Comparative Study of Different Numerical Alternatives for Modeling Two-Phase Flows in Naturally Fractured Reservoirs

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Abstract. Two-phase flows of water and oil in heterogeneous and anisotropic porous media can be described by a system of nonlinear partial differential equations that comprises an elliptic pressure equation and a hyperbolic saturation equation coupled through the total velocity field. These problems are difficult to model, due to physical and geometric complex features of oil reservoirs, which can include fractures, inclined layers and directional wells. In such cases, it is important to use formulations that can deal with full permeability tensors on unstructured grids. Here, we present a comparative study of different numerical formulations in the context of modeling two-phase flows in naturally fractured reservoirs. Our simulation tool incorporates different Finite-Volume Methods with Multi-Point Flux Approximation (MPFA) schemes to obtain implicitly the solution of the pressure equation. Among the different MPFA approaches we used some non-conventional alternatives proposed by our research group, namely: MPFA with a diamond stencil (MPFA-D); MPFA with a quasi-local stencil (MPFA-QL); MPFA with harmonic points (MPFA-H) and the Nonlinear Finite Volume Method that Preserves Positivity (NLFV-PP). A Hybrid-Grid Method (HyG) was employed to model the fractures and the classical First Order Upwind Method (FOUM) was adopted to solve the saturation equation applied in its implicit version.

Keywords: Two-phase flows of oil and water, Heterogeneous and anisotropic reservoirs, Naturally fractured reservoirs, Hybrid-grid method, Linear and Non-Linear Finite Volume Methods.

1 Introduction

In the last few years, naturally fractured reservoirs have received increasing attention by the oil industry because most of the remaining exploitable fields are estimated to be in such reservoirs [1]. The objective of this paper is to present a comparative study of some non-orthodox locally conservative formulations for the numerical simulations of single and two-phase flows in naturally fractured reservoirs. The solution of the two-phase problem is performed by a segregated Sequential Implicit (SEQ) approach. To solve the elliptic pressure equation we have used proper adaptations of different linear Finite Volume Methods with Multi-Point Flux Approximation (MPFA) and a Non Linear Finite Volume method that preserves the positivity of the numerical solution [2–5]. The hyperbolic saturation equation is solved implicitly using the classical First Order Upwind Method (FOUM) [6]. The treatment of the fractures is made by the Hybrid-Grid Method [6–8]. The hybrid-grid methods (HyG) represents fractures explicitly as (n-1)-dimensional cells in a n-dimensional domain (for example, as 1-D lines in 2-D domains). In this case, the elements faces of the mesh must fit the positions of the fractures. Later, these fractures are expanded to n-D in the computational domain and the equations for the fractures and the rock matrix are discretized together, which makes the HyG method more adequate to take full advantage from full pressure support methods [6]. Among the different existing MPFA approaches, we have used some non-conventional and recently proposed finite volumes methods: the MPFA-D [2,3], which has the "diamond" stencil and is capable to solve diffusion problems in heterogeneous and anisotropic media using general polygonal meshes; the MPFA-QL [4,5], which has a "quasi-local" stencil and is inspired by a set of non-linear methods used to solve diffusion problems in strongly anisotropic and heterogeneous media; the MPFA-H [4], which uses the so called "harmonic points" that were initially proposed to solve diffusion problems in heterogeneous and anisotropic media over any polygonal mesh; and finally the Non-Linear Finite Volume Method that Preserves Positivity (NLFV-PP) [4], which is a locally conservative method, that preserves the positivity of numerical solutions. The analyzed formulations are compared through the solution of some problems found in the literature.

2 Mathematical model

For the modeling a two-phase flow of water and oil in oil reservoirs, the mass conservation equation and the Darcy's Law are manipulated in such a way that the flow can be described by an elliptical pressure equation, Eq. (1), and a hyperbolic saturation equation, Eq. (2) coupled by the velocity field. We have assumed that the fluids, water and oil, and the rock are incompressible and that the flow is isothermal, in a fully saturated medium. Furthermore, the effects of gravity and capillarity were neglected.

$$\vec{\nabla} \cdot \vec{v} = Q \ \ (with\ \vec{v} = -\lambda K \vec{\nabla} p) \tag{1}$$

$$\phi \frac{\partial S_w}{\partial t} + \vec{\nabla} \cdot (f_w \vec{v}) = Q_w \tag{2}$$

In Eq. (1) we have $\vec{\nabla}$ as the gradient operator, $\vec{v} = \vec{v}_w + \vec{v}_o$ is the total velocity and the specific total flow rate that is $Q = Q_w + Q_o$, K is the absolute permeability tensor, $\lambda = \lambda_w + \lambda_o$ is the total mobility and p is the total pressure, with subscript o, w refers to oil and water, respectively. In Eq. (2) ϕ is the porosity, S_w is the water saturation, and $f_w = \lambda_w/\lambda$ is the fractional flow of water. Equations (1) and (2) are coupled by the total velocity field $\vec{v}$. Finally, in order to get a complete description of the problem, it is necessary to define appropriate initial and boundary conditions.

3 Numerical formulation

In this work, we have used a segregated Sequential Implicit Scheme (SEQ), in which the saturation equation was solved using the FOUM, in its implicit version [6], and the pressure equation was solved by several strategies. Since we are using finite volume methods, the discretization of the pressure equation is performed by integrating Eq. (1) into a control volume (CV) and by using the divergence theorem; we have:

$$\int_{\Gamma_i} \vec{v} \cdot \vec{n}\, d\Gamma_i = \int_{\Omega_i} Q\, \partial\Omega_i \Rightarrow \sum\nolimits_{IJ \in \Gamma_i} \vec{v}_{IJ} \cdot \vec{N}_{IJ} = \bar{Q} \tag{3}$$

where $\vec{n}$ is the outward unitary normal vector to the control surface Γ_i, Ω_i is the CV volume and $\bar{Q}$ is the mean source term in it. In 2-n, $\vec{N}_{IJ}$ is the outward area vector, and $\vec{v}_{IJ}$ the velocity vector for the edge IJ. In equation (3), the flow rate $\vec{v}_{IJ} \cdot \vec{N}_{IJ}$ can be approximated in many ways, each one leading to different schemes. In case of MPFA-D (Multipoint flux approximation with diamond stencil) scheme the flow rate $\vec{v}_{IJ} \cdot \vec{N}_{IJ}$ is computed by:

$$\vec{v}_{IJ}.\vec{N}_{IJ} = \tau_{IJ}\left[p_{\hat{R}} - p_{\hat{L}} - \upsilon_{IJ}\left(p_J - p_I\right)\right] \tag{4}$$

where $p_{\hat{R}}$ and $p_{\hat{L}}$ are the pressures on the collocation points of the CVs sharing the edge IJ, p_I and p_J are the pressures on the vertices comprising the edge IJ (to be interpolated through a linear preserving weighting strategy), τ_{IJ} is the equivalent transmissibility term and υ_{IJ} is the cross-diffusion term [2,3]. On the other hand, if we use the MPFA-H (Multipoint flux approximation using harmonic points) the flow rate $\vec{v}_{IJ} \cdot \vec{N}_{IJ}$, is given by:

$$\vec{v}_{IJ}.\vec{N}_{IJ} = A_{\hat{L},IJ}p_{\hat{L}} - A_{\hat{R},IJ}p_{\hat{R}} + \sum_{\gamma=i,j} D_{\hat{R},\gamma(IJ)}p_{\hat{R},\gamma(IJ)} - D_{\hat{L},\gamma(IJ)}p_{\hat{L},\gamma(IJ)} \tag{5}$$

where $p_{\hat{R},\gamma(IJ)}$ and $p_{\hat{L},\gamma(IJ)}$ are the auxiliary variables localized in the harmonic points, see Fig. 1 (center). And $A_{\hat{L},IJ}$, $A_{\hat{R},IJ}$, $D_{\hat{R},\gamma(IJ)}$ and $D_{\hat{L},\gamma(IJ)}$ are physical-geometrical parameters [5]. When we use the MPFA-QL (Multipoint Flux Approximation with Quasi-Local stencil) scheme, the flow rate is similar to Eq. (5) but, in this case, the $P_{\hat{R},\gamma(IJ)}$ and $P_{\hat{L},\gamma(IJ)}$ are the auxiliary variables localized in the vertices that do not necessarily belong to the same edge shared by the adjacent cells, obtained using the co-normal vector [4,5], see Fig. 1 (right). Finally, if we use the NLFV-PP (Positivity-preserving Non-linear finite volume) scheme the flow rate on the edge IJ is approximated by:

$$\vec{v}_{IJ}.\vec{N}_{JI} = A_{\hat{R},IJ}(p)p_{\hat{R}} - A_{\hat{L},IJ}(p)p_{\hat{L}} \tag{6}$$

where $A_{\hat{R},IJ}(p)$ and $A_{\hat{L},IJ}(p)$ are parameters involving the pressure localized in the vertices and physical - geometrics arguments [4], leading to a non-linear formulation.

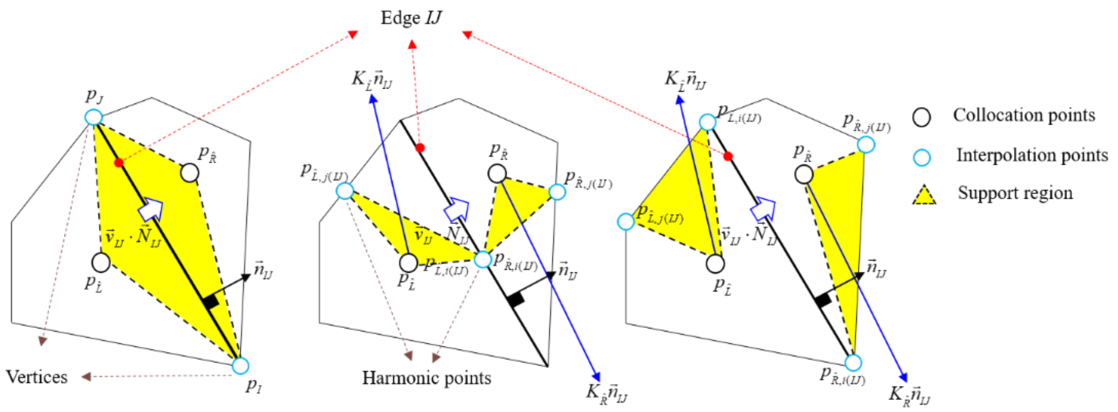

Figure 1. Some physical-geometric parameters and the support region: MPFA-D (left), MPFA-H (center), MPFA-QL and NLFV-PP (right).

4 Adaptation for fractures

The adaptation of these methods to deal with fractures is made by using the Hybrid-Grid Method (HyG), which consists of, geometrically, representing the fractures explicitly, as (n-1)-D entities (edges of the original mesh) and expanding them to n-D in computational domain, so that equations for fractures and for the rock matrix can be discretized together [6–8]. Considering the original grid (Fig. 2a), in the vicinity of each fracture-edge ($\hat{f}_1$ and $\hat{f}_2$, in Fig. 2), two parallel straight auxiliary lines must be drawn, each one at the distance of half-aperture of that fracture from the edge. The intersection of these auxiliary lines will generate the new points (I' and I'' in Fig. 2b) that will form the expanded fracture polygonal cell in the computational grid (Fig. 2b). A more detailed description of this procedure can be found in the work of Cavalcante et al. [6].This allows a straight extension of the MPFA and NLFV schemes to deal with fractured reservoir

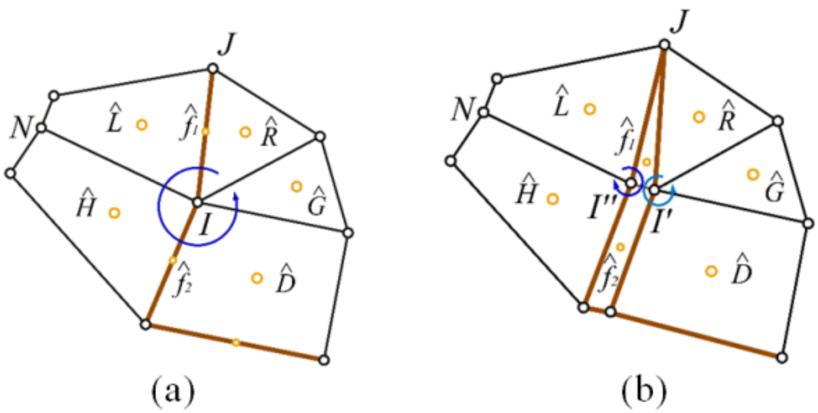

Figure 2. Hybrid-Grid Construction: (a) the original mesh; (b) the mesh with expanded fractures.

5 Results

The results obtained in one of the tests performed are presented below. Figure 3 shows the pressure and saturation fields for the problem of the two-phase flow in a reservoir with a ¼ five-spot configuration. The domain is defined as [0.1] x [0.1] and it is discretized by a 454 triangular mesh. Considering a dimensionless model, the pressure in the injector well is $p_I = 1$ and in the producer well it is $p_P = 0$ and the flow is zero throughout the domain external boundaries. The water saturation is set to 1 in the injector well and 0, initially, in the rest of the domain. The viscosity of water is 1 and the viscosity of oil is 0.45. The permeability of the rock matrix (K_m) is such that $K_{xx} = K_{yy} = 2$ and $K_{xy} = K_{yx} = 1$, while the fracture permeability is given by $10^4 K_m$. Finally, the fracture aperture is 10^{-3}. The saturation equation was solved implicitly with CFL = 4.

The simulation was performed using the MPFA-D, MPFA-QL, MPFA-H and NLFV-PP formulations in the context of the HyG method. In Figure 3, it was not possible to observe significant differences among the results obtained with the different methods. However, in Figure 4, which shows the production report, it is possible to observe some differences for the water cut and the cumulative oil. The MPFA-H method predicts a higher oil production, as a result of a later watercut.

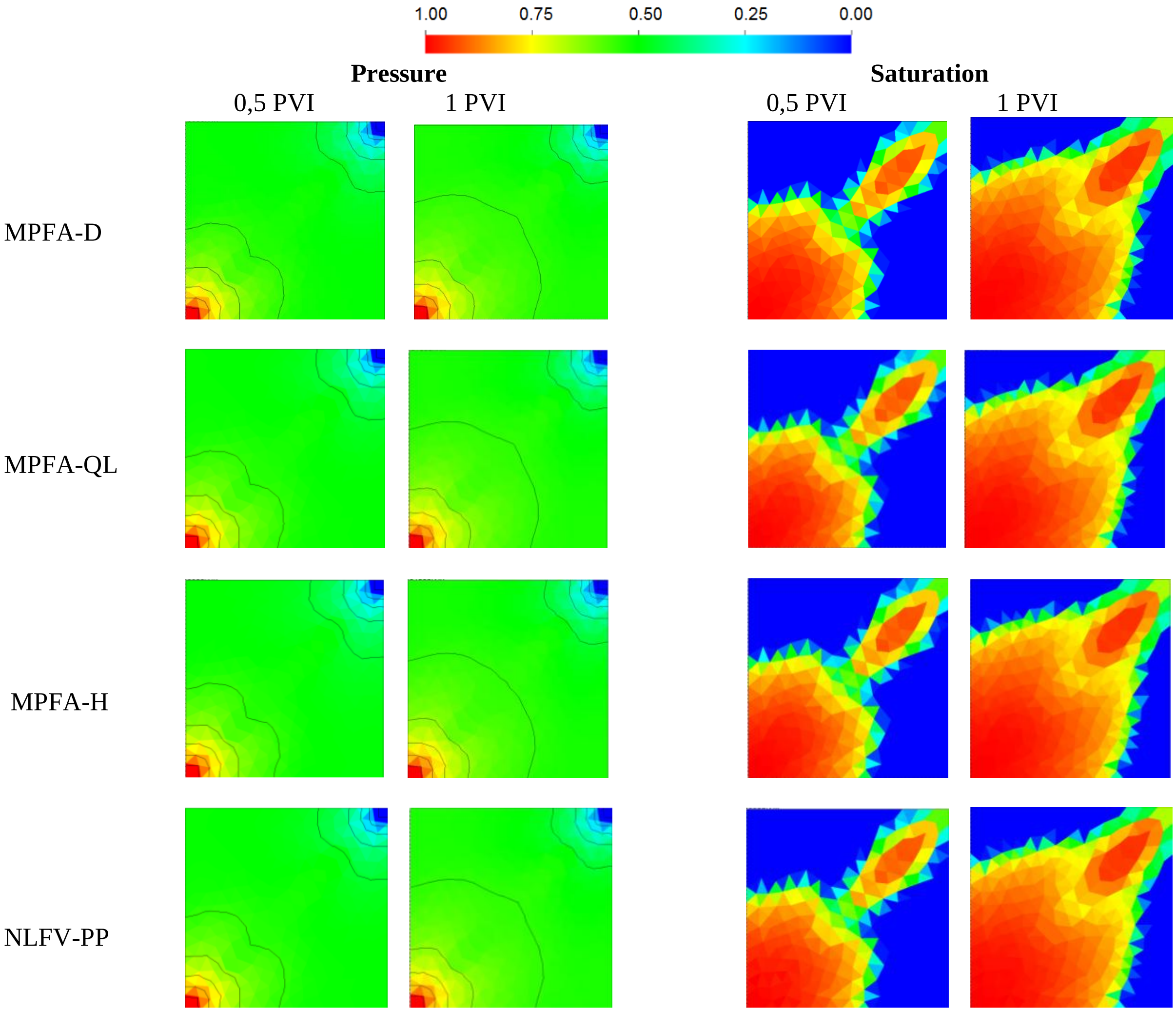

Figure 3. Pressure and saturation field for the two-phase flow in a reservoir with a diagonal fracture at 0.5 and 1 PVI.

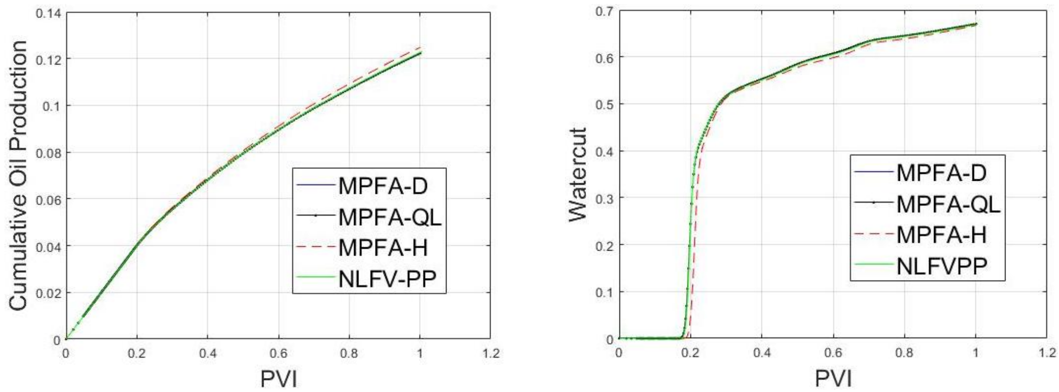

Figure 4. Cumulative oil (left) and watercut (right) for the problem of a two-phase flow in a reservoir with a diagonal fracture.

6 Conclusions

In this work, we have conducted a comparative study among different non-orthodox locally conservative formulations that were extended to deal with naturally fractured reservoirs in 2-D. Linear and non-linear finite volume formulations coupled to the Hybrid Grid method (HyG) were evaluated. Based on the examples we have solved, it was not possible to observe significant differences between the results obtained using the MPFA-D, MPFA-QL, MPFA-H and NLFV-PP, which shows that these methods provide accurate and robust solutions for the problem analyzed. Currently, we are implementing the Embedded Discrete Fracture Model (EDFM) coupled with the MPFA-D, as a flexible alternative to those using fractures aligned with the control surfaces.

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