8

Integration	Displacement (m)	Rotation (rad)	Error (Displacement)	Error (rotation)
Reduzida	$-9,185e^{-5}$	$-2,466e^{-5}$	$1,19\%$	$14,42\%$
Normal	$-9,263e^{-5}$	$2,449e^{-5}$	$2,05\%$	$15,01\%$

It was observed that so that the values produced by the reduced integration were closer to the analytical values. In this case, it was seen that in order to converge the values of normal integration to those obtained in reduced integration, the number of elements must be increased.

The extrapolated and smoothed results are those that make the results compatible for the meeting of each element. When the results are only extrapolated, the value curves are discontinued, as described by Assan [4]. We noticed that, due to smoothing, the values in the nodes are smaller than those measured in the Gauss points.

4 Conclusion

It was consolidated with this work that the spherical domes of reinforced concrete are complex structures and that need special structural analysis. While admitting one of a prior sensitivity of the behavior of the same and how the efforts work, it can be made stable, safe, well designed and economical.

Thus, it is confirmed that the adherence of the type of element is important for approximations with the least numbers of elements possible, however, for this type of element, triangular types with reduced integration yielded better adherence, surpassing the results of the specific element for shells and generating approximation results with minor errors in the displacement analysis.

In addition, we conclude that numerical simulations carried out using the ABAQUS software assist in the analysis and preparation of pre-dimensioning formulas for domes, making the response faster with the change in geometry and in the parameters that define the structures, with accurate and reliable results.

Type your conclusions or closing remarks here. Please be as concise and objective as possible. Do not make a "summary" of the paper, but instead list the main findings and results, even if these are only partial conclusions so far.

Authorship statement. The text should be exactly as follows: The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] Billington, D. P., 1982. *Thin shell concrete structures*. McGraw-Hill.

[2] Silva, R. L. C., 2017. *Contribuição ao projeto de reservatórios cilíndricos de concreto armado apoiados em base elástica*. PhD thesis, Universidade Federal de Alagoas, Maceió, Brasil.

[3] Timoshenko, S. & Woinowsky-Krieger, S., 1959. *Theory of plates and shells*. McGraw-Hill Kogakusha LTDA, New York.

[4] Assan, A. E., 2003. *Métodos dos Elementos Finitos: Primeiros Passos*. Editora da Unicamp, Campinas.

[5] Soriano, H. L., 2003. *Método de Elementos Finitos em Análise de Estruturas*. Editora da Universidade de São Paulo, São Paulo.

[6] Bathe, K. J., 1982. *Finite element procedures in engineering analysis*. Prentice-Hall, New Jersey.

[7] Marques, G. B., 2019. *Notas de aula da disciplina Placas e Cascas*. PhD thesis, Programa de Pós-Graduação de Engenharia Civil – Universidade Federal de Alagoas, Maceió, Brasil.

[8] Marques, S. P. C., 1982. *Finite element procedures in engineering analysis*. Prentice-Hall, New Jersey.

[9] Lustosa, I. A. A., 2011. *Um estudo comparativo analítico-numérico de esforços e deslocamentos em cascas cilíndricas abertas ou com conexões de borda*. PhD thesis, Universidade de Brasília, Distrito Federal.

[10] Bazant, P. Z. & Cedolin, L., 2010. *Stability of structures: elastic, inelastic, fracture and damage theories*. WSPC.

[11] Bezerra, P. H. A., 2013. *Análise de estruturas de superfície delgadas axissimétricas via método dos elementos finitos com utilização de elemento retilíneo*. PhD thesis, Programa de Pós-Graduação de Engenharia Civil – Universidade Federal do Rio Grande do Norte, Natal, Brasil.