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A computational optimization procedure aiding the design of vehicular suspension systems

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Abstract. Aiming at the design of formula-style vehicles, this manuscript presents a computational procedure based on optimization principles for the concept of vehicular suspension systems. The design criteria seek to find the three-dimensional location of both upper and lower ball joints of a double wishbone suspension in view of particular relationship between the camber gain and the steering angle. The studied case investigated concerns in optimize the performance of the vehicle in view of the well-known *skidpad* test. Geometric constraints imposed by components of the suspension system are also considered within the formulation, which in turns define the search space. Due to the small amount of design variables, the *Particle Swarm Optimization* (PSO) is the heuristic algorithm chosen to solve the optimization problem. The achieved results show that the proposed design criterium was reached and the constraints were fulfilled within less than one second in an ordinary laptop computer, showing the effectiveness of the numerical procedure.

Keywords: Optimization; Vehicle suspension system; Camber gain; Formula SAE.

1 Introduction

Among the several roles of the vehicular suspension system, one can emphasize those related to promote ride comfort, effective handling, driving stability and road adhesion, resulting in a safer and more comfortable vehicle. However, to conceived a suspension system that accounts for all the aforementioned requirements is not a trivial task, once in general they are conflicting in nature. Accordingly, within an optimization framework, this means that a broad range of objective functions and design variables can be proposed according to the desired vehicle behavior. This reflects the reason why such investigations remain a current research topic, where a broad range of computational optimization techniques has being investigated in order to improve the design of both active and passive suspension systems [1, 2, 3].

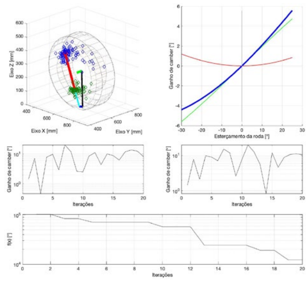
Among the different vehicle types and their applications, this study is focused on the design of formula-style ones, manly those involved with the *Formula SAE*® competition. Briefly, this is an undergraduate level competition that challenges teams around the globe to conceive, design, manufacture and compete with small formula-style vehicles [4]. An example of those vehicles is depicted in Fig. 1a. During the competition, the vehicles are tested regarding performance and efficiency in a set of events, *e.g.*: acceleration, *skidpad* and endurance. Therefore, the design criteria involved throughout the vehicle concept must aim the previous mentioned events.

Based on this, the main objective of the present manuscript is to provide a computational optimization procedure and related numerical code in order to aid the conceptual design of the suspension system of formula-style vehicles, reducing, therefore, the time spent within the project.

Since double wishbone (double A-arms) suspension types are commonly used in these vehicles, the three-dimensional positions of the upper and the lower ball joints are chosen to be the design variables. Due to the several variables and nonlinearities involved within such systems, focus is given in a subsystem of them, *i.e.*, the design criterium seeks to find a particular relationship between the total camber gain and the steering angle in order to improve the vehicle performance in view of the *skidpad* test. Moreover, geometrical restrictions imposed by other components of the suspension system are also considered within the formulation in order to achieve a feasible search space. Due to the small amount of design variables (six in the present case), the *Particle Swarm Optimization* (PSO) is chosen to solve the optimization problem [5, 6].

This study should be considered as a first step aiming the design of suspension systems, where a broad range of objective functions and more complex dynamic phenomena could be included in order to improve the algorithm prediction. Moreover, for the interested ones – mainly those who participate of the *Formula SAE*® competition - the *Matlab*® code developed from this study is freely available for download by the QR code shown in Fig. 1b.

a)



b)

Figure 1. a) Example of a small formula-style vehicle conceived for the *Formula SAE*® competition (*Fórmula Uno* team, 2018 prototype). b) QR code to download the *Matlab*® algorithm developed from this study.

2 Methods

2.1 Geometric analysis of the suspension system

As highlighted previously, this work intends to provide a simplified optimization strategy aiming the conceptual design of suspension systems. Therefore, the analysis approached herein only considers the camber gain due to the steering of wheels. Based on this, and following the developments of [7], the total camber gain γ during steering could be defined as the sum of the contributions provided by the caster angle and the king pin inclination (KPI), *i.e.*,

$$\gamma \overset{\text{def}}{=} \gamma_{caster} + \gamma_{kpi} \quad \Rightarrow \quad \begin{cases} \gamma_{caster} \overset{\text{def}}{=} \cos^{-1}[\sin(\theta_c)\sin(\theta_s)] - 90° \\ \gamma_{kpi} \overset{\text{def}}{=} \theta_k + \cos^{-1}[\sin(\theta_k)\sin(\theta_s)] - 90° \end{cases} \tag{1}$$

where γ_{caster} is the camber gain due to the caster and γ_{kpi} is the camber gain due to the king pin inclination. The variables θ_c, θ_k and θ_s are the caster, the king pin and the steering angles (in degrees), respectively.

In the present investigation, the three-dimensional positions of the upper $\mathbf{p}_\mathrm{u}$ and the lower $\mathbf{p}_\mathrm{l}$ ball joints of a double wishbone suspension type are chosen to be the design variables (see Fig. 2c and further discussion in Section 2.2). In this regard, Eq. (1) was parametrized in function of $\mathbf{p}_\mathrm{u}$ and $\mathbf{p}_\mathrm{l}$ in order to provide the following functional form: $\gamma = \gamma(\mathbf{p}_\mathrm{u}, \mathbf{p}_\mathrm{l})$. Such parametrization is performed in view of intricate geometric equations, where technical details can be found in [8]. This particular parametrization was performed since the function γ is used to define the objective function. This issue is better explored in the sequence.

2.2 Computational optimization procedure

This section describes the technical details related to the computational optimization procedure employed in this work. Moreover, the particular case investigated and related design variables, objective function and constraints are also discussed.

Remark 1: Optimization problem and solution algorithm. A general optimization problem could be formally presented as a variational principle:

$$\mathbf{x}^{\mathrm{opt}} = \underset{\mathbf{x} \in \mathcal{S}}{\operatorname{argmin}} f(\mathbf{x}) \tag{2}$$

where $\mathbf{x}^{\mathrm{opt}}$ is the argument the minimizes a given objective function $f(\mathbf{x})$ subjected to constraints $\mathcal{S}$. The space $\mathcal{S}$

is usually called the *search space*, and it constrains the possible values the design variables $\mathbf{x}$ are allowed to assume. In other words, all the problem constraints are included within the search space $\mathcal{S}$, *i.e.*, side, inequality and equality ones [9].

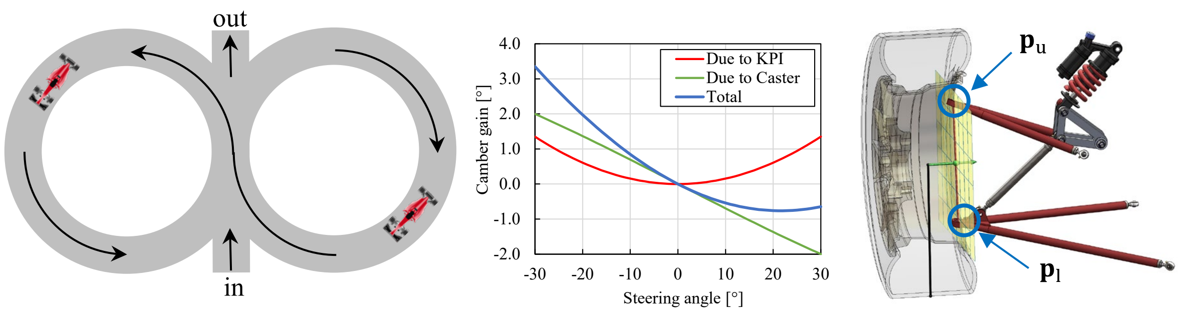
Due to the small amount of design variables (six in the present case, see *Remark 2*), the *Particle Swarm Optimization* (PSO) is chosen to solve the optimization problem [5, 6]. This method has been proven to be a sound heuristic algorithm, avoiding local minima and presenting considerable convergence rates for different optimization problems. The variant of the PSO algorithm used in the present study is that implemented within the *Matlab®* library *Global Optimization Toolbox*. In the present approach, the constraints included into the space $\mathcal{S}$ are taking into account by the classical *Penalty Method* (see [9] for further details in this regard).

Remark 2: Studied case, design variables and objective function. The case under consideration concerns in optimize the performance of a small formula-style vehicle in view of the *skidpad* test (Fig. 2a). This is a well-known test in the automotive field, where important information regarding both the vehicle performance and safety can be retrieved. Moreover, in view of the *Formula SAE®* competition [4], this test consists of a relevant part of the event. Looking for a better performance in this test, the proposed objective function relies on the sum of two separated functions, *i.e.*:

$$f(\mathbf{x}) \stackrel{\text{def}}{=} w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}); \qquad f_1(\mathbf{x}) \stackrel{\text{def}}{=} \left|\gamma_{left} - \bar{\gamma}_{left}\right|; \qquad f_2(\mathbf{x}) \stackrel{\text{def}}{=} \left|\gamma_{right} - \bar{\gamma}_{right}\right| \tag{3}$$

where f_1 and f_2 are the absolute values of the difference between the calculated total camber gain γ and a prescribed one $\bar{\gamma}$ for both the left and right wheels (Fig. 2b). The variables w_1 and w_2 are weighting factors chosen heuristically in order to prevent one function prevails upon the other in the objective function. In the present investigation, the design variables $\mathbf{x}$ rely on the three-dimensional location of the upper $\mathbf{p}_\mathrm{u}$ and the lower $\mathbf{p}_\mathrm{l}$ ball joints, which is defined by the set $\mathbf{x} \stackrel{\text{def}}{=} \{\mathbf{p}_\mathrm{u}\ \mathbf{p}_\mathrm{l}\}$ (Fig. 2c).

In order to improve the road adhesion during the *skidpad* test, the following values are assumed in the present investigation: $\bar{\gamma}_{left} = 1.63°$ and $\bar{\gamma}_{right} = -1.32°$ for steering angles of $-30°$ and $25°$, respectively.



a) b) c)

Figure 2. a) Schematic representation of the s*kidpad* test employed in the *Formula SAE®* competition. b) The objective function is defined based on particular relationships between the total camber gain (blue curve) and the steering angle for both the left and right wheels. c) Three-dimensional view of the suspension system and wheel showing the design variables*, i.e.*, the positions of the upper $\mathbf{p}_\mathrm{u}$ and the lower $\mathbf{p}_\mathrm{l}$ ball joints.

Remark 3: Search space. In the present optimization problem, the search space depends on the three-dimensional free volume provided by the arrangement and geometry of the components of the suspension system. Based on this, the search space $\mathcal{S}$ is formally defined by

$$\mathcal{S} \stackrel{\text{def}}{=} \{\, \mathbf{x} \stackrel{\text{def}}{=} \{\mathbf{p}_\mathrm{u}\ \mathbf{p}_\mathrm{l}\};\ \ \mathbf{p}_\mathrm{u}, \mathbf{p}_\mathrm{l} \in \mathcal{R}^3 \ \mid\ \mathbf{x} \in \mathcal{K} \} \tag{4}$$

where $\mathcal{R}^3$ is the three-dimensional *Euclidian* space and $\mathcal{K}$ represents the space resulting from the *Boolean* operation (intersection) among the geometric constraints that surround the ball joints (Fig. 3). It is important mentioning that space $\mathcal{K}$ varies from system to system, once it depends on the geometry of the components of the suspension system. Moreover, additional ranges for the caster angle (0º to 5º), king pin inclination (0º to 5º), caster trail (10 to 20 mm) and scrub radius (10 to 40 mm) are also considered within the space $\mathcal{K}$ in order to achieve a feasible search space.

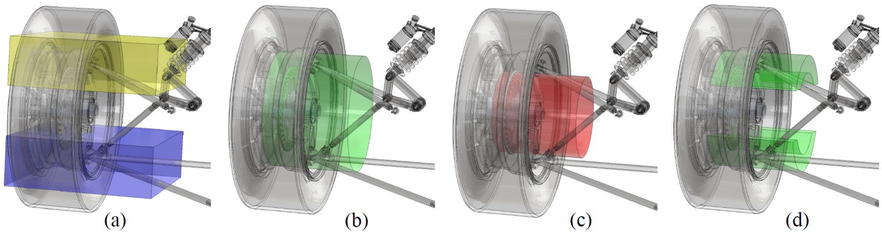


Figure 3. Geometrical constraints within the wheel. The *Boolean* operation (intersection) among the volumes (a), (b) and (c) defines the search space (d).

3 Results and discussion

The first step of this work consisted in verifying the accuracy of the analysis algorithm discussed in Section 2.1. This was performed comparing the results provided by the mentioned algorithm to those retrieved from the commercial software *Lotus Suspension Analysis®* for the same input data. It was verified that the results achieved for both codes are nearly identical, validating the implemented analysis algorithm. Further technical details about this particular verification can be found in [8].

The computational optimization procedure described in Section 2.2 was run with a swarm of 100 particles, and the results are discussed as follows. In an ordinary laptop computer, the optimization algorithm took less the one second to achieved a normalized objective function in the order of magnitude of 10^{-13} in about 130 iterations, being this considered the converged point. The design variables (upper and lower ball joints) at the convergence are given by: $\mathbf{p}_\mathrm{u} = [639{,}3706;\ 592{,}3901;\ 429{,}9799]$ mm and $\mathbf{p}_\mathrm{l} = [621{,}3020;\ 593{,}9692;\ 105{,}4320]$ mm. Based on these values, the optimized geometries of the suspension system under investigation are given by: scrub radius of 30.5177 mm; caster trail of 14.5591 mm; caster angle of 3.1865º; king pin inclination of 0.2788º. Based on these data, one can verify that all the imposed constraints were indeed fulfilled by the optimization procedure (see *Remark 3* of Section 2.2). These results clearly demonstrate the effectiveness of the present optimization strategy.

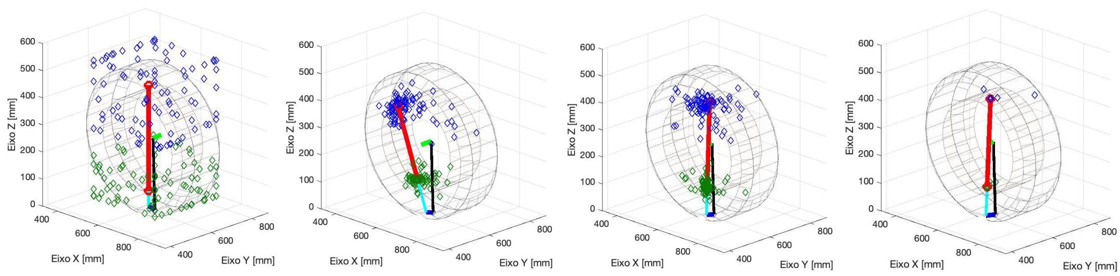
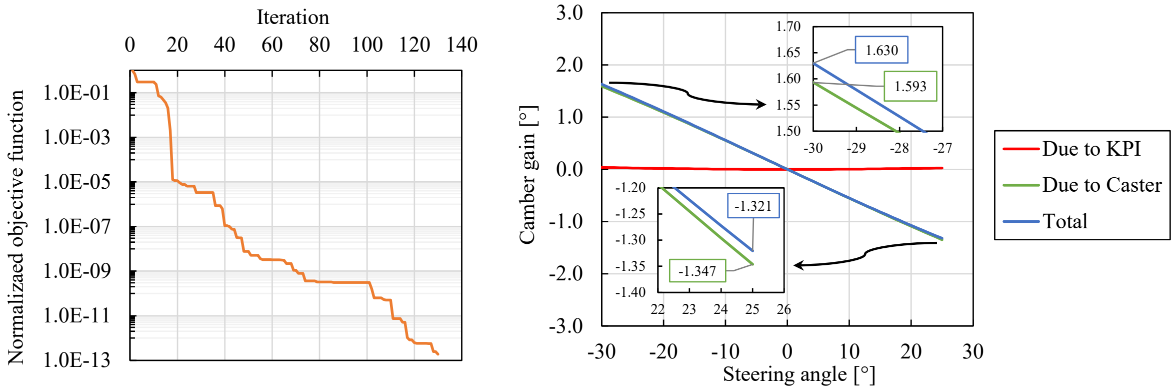


Figure 4. Dynamics of the particle swarm seeking for the best spatial location for the upper and lower ball joints. From left to right, the sketches correspond to the following iterations: 0, 20, 40 and 80, respectively.

The dynamics of the swarm throughout the optimization procedure is illustrated in Fig. 4 for the following iterations: 0, 20, 40 and 80. Moreover, the history of the normalized objective function is plotted in Fig. 5a. Discussion regarding these data are twofold. Firstly, one can see that, up to 30 iterations, the objective function decreases at a high rate followed by a nearly constant decay rate until the converged point. This behavior reflects the high convergence rate characteristic generally provided by the PSO algorithm [10]. Secondly, even though a normalized value of 10^{-13} was defined as the convergence threshold, it was verified that the total camber gain at iteration 80 shows no further improvements on the design criterium proposed and, therefore, a higher stop value is enough in this case. This emphasize the importance of a real-time observation on how the objective function evolves throughout the optimization procedure, manly when heuristic algorithms are used, since is well-known that no formal mathematical criteria define the convergence of such class of algorithms.

Concerning the particular case investigated, one can see in Fig. 5b that the optimum geometry of the suspension is mainly ruled by the camber gain due to the caster. Nevertheless, the small KPI of 0.2788° resulting from the optimization procedure becomes important to exactly achieve the proposed objective function. This is highlighted in the inside graphs of Fig. 5b.



a) b)

Figure 5. a) History of the normalized objective function (logarithm scale) throughout the optimization process. b) Camber gain due to starring angle at the convergence point (iteration 130).

4 Conclusions

This work presented a computational optimization procedure in order to assist the design of vehicular suspension systems. Aiming at the *Formula SAE*® competition, the particular case investigated concerns in optimize the performance of a small formula-style vehicle in view of the *skidpad* test. The results show that all the proposed design criteria were achieved within less than one second in an ordinary laptop computer, showing the effectiveness of the numerical procedure. This study should be considered as a first step aiming the design of suspension systems, where other objective functions and more complex dynamic phenomena could be included in order to improve the algorithm prediction. Moreover, for the interested ones – mainly those who participate of the *Formula SAE*® competition - the *Matlab*® code developed from this study is freely available for download by the QR code shown in Fig. 1b.

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References

[1] A. Baumal, J. McPhee and P. Calamai, "Application of genetic algorithms to the design optimization of an active vehicle suspension system," *Computer Methods in Applied Mechanics and Engineering,* vol. 163, 1998.

[2] E. Grotti, D. M. Mizushima, A. D. Backes, M. D. d. F. Awruch and H. M. Gomes, "A novel multi-objective quantum particle swarm algorithm for suspension optimization," *Computational and Applied Mathematics,* vol. 39, no. 105, 2020.

[3] L. Zhang, J. Liu, F. Pan, S. Wang and X. Ge, "Multi-objective optimization study of vehicle suspension based on minimum time handling and stability," *Journal of Automobile Engineering,* vol. 234, no. 9, 2020.

[4] "Formula SAE Rules," 2020. [Online]. Available: https://www.fsaeonline.com/cdsweb/gen/DocumentResources.aspx.

[5] J. Kennedy and R. C. Eberhart, "Particle swarm optimization," in *Proceedings of IEEE International Conference on Neural*, 1995.

[6] M. E. H. Pedersen, "Good Parameters for Particle Swarm," Hvass Laboratories, 2010.

[7] D. Seward, Race car design, Macmillan International Higher Education, 2014.

[8] A. V. Farias, "Computational optimization applied to the conceptual design of the suspension of a Formula SAE vehicle (in portuguese)," Undergraduate Monograph, Chapecó, 2020.

[9] J. S. Arora, Introduction to optimum design, Elsevier, 2004.

[10] C. E. L. S. J. Vaz Jr M., "Particle swarm optimization and identification of inelastic material parameters," *Engineering Computations,* vol. 30, no. 7, p. 936–960, 2013.