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Topology optimization of 2D truss structures using the BESO method

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Abstract. Topology optimization is the process of determining the optimal material layout within a given design domain, for a set of loads and boundary conditions. It is an important engineering tool that allows the design of lighter structures with improved strength. Among the many optimization techniques, the Bidirectional Evolutionary Structural Optimization (BESO) method is a robust and computational efficient algorithm that can be applied in designing optimized structures. In this work, the BESO method was applied to the design of optimized 2D truss structures. From a group of reference numerical examples (Michell Structures), the method capability to achieve optimal solutions was researched. These case studies were also evaluated on the manufacturability of the final topologies to assess if the BESO method can be used in structural designs of steel framework structures.

Keywords: Topology Optimization, Bidirectional Evolutionary Structural Optimization, BESO, Truss Structures, Michell Structures.

1 Introduction

Truss structures are widely used in many engineering applications because they allow the construction of strong structures with low weight. Some constructions, such as transmission towers, would not be possible without trusses because they would collapse under their own weight. So, techniques of topology optimization are in constant development, with new solutions and methods.

In order to produce a well design with high performance and capable of supporting high loads and stresses with low structural displacements, topology optimization techniques, such as the Bi-directional Evolutionary Structural Optimization Method (BESO), are currently in development to solve these problems. The BESO method, developed by Xie and Huang [1], adds and removes material from the structure based on the element sensitivity to achieve an optimized design.

Structural optimization generates new structures with good performance to solve various limitations of mechanical projects. These structures permit to explore a structural design with high-performance and low-cost, so it can be technologically competitive according to Xie and Huang [1].

Michell Structures according to Lewiński et al. [2] were the first studies focused on stress optimization of trusses. They are designed with analytical solutions and provide good results that serve as benchmark references for topology optimization codes. Michell Structures, based on the small deformation theory of bars, present the best optimization results and they were used in this work as reference results.

Recent works of truss structures optimization were developed by He et al. [3], which it was applied a numerical layout optimization employing an adaptive member addition. This solution proposes an efficient means of generating optimum trusses structures. Tomšič and Duhovnik [4] implemented the evolutionary structural optimization for simultaneous topology and size optimization of 2D and 3D trusses. Lüdeker and Krigesman [5] applied a multi-model approach in which the fail-safe requirement is an optimization constraint including redundancies to solve truss optimization problems.

The objective of this article is the implementation of the BESO algorithm to optimize 2D truss structures.

This in-house algorithm was developed and tested using MATLAB.

2 Topology Optimization Problem Formulation and Implementation

This work aims to obtain a final structure with maximum stiffness limited by a desired volume. So, the optimization problem objective is to minimize the structure compliance:

$$\begin{cases} \min \quad C = \frac{1}{2} u^T K u, \\ s.t. \quad Ku = F, \\ \qquad V_F - \sum_{i=1}^{n} V_i x_i = 0, \\ \qquad x_i = x_{\min} \;\; or \;\; 1. \end{cases} \tag{1}$$

where u, K and F are the displacement vector, the stiffness matrix and the load vector, respectively. V_f is the desired volume, V_i is the volume of a single element and x_i is the design variable. The design variable is used to represent an active or inactive element in the structural problem and it is limited by a very small value ($x_{min} = 0{,}001$ – inactive element) or 1 (active element).

This work represents an inactive element by implementing a material penalization scheme which is also widely used in the SIMP method by M. P. Bendsoe and O. Sigmund [6]. This method represents the Young's modulus of an element based on its density, according to the following equation:

$$E(x_i) = E_0 x_i^3 \tag{2}$$

where E_o is the Young's modulus of the solid material.

The addition and removal of elements is based on the element sensitivity, where elements with the smallest sensitivity values are removed from the structure, while the highest sensitivities are added. The sensitivity number (α_i^e) of the elements can be defined as

$$\alpha_i^e = \frac{\left(\frac{1}{2} u_i^T K_i u_i \right)}{V_i} \tag{3}$$

where u_i is the displacement vector of the ith element, K_i is the stiffness matrix of the ith element and V_i is the volume of the ith element.

The optimization process ends when the desired volume is reached and the objective function stabilizes. This last criterion is evaluated by the following expression:

$$error = \frac{\left| \sum_{i=1}^{N} C_{j-i+1} - \sum_{i=1}^{N} C_{j-N-i+1} \right|}{\sum_{i=1}^{N} C_{j-i+1}} \le \tau \tag{4}$$

where the structure converges if $error \le \tau$. N is an integer number (N=5 for this work), j is the current iteration number, τ is an allowable convergence tolerance and C is the compliance. Fig. 1 shows the complete BESO algorithm implemented for this work.

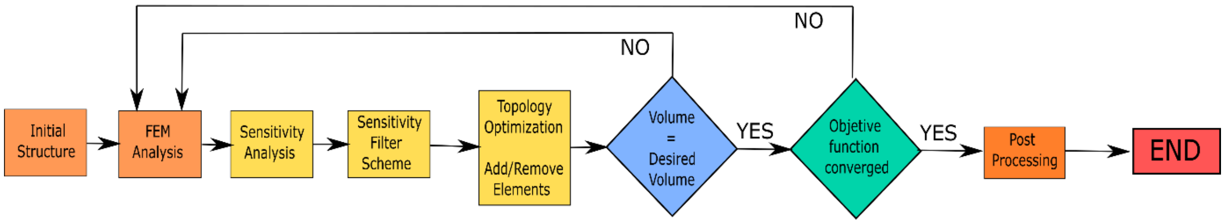

Figure 1. Topology optimization algorithm.

2.1 Initial structure problem

The initial structure for all the examples implemented in this work is a full structural domain comprised of a group of trusses organized in a square fashion with a middle node (Fig. 2). So, the problem can be defined by the number of nx groups in the X axis and ny groups in the Y axis

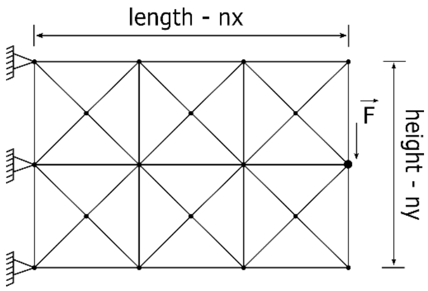

Figure 2. Example of an initial structure. In this problem nx=3 and ny=2.

2.2 Michell problem formulation

The Michell problem used to validate the implementation is the Cantilever beam (Fig. 3a) and the half-plane (Fig. 3b). The problem dimensions are presented by Fig.3 and the element properties are the following: cross section area of 0.000025m, Young's modulus of 200 Gpa and point load of 100 N for all cases.

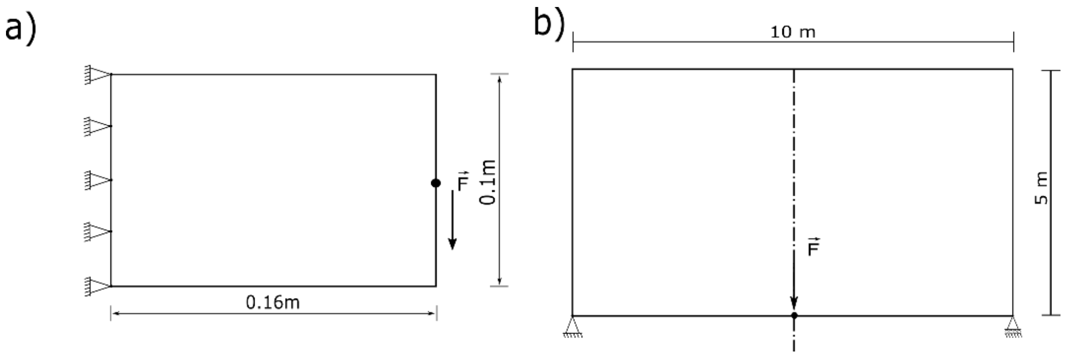

Figure 3. Evaluated optimization problems: a) Cantilever beam problem; b) Half-plane problem.

2.3 Results

The cantilever beam results are presented in Fig. 4 for different volumes. The final topology can be compared with Michell's Cantilever design with shows a similar structural topology. It was adopted for this simulation an evolution rate (ER) of 1%, a maximum element addition rate (AR) of 5%, a convergence tolerance (τ) of 0.01% and the desired objective volume is 50%. Tab. 1 shows the values of volume and compliance for some iterations.

Table 1. Cantilever: Compliance and Volume for some iteration steps of the optimization.

Iteration	Figure 4	Volume (%)	Compliance (N.m)
1		99	3.5580e-04
10	a	90	3.6008e-04
30	b	70	4.3602e-04
50	c	50	5.7014e-04
74	d	50	5.8218e-04

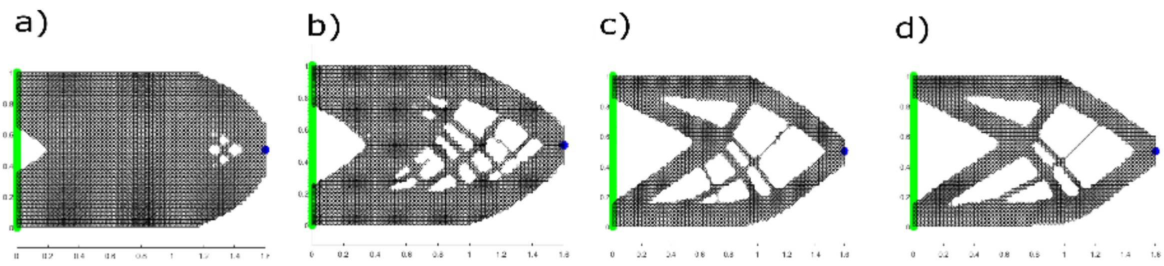

Figure 4. Cantilever topology optimization evolution

Fig. 5 shows the cantilever evolution histories of the volume and the mean compliance. The final topology achieved is also presented.

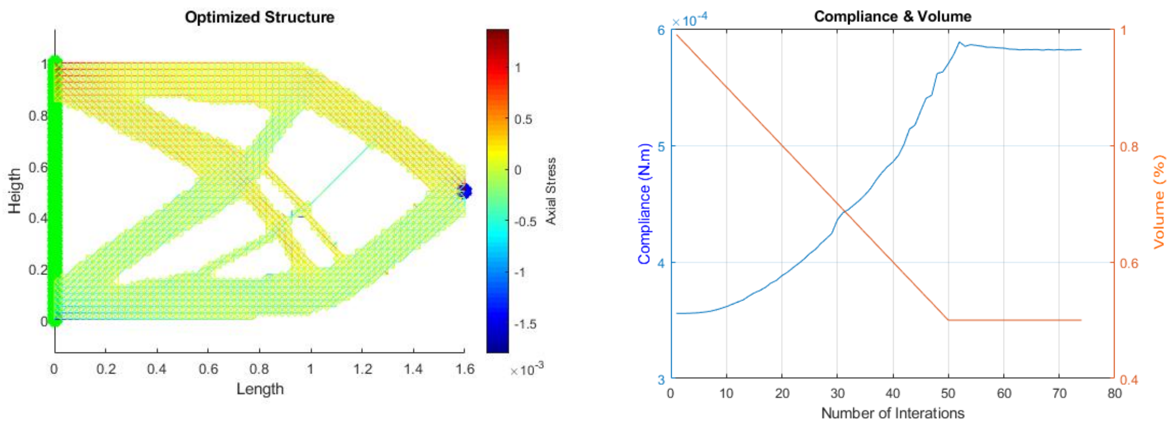

Figure 5. Cantilever: Optimized Structure with axial stresses and evolution of compliance and volume.

The half-plane results are presented in Fig. 6 with similar results to Michell’s half-plane design. It was adopted for this simulation an evolution rate (ER) of 1%, a maximum element addition rate (AR) of 5%, a convergence tolerance (τ) of 0.01% and the desired objective volume is 30%. Tab. 2 shows the values of volume and compliance for some iterations.

Table 2. Half-plane: Compliance and Volume for some iteration steps of the optimization.

Iteration	Figure 6	Volume (%)	Compliance (N.m)
1		99	9.6939e-04
10	a	90	9.8109e-04
30	b	70	0.0011
60	c	40	0.0018
84	d	30	0.0028

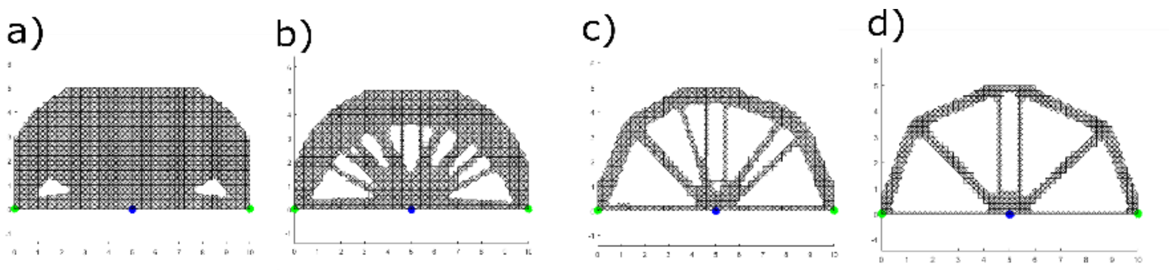

Figure 6. Half-plane topology optimization evolution

Fig. 7 shows the half-plane evolution histories of the volume and the mean compliance. The final topology achieved is also presented.

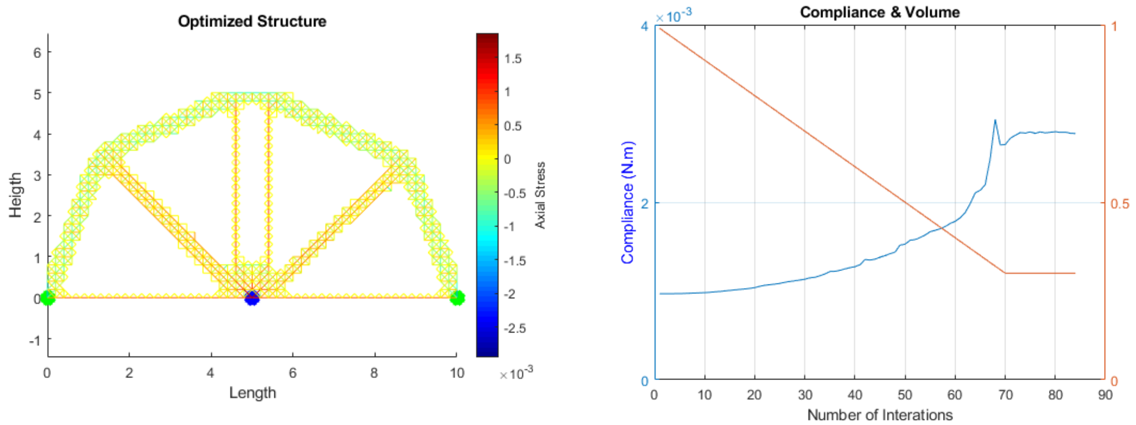

Figure 7. Half-plane: Optimized Structure with axial stresses and evolution of compliance and volume.

3 Conclusions

The objective of this work is to apply the BESO method to solve truss optimization problems. The benchmark problems were compared with Michell Structures design and they presented similar topologies. Some problems encountered during the development of this work war related to symmetry results, which were probably caused by the sensitivity filter scheme. Although these preliminary results are promising, further research will be aimed to develop and test new methods for the sensitivity filter scheme and also to modify the implementation from truss elements to frame elements.

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