An Embedded Discrete Fracture Model (EDFM) Approach Using a Multipoint Flux Approximation (MPFA) Formulation in Naturally Fractured Reservoirs

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Abstract. Naturally Fractured Reservoirs (NFR) form part of major water and energy sources around the world. However, due to their geological complexity and high-contrast permeability, it is difficult to obtain accurate forecasts and estimates of their production behavior. The numerical modeling and simulation of such reservoirs, involving fractures of different scales is a great challenge from a mathematical and numerical point of views. In this context, a very interesting way of representing fractures is through the Embedded Discrete Fracture Model (EDFM). The strategy was initially developed as a technique that directly incorporates fractures in a conventional structured mesh, bypassing the additional computational cost of using unstructured meshes, and remaining compatible with the complex fracture geometries, such as non-planar fractures and fractures with variable aperture. In this context, our simulation tool incorporates a Multi-Point Flux Approximation method via a diamond stencil (MPFA-D), which is a very flexible and robust formulation capable of handling highly heterogeneous and anisotropic domains using general polygonal meshes to solve the pressure equation. To verify our formulation, we solve some representative problems found in the literature.

Keywords: Naturally fractured reservoirs. Embedded Discrete Fracture Model (EDFM). Multiple Point Flux Approximation (MPFA). One-Phase Flow. Heterogeneous and anisotropic reservoirs.

1 Introduction

Naturally Fractured Reservoirs (NFR) are estimated to be the majority of the remaining exploitable fields [1], however, due to their geological complexity, the high contrasts of permeabilities and dimensions, it very is difficult to obtain accurate forecasts and estimates of their production behavior. Over the years, several studies have been carried out, aiming to develop models that can handle this type of reservoir [2,3]. Among the strategies presented in recent years, stands out the Embedded Discrete Fracture Modeling (EDFM) [4–6]. In this model, the fracture planes are embedded in the rock matrix, but their discretization is done separately and independently, being connected via a non-neighbor connections (NNCs) transmissibility. Therefore, EDFM was developed as a technique that directly incorporates the influence of fractures in a conventional structured mesh, bypassing the additional computational cost of unstructured meshes and remaining compatible with the complex fracture geometries, such as non-planar fractures and fractures with a variable opening [2,7,8].

Our objective in this work is to model the one-phase fluid flow in naturally fractured reservoirs. In this context, we have used the non-orthodox Multipoint Flux Approximation) [9–11] via a diamond stencil (MPFA-D) finite volume scheme to solve the elliptic pressure equation [12, 13]. Therefore, we combine the flexibility of the MPFA-D method, which is capable of deal with general full permeability tensors, with the low computational cost of EDFM approach to model the one-phase flow in fractured reservoirs.

2 Mathematical model

2.1 Governing Equation

The elliptic pressure equation for one-phase fluid flow can be obtained from the proper manipulation of the general mass balance equation and Darcy's Law resulting in equation (1), where $\vec{\nabla}$, $\vec{v}$, Q, K and p represent the gradient operator, total velocity, total flow, absolute permeability tensor, and global pressure, respectively. The model considers an isothermal, and incompressible fluid flow through porous media, where the effects of gravity and capillarity can be neglected.

$$\vec{\nabla}.\vec{v} = Q \quad \left(with\ \vec{v} = -K\vec{\nabla}p\right) \tag{1}$$

3 Numerical Formulation

The key idea of the EDFM approach is to split the computational domain into a rock matrix domain (Ω_m) and a fracture domain (Ω_f) using two different grids, separately, the coupling is carried out via a transfer function applied through non-neighbor connections (NNCs) [5,6,8].

3.1 Fracture-Matrix Coupling and Connectivity Index

Li and Lee [4] showed that the matrix and fracture grids could be coupled using a transfer function. From a mathematical point of view the transfer function is a source term between the fracture f and the matrix m, which is defined as:

$$q_{f,m} = CI.K.\left(p_f - p_m\right) \tag{2}$$

with CI being the 'connectivity index' between the matrix and the fracture defined as the area fraction of the fracture element (z) in the matrix cell (x,y), i.e., $(A_{xy,z})$, divided by the average distance $\langle d\rangle_{xy,z}$ between the fracture element (z) and the matrix cell (*x,y*), calculated numerically in the following form or using an analytical solution [14]:

$$CI_{(x,y),z}^{m-f} = \frac{A_{xy,z}}{\langle d\rangle_{x,y\sim z}}, \qquad with\ \langle d\rangle_{x,y\sim z} = \frac{\int_{A_{x,y}} x\, dA}{A_{x,y}} \tag{3}$$

3.2 Discretized Linear System for the Elliptic Pressure Equation

In this work, we consider 2D domains, but, in Ω_f, we deal with a simplified 1D flow, therefore, in this domain, the MPFA-D becomes the classical Two Points Flux Approximation (TPFA) method [12].

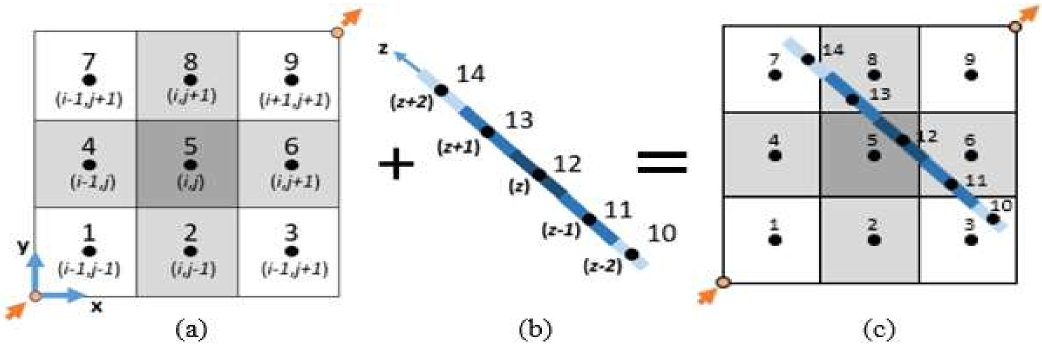

Figure 1 - (a) The discrete rock matrix mesh with a central cell 5. (b) The discrete fracture grid with a central cell 12. (c) An example of a grid with one fracture.

If we analyze the flow in cell 5, in the rock matrix grid (fig. 1a), and in cell 12, in the fracture grid (fig. 1b), we have: The F_5^m is understood to be the flow in the cell 5 of the rock matrix domain and $F_{12}^f$the flow in the cell 12 of the fracture domain.

$$\sum F_5^m = F_{2-5}^{m-m} + F_{4-5}^{m-m} + F_{5-6}^{m-m} + F_{5-8}^{m-m} \tag{4}$$

$$\sum F_{12}^f = F_{11-12}^{f-f} + F_{12-13}^{f-f} \tag{5}$$

To complete the formulation of the scheme, and satisfy the principle of mass conservation for cell 5 and cell 12, we need an NNC, as follows:

$$\sum F_5^{NNC} = F_{5-12}^{m-f} = -F_{12-5}^{f-m} \tag{6}$$

Thus, the total flow in cell 5 can be written as a combination of eq. (4) and eq. (6). The same argument is valid for the total flow in cell 12, by combining eq. (5) and eq. (6), honoring the flow direction. The discretization of eq. (4) and eq. (5) is obtained via the MPFA-D and the TPFA schemes, respectively. However, for eq. (6 the connectivity index should be used, as presented in eqs. (2) and (3).

3.3 Fracture Intersection

If there is an intersection of fractures, their mutual transmissibility can be calculated using the star-delta transformation, as utilized for electrical circuits [15]. The general expression for the transmissibility between fractures o and p, sharing a node connected by n fractures, each one with an aperture w_f^k, and length L^k, is given by:

$$T^{o-p} \simeq \frac{\alpha_o \alpha_p}{\sum_{k=1}^{n} \alpha_k} \qquad (with\ \alpha_k = -\frac{K_f^k w_f^k}{\frac{1}{2} L^k}) \tag{7}$$

3.4 Solution Strategy

The linear system $Tp = F$ is constructed according to the MPFA-D/TPFA discretization scheme:

$$\begin{bmatrix} T^{m-m} & T^{m-f} \\ T^{f-m} & T^{f-f} \end{bmatrix} \begin{bmatrix} p^m \\ p^f \end{bmatrix} = \begin{bmatrix} F^m \\ F^f \end{bmatrix} \tag{8}$$

where, the sub-matrix blocks T^{m-m} and T^{f-f} contain the matrix-matrix and fracture-fracture transmissibility, respectively. The off-diagonal sub-matrices, i.e., T^{m-f} and T^{f-m}, contain the transmissibilities between the fractures and matrix. It is worthwhile to highlight that the linear system above is solved using a sparse iterative linear solver to obtain the pressure field.

4 Results

In this work, we have implemented the Embedded Discrete Fracture Model (EDFM) using the MATLAB programming language (MATLAB R2019a) to model the one-phase flow of oil and water in 2D petroleum reservoirs. Using a Hybrid-Grid Method (HyG) coupled with the MPFA-D scheme [3], we have verified the EDFM for single-phase flow problems. One fracture network configuration obtained from H. Hajibeygi, et al [14] was considered. For this case, the HyG/MPFA-D employs over fifty-nine thousand grid cells to capture both, the fractures and resolved channels of high conductivities. Following the work by Hajibeygi et al. [14], two fractures with an aperture of 1/250 were considered over a 9x9 rock matrix. The Fig. 2.a illustrates the configuration of the fracture network in the reservoir. No-flow boundary conditions are prescribed on both the bottom and top faces of the reservoir. On the left and right sides of the reservoir, the prescribed pressure is P=1 and P=0, respectively. A fracture-matrix permeability ratio Kf/Km = 10^3 has been used in this case.

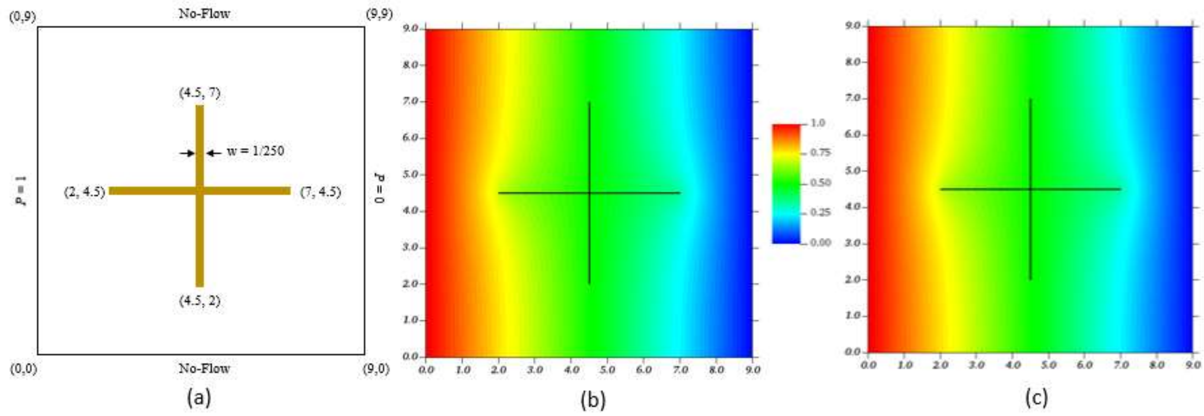

Figure 2 - (a) Domain and boundary conditions of the test case. (b) Pressure field by EDFM/MPFA-D.
(c) Reference pressure field by HyG/MPFA-D.

The HyG/MPFA-D employs a mesh with 59800 grid cells and 225 small-scale grid cells are placed inside the fractures to better represent the problem. The cM and cF represent the number of matrix and fracture grid cells, respectively. Figure 2.b shows the pressure field calculated by the EDFM starting with 81 cM and 16 fM grid cells and in fig.2.c we present the result obtained with the HyG. In fig 2 (b and c), we can see that the results are considerably similar, showing the accuracy of the EDFM working with MPFA-D in the matrix and the TPFA in the fractures.

In figure 3, taking the HyG/MPFA-D as the reference solution, the pressure field of EDFM meshes with sizes starting from 81 cM and 16 cF and gradually being refined to 59049 cM and 252 fM, which shows the pressure in the cells of the rocky matrix of all the generated cases at y = 4.5 cross-section. As it can be clearly seen, the pressure fields obtained by the EDFM method converges to the reference case.

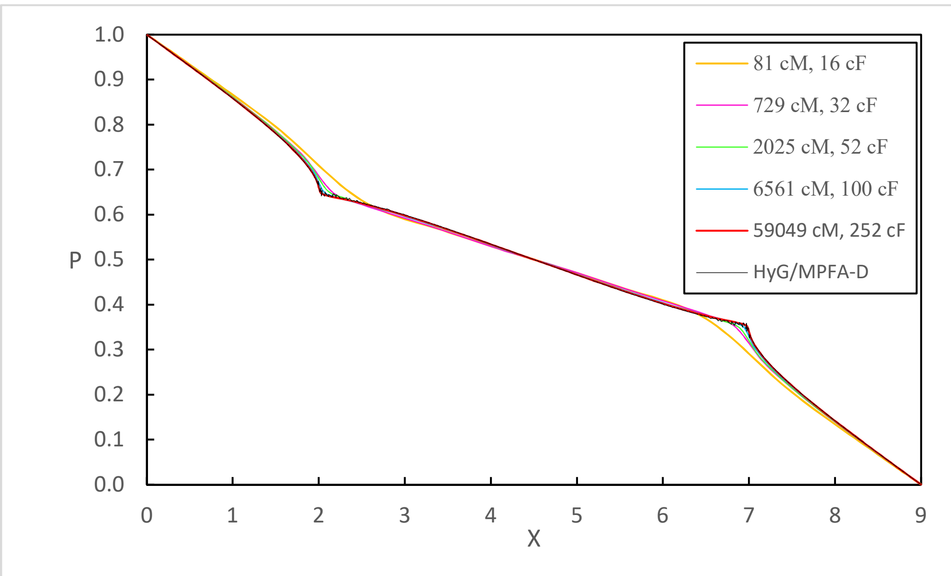

Figure 3 - Matrix pressure of EDFM on test case at y = 4.5 for different mesh refinement levels.

5 Conclusions

In this work, we have presented, for the first time in literature, the use of the Embedded Discrete Fracture Model (EDFM) method with the Multipoint Flux Approximation via a "diamond stencil" (MPFA-D) formulation to calculate the pressure field in the rock matrix and the TPFA scheme in the fractures. To verify the accuracy of our proposal, we have used a benchmark problem and we have compared our solution with the one obtained by the Hybrid-Grid Method (HyG). The results showed the efficiency and the accuracy of the EDFM that was able to give a very accurate solution using a much coarser mesh than the HyG approach, reducing the computational cost and obtaining a better efficiency. In the near future, we intend to use the EDFM+MPFA-D method together a streamline solver to model two-phase flow problems in naturally fractured porous media.

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