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Simplified numerical model for analysis of steel-concrete composite beams with partial interaction

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Abstract. The present study refers to the composite steel-concrete beams analysis considering the various non-linear effects. These effects generate complexity to the design requiring computational methodologies for the accurate measurement of structural behavior. With good numerical efficiency, the Refined Plastic Hinge Method also stands out for its simplicity. This methodology will be used considering rotational pseudo-springs at the finite elements ends for the simulation of plasticity. However, this approach was developed for isotropic materials with elastic-perfectly-plastic behavior, implying loss of precision in the analysis of structures containing concrete in their composition. Furthermore, the effects of partial interaction can not be simulated by the inherently rotational behavior of the pseudo-springs. Thus, the introduction of the cracking and partial interaction effects will be approached through effective moment of inertia defined by normative criteria for partial shear connection and using the Patel model for cracking simulation in concrete slab. In addition, geometric non-linearity will be introduced to the model considering a corrotational formulation, dismembering from the rigid body displacements those that actually cause deformation to the element. The validation of the implementations will be done based on the comparison with numerical and experimental data present in the literature.

Keywords: Partial shear connection, Steel-concrete composite beams, cracking, Concentrated plasticity.

1 Introduction

Steel and concrete are materials that have plenty of applications in the construction area, whereas both satisfy several structural problems because of their properties (mechanical and physical) and can be produced relatively easily. Structures that are formed by the combination of both show an improvement in diverse areas, as such resistant capacity, rigidity, protection of metallic elements, among others [1]. Steel-Concrete composites structures have a high degree of complexity to be calculated and analyzed. Thus, computational methodologies are the best option to overcome this problem [1, 2], because it has a design code simplification [3], making it easier check the safety of the structure.

The Refined Plastic Hinge Method [4] is one of the methods available to have an efficient analysis, it consists in dealing with the plasticity in a concentrated form, several papers evaluete the inelastic effect through this method [5–7]. RPHM consist in the insertion of a fictional spring in the nodal point of study, which will simulate the non-linear effects of the material degradation.

Patel et al. [8] introduced a new approach to estimate the moment of inertia of reinforced concrete structures, through an explicit equation was possible to observe a improved behavior adjustment of reinforced concrete beams when compared with experimental values.

This paper aims to use the equation provided by Patel et al. [8] to study the behavior of steel-concrete composite beams with partial share connections together with computational analysis, comparing with experimental values existing in the literature and getting quick answers without losing accuracy.

2 Finite element formulation

In the present work, the displacement-based formulation with concentrated plasticity in the nodal points is applied. In this case, the axial and flexural stiffness degradation occurs exclusively at the FE nodes. Then, the method is presented, introducing the material nonlinearity only. Some considerations and simplifications of this formulation can be seen in [1, 4].

In the structural system modelling, the hybrid beam-column finite element of length L, delimited by nodal points i and j (Figure 1), is used. This element has zero-length pseudo rotational springs at its ends, which are responsible for the plasticity simulation by means of the parameter S_p, discussed in Section 3. The finite element is referenced to the co-rotational system where the degrees of freedom are the rotations at nodes i and j, given by θ_i and θ_j, and the axial displacement in j, δ. The terms M_i, M_j and P represent the bending moments and the axial force in the respective degrees of freedom.

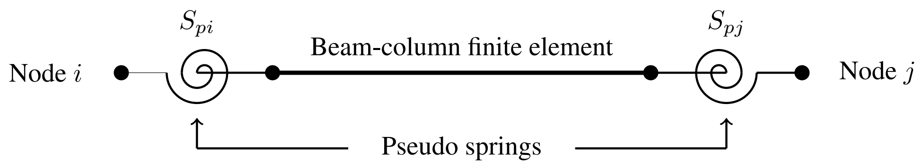


Figure 1. Finite element with pseudo-springs

$$\begin{Bmatrix} \Delta N \\ \Delta M_{pi} \\ \Delta M_{pj} \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & S_{pi} - \dfrac{S_{pi}^2 \left(S_{pj} + k_{33}\right)}{\beta} & \dfrac{S_{pi} k_{23} S_{pj}}{\beta} \\ 0 & \dfrac{S_{pj} k_{32} S_{pi}}{\beta} & S_{pj} - \dfrac{S_{pj}^2 \left(S_{pi} + k_{22}\right)}{\beta} \end{bmatrix} \begin{Bmatrix} \Delta\delta \\ \Delta\theta_{pi} \\ \Delta\theta_{pj} \end{Bmatrix} \tag{1}$$

in which $\beta = \left(S_{pi} + k_{22}\right)\left(S_{pj} + k_{33}\right) - k_{32}k_{23}$.

The terms k_{11}, k_{22}, k_{23}, k_{32}, and k_{33} are components of the beam-column stiffness matrix element, without the pseudo-springs, described as [1]:

$$\begin{aligned} k_{11} &= \frac{E_s A}{L} & k_{22} &= \frac{E_s\left(3I_{eff,i} + I_{eff,j}\right)}{L} \\ k_{23} = k_{32} &= \frac{E_s\left(I_{eff,i} + I_{eff,j}\right)}{L} & k_{33} &= \frac{E_s\left(I_{eff,i} + 3I_{eff,j}\right)}{L} \end{aligned} \tag{2}$$

where E_s is the steel modulus of elasticity, A is the homogenized area of the section, I_{eff} is the modulus of inertia as discussed on Section 4, measured in nodes i and j, and L is the finite element length.

3 Pseudo springs flexural stiffness

The limits of uncracked, elastic or plastic states are defined by the by the moment-curvature relationship [1]. In this nonlinear procedure, the initial cracking moment M_{cr}, the initial yield moment M_{er} and the full yield moment M_{pr} can be easily obtained.

According to the classical RPHM, three equations define the pseudo-spring stiffness for the previously mentioned regions. In regions 1 and 2, it is observed that the section is in an elastic regime. In regions 3 and 4, there can be noticed that the section is in a stiffness degradation process due to plastic strains. And finally, for when the fully plastified section occurs (region 5). For a given axial force-bending moment combination, S_p is defined as follow:

$$\begin{aligned} &\text{if} \quad M \leq M_{er}: \quad S_p = 1 \times 10^{10} \\ &\text{if} \quad M_{er} \leq M \leq M_{pr}: \quad S_p = \frac{E_s I_{eff}}{L}\left(\frac{M_{pr} - M}{M - M_{er}}\right) \\ &\text{if} \quad M_{pr} \leq M: \quad S_p = 1 \times 10^{-10} \end{aligned} \tag{3}$$

in which L is the finite element length and $E_s I_{eff}$ is the section's flexural stiffness, considering the cracking, as discussed below.

Note that, by the value described in Eq. 3, there is no possibility of simulating cracking and partial shear connection in the elastic regime. This adjustment is made in the following section.

4 Moment of inertia

Patel et al. [8] proposed an explicit equation for the effective moment of inertia evaluation of RC sections in a cracking state. The effective moment of inertia, $I_{eff,c}$, is given by:

$$I_{ef} = \frac{3I_c}{1 + e^{-\left[7.4688 + \sum_{k=1}^{6} \left(\frac{a_k}{1 + e^{H_k}} \right) \right]}} \tag{4}$$

where:

$$H_k = b_k \chi_t + c_k \frac{I_{cr}}{I_c} + d_k \frac{M_{cr}}{M} + e_k \tag{5}$$

being M_{cr} and M, respectively, the initial cracking bending moment and the bending moment acting on the section, I_c is the intact section moment of inertia, I_{cr} is the cracked moment of inertia of the section evaluated in the critical point of moment-curvature relationship [1], a_k, b_k, c_k, d_k and e_k described in the table 1 and χ_t the tractioned armor rate.

Table 1. Parameters of Equation 4 and 5

Parameter	k					
	1	2	3	4	5	6
a	8.7116	-0.3754	11.6985	-10.7167	0.6177	22.9397
b	-0.1978	4.3806	2.8322	3.0191	10.1889	-3.7310
c	1.2333	-22.0048	-4.1654	-4.3927	-15.7592	5.4520
d	0.0011	-0.1823	9.4775	9.7598	5.0682	-0.0189
e	-0.0386	6.2396	-6.7756	-7.1914	-3.2443	-2.9660

Considering the partially conjunct action with concrete slab and steel section, the effective moment of inertia, I_{eff}, can be determined as a directly function of degree of interaction, η_i. Thus [3]:

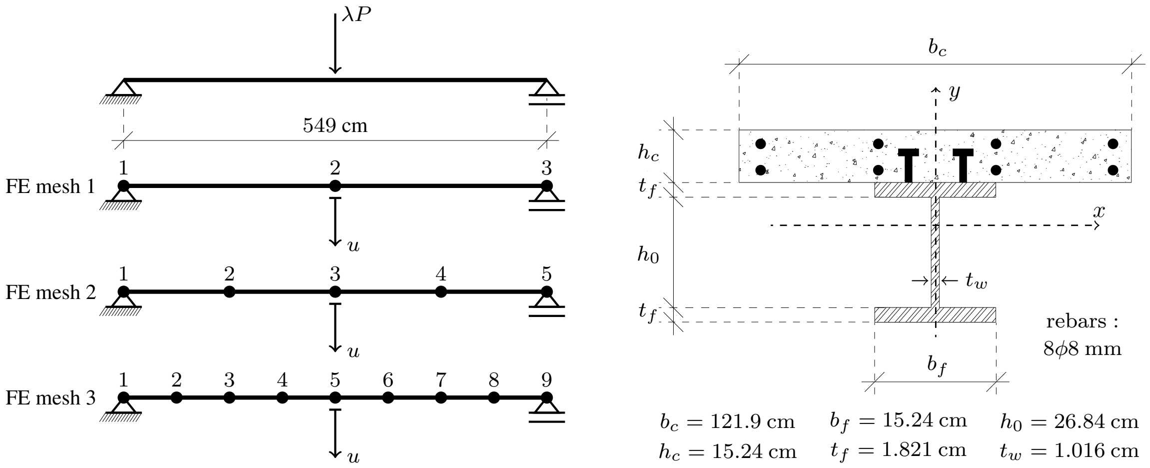
$$I_{eff} = I_{steel} + \sqrt{\eta_i}\,(I_{tr} - I_{steel}) \tag{6}$$

in which I_{steel} and I_{tr} are moment of inertia of steel and homogenized cross sections, respectively. The homogenized moment of inertia is calculated by the direct relation of $I_{eff,c}$ and I_{steel}.

5 Numerical application

In this section the numerical procedure described in this paper will be tested. Chapman and Balakrishnan [11] tested simply supported composite beams with partial interaction. In this analysis, the $E1$ beam [11], illustrated in Fig. 2, is simulated using the proposed formulation. In this same figure, loads, geometry, FE meshes and the cross-section are showed. The partial shear connection is made by equally spaced 50 rows with a couple of stud-bolt connectors per row. The material data of this beam are presented in Tab. 2.

In Figure 3 the equilibrium paths for finite element meshes 1,2 and 3 are plotted and compared with the experimental results [11]. As can be seen in this figure, the data of numerical analysis tends to distance itself from the experimental values,thus showing that is overestimated, however in all cases it is seen a good precision in both initial stiffness and final bearing capacity. In this same figure it can be observed that after the beginning of the stiffness degradation the most refined meshes present a more rigid behavior.



(a) Geometry, loads and adopted FE meshes (b) Cross section

Figure 2. Simply supported beam with partial shear connection

Table 2. Material data of simply supported composite beam with partial interaction (in kN, cm)

Concrete			Connectors	Steel		Rebars	
f_c	ε_{ci}	ε_{cu}	η_i	f_y	E_s	f_{yr}	E_{sr}
3.268	-0.0022	-0.00395	0.929	25.82	20200	32	20500

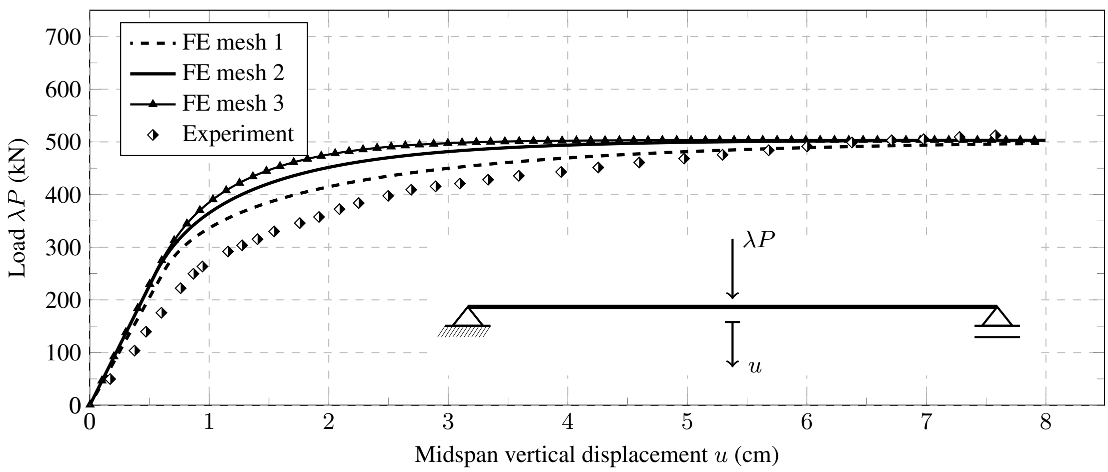


Figure 3. Equilibrium path of simply supported composite beam

6 Conclusions

This paper presents a concentrated plasticity-based formulation using the finite element method for material nonlinear analysis of steel-concrete composite beams with partial interaction. The classical Refined Plastic Hinge Method was applied considering the explicit modification of the effective moment of inertia. For this, the cracking effect of the slab was introduced using the Patel et al. [8] formulation. Associate to this, the moment of inertia was reduced by the degree of interaction of concrete slab and steel section.

The simply supported beam simulated in this paper presented consistent initial stiffness and final bearing capacity with the experimental data. It is important to highlighted that low refinement meshes were sufficient for a satisfactory global response in the tested example.

Thus the proposal of union of the classical RPHM with the effective moment of inertia equation (originally designed for reinforced concrete),considering cracking and partial interaction, provided satisfactory results in the context of material nonlinear analysis of steel-concrete composite beams. But this results can be improve with a better study of pseudo-springs stiffness degradation.

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Authorship statement

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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