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Numerical Simulation of Batch Settling in Non-Newtonian Fluids

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Abstract. The study of particle sedimentation in viscous fluids plays a fundamental role in several applications of different scientific and industrial branches. Furthermore, it is crucial to emphasize the relevance of this procedure in the control of pressures inside oil well. For instance, the transportation and suspension of sediments during the disable operation and diminishing the hydrostatic pressure on the annular to avoid the oil well collapse. Although the extensive applicability of the sedimentation phenomenon, the problem complexity is related to provide reliable predictions. Hence, mathematical models accomplish a considerable approach in experimental and theoretical researches. The highly nonlinear characteristics of the model and the development of rarefaction and shock waves in concentration profiles require appropriate numerical schemes for prediction solutions. Nonetheless, the employment of implicit methods is infeasible due to high computational effort. Moreover, there is an absence of numerical-theoretical studies of the sedimentation phenomenon in non-Newtonian fluids. The present research employs the non-oscillatory corrector-predictor explicit method developed by Nessyahu and Tadmor for the numerical simulation of batch settling in viscoelastic fluids.

Keywords: Finite Volume Method, Non-Oscilatory Methods, Batch Settling and Non-Newtonian Fluid.

1 Introduction

Several industrial and scientific branches encompass sedimentation phenomena. The sedimentation machines in the mining industry, batch settling in geophysical researches, and drilling fluids analysis in the petroleum field are some examples. Notably, the solid particle sedimentation in a drilling fluid confined in the annular between the cement and an isolation tool might induce severe operational problems [1]. For instance, the *Annular Pressure Build-Up* (APB), i.e., when the pressure in the annular increases abruptly because of the heat gradients in the wells production phase [2]. This process can destroy the oil well causing a catastrophe. Hence, solid particle sedimentation has been widely investigating due to immense challenges in its accurate prediction [3].

The initial investigations of the sedimentation phenomenon occurred at the beginning of the 20th century [4]. However, only about 50 years later, it was developed the first simplified theory for the batch sedimentation. The research proposed a kinematic sedimentation theory based on the propagation waves' ideal monodisperse suspension [5]. Thereby, mathematical models have been widely developing for the comprehension of the phenomenon [6]. Extensions of the theory cited were proposed, as in the study of continuous sedimentation of a suspension with a nonconvex flux law presented by Petty [7] and in the development of an entropy weak solution to determine the physically relevant solutions by Bustos et al. [8].

Appropriate numerical methods became extremely important for the predictions of batch settling phenomena. Nonetheless, the non-linear characteristics of the mathematical formulation and the high gradients in the concentration profiles generate some numerical challenges to yield accurate solutions. The Finite Difference Method (FDM) has been employing in the sedimentation simulation, but numerical instabilities might corrupt the solution through undesirable oscillations [9]. On the other hand, a polydisperse model has been solving with a non-oscillatory difference scheme, called Nessyahu-Tadmor (NT) method [10].

Recently, an adaptative multi-resolution (WENO) and implicit-explicit (IMEX) schemes were implemented in sedimentation models to reduce costs and inflexibility [11, 12]. Although many authors investigated the prediction of settling suspensions through mathematical modeling, there is literature lacks a phenomenological model to predict the particle settlement in a non-newtonian fluid. Thus, the present research employs the non-oscillatory corrector-predictor explicit method developed by Nessyahu and Tadmor [10] for the numerical simulation of batch settling in viscoelastic fluids model developed by Rocha [3].

2 Mathematical Modeling

The mathematical modeling of the unidimensional sedimentation in drilling fluids can be described by a system of partial differential equations, composed by mass conservation and linear motion equations, besides empirical and constitutive correlations to describe interaction forces and the permeability [3].

In the one-dimensional settlement in the direction z, the continuity and the movement equation to the solid phase, considering the solids density constant, are respectively given by [3]:

$$\frac{\partial \epsilon_s}{\partial t} + \frac{\partial(\epsilon_s \upsilon_s)}{\partial z} = 0 \tag{1a}$$

$$\rho_s \epsilon_s \left(\frac{\partial \upsilon_s}{\partial t} + \upsilon_s \frac{\partial \upsilon_s}{\partial z} \right) = \frac{\partial T_s}{\partial z} + m + \epsilon_s(\rho_s - \rho_l) g \tag{1b}$$

$$\text{for} \quad 0 \leq z \leq L_0 \quad \text{and} \quad t \geq 0$$

where L_0 is the height of the suspension and the following boundary conditions:

$$q_s = \epsilon_s \upsilon_s(z = 0, t) = 0, \qquad q_s = \epsilon_s \upsilon_s(z = L_0, t) = 0 \tag{1c}$$

$$\epsilon_s(z, t = 0) = \epsilon_{s0}, \tag{1d}$$

The set of equations 1 are based on the Kynch's sedimentation theory [5]. It is valuable to emphasize the importance of his assumption that the local sedimentation velocity is a function only of the local volumetric solids concentration to solve the problem [13]. In both equations, t, z and g refer to time, the axial position of the sedimentation column and to gravity acceleration, respectively. The volumetric concentration, density, velocity and solids tension are represented by $\epsilon_s, \rho_s, \upsilon_s$ and T_s. Moreover, the resistive force is characterized by m and the liquid density by ρ_l. The homogeneity of the suspension is demonstrated by the initial condition, where ϵ_{s0} is the initial concentration. The nullity of the solids flux, q_s, in the bottom and the top of the sedimentation column is expressed in the two boundary conditions.

After the introduction of the rheological power-law model and constitutive hypothesis, the balance force in the system has been define completely which yield the expression to calculate the sedimentation velocity in the solid phase [3]:

$$\upsilon_s = \left\{ \frac{K}{M(1-\epsilon_s)^{n-1}} \left[\frac{d_p}{\Theta(\phi)} \right]^{n-1} \left(\frac{\rho_{susp}}{\rho_{susp} - \rho_s \epsilon_{s0}} \right) \left[\epsilon_S(\rho_S - \rho_f) g - \frac{dP_s}{d\epsilon_s} \frac{\partial \epsilon_s}{\partial Z} \right] \right\}^{\frac{1}{n}} \tag{2}$$

which the sphericity, permeability of the environment and the pressure gradient are considered, respectively [3]:

$$\Theta(\phi) = -3,45\phi^2 + 5,25\phi - 1,41 \tag{3a}$$

$$K = K_0 d_p^2 \left(\frac{\epsilon_{sm}}{\epsilon_s} - 1 \right)^{\Lambda} \tag{3b}$$

$$\frac{dP_s}{d\epsilon_s} = \frac{P_{sref}\beta}{\epsilon_s^2} exp\left[-\beta \left(\frac{1}{\epsilon_s} - \frac{1}{\epsilon_{sref}} \right) \right] = \frac{A}{\epsilon_s^2} exp\left[-\beta \left(\frac{1}{\epsilon_s} - \frac{1}{\epsilon_{sref}} \right) \right] \tag{3c}$$

where ϕ is the sprericity of the particle. K and d_p represents the environment permeability and the medium diameter of the particles, respectively. K_0 and β are parameters. Moreover, P_s and P_{sref} are the solids pressure and the solids pressure in a reference concentration ϵ_{sref}.

The dimensionless concentration and parameters are given as

$$\xi = \frac{t}{tf}; \quad \eta = \frac{Z}{L_0}; \quad \theta_s = \frac{\epsilon_s}{\epsilon_{s0}};$$

$$u_s = \frac{\upsilon_s}{\upsilon_{stk}}; \quad S = \frac{\upsilon_{s0} t_f}{L_0} \tag{4}$$

where ξ and η are dimensionless versions of t and z, respectively, θ_s is the dimensionless solid concentration, u_s is the dimensionless solid velocity, S is a dimensioless parameter and average velocity υ_{stk} is the terminal velocity of an isolated particle [10]:

$$\upsilon_{stk} = \frac{d_p^2(\rho_s - \rho_f)g}{18\mu_{susp}(\lambda*)} \tag{5}$$

The dimensionless form of the mathematical model is given by:

$$\frac{\partial \theta_s}{\partial \xi} + S\frac{\partial(\theta_s u_s)}{\partial \eta} = 0 \tag{6a}$$

$$u_s = \frac{1}{\upsilon_{s0}}\left\{\frac{K}{M(1-\theta_s\varepsilon_{s0})^{1-n}}\left[\frac{d_p}{\theta(\phi)}\right]^{n-1}\left(\frac{\rho_{susp}}{\rho_{susp}-\rho_s\varepsilon_{s0}}\right)\left[\theta_s\varepsilon_{s0}(\rho_s-\rho_f)g - \frac{\varepsilon_{s0}}{L_0}\frac{dP_s}{d\varepsilon_s}\frac{\partial\theta_s}{\partial\eta}\right]\right\}^{\frac{1}{n}} \tag{6b}$$

$$\text{for} \quad 0 \leq \eta \leq 1 \quad \text{and} \quad \xi \geq 0$$

with the following boundary conditions:

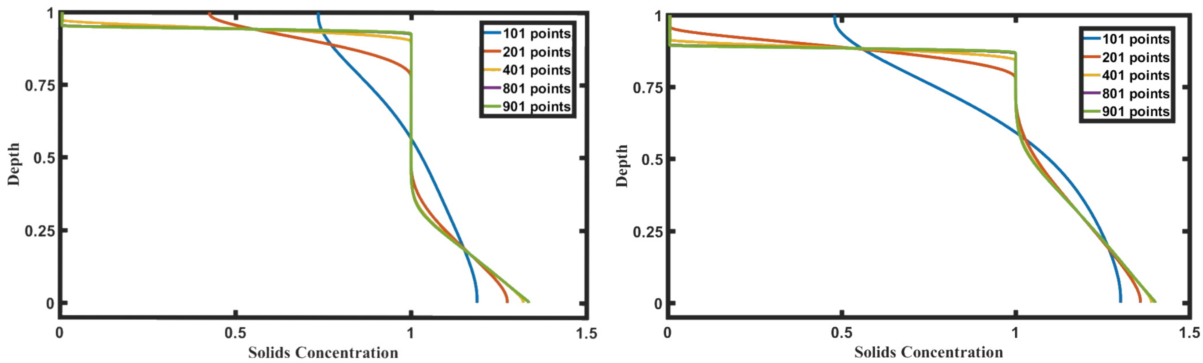
$$F_s = \theta_s u_s(\eta = 0, \xi) = 0, \qquad F_s = \theta_s u_s(\eta = 1, \xi) = 0 \tag{6c}$$

$$\theta_s(\eta, \xi = 0) = 1$$

The dimensionless equations 6 are highly non-linear model. Thus, the model has been solved with non-oscillatory Nessyahu-Tadmor method [10].

3 Results and Discussions

In this section we present the results of the numerical predictions. The input parameters used in each studied case are listed in Table 1 as described by Rocha [3]. Firstly, a mesh convergence was analysed for different dimensionless time. The Figures 1a and 1b show the axial dimensionless concentration profile θ_s for different dimensionless time ξ while Figure 1a was obtained for $\xi = 0.5$ and Figure 1b for $\xi = 1.0$. Each line color present the solution for a grid size. As can be observed, the mesh convergence was obtained for 801 points in the mesh. The results were constructed with the converged mesh.



(a) (b)

Figure 1. Axial concentration profiles for different grids with dimensionless time (a) $\xi = 0.5$ and dimensionless time (b) $\xi = 1.0$.

The Figure 2 shows behavior of the dimensionless solid concentration profile for different time. As can be seen, each profile is divided in two regions along the time. This result presents the formation of a clarified zone in the top of the column and a concentrated zone in the bottom. As presented by Kynch [5], solids concentration

Table 1. General operation conditions parameters following Rocha [3].

General operation data	Value
K_0	$3.66600 \cdot 10^{-2}$ m^2
A	5.33445
β	2.13699
Fluid viscosity (μ)	4.50000
Fluid density (ρ_l)	$8.91230 \cdot 10^2$ kg.m^{-3}
Solids density (ρ_s)	$2.71080 \cdot 10^3$ kg.m^{-3}
Medium diameter of the particles (d$_p$)	$4.08030 \cdot 10^{-5}$ m
Initial concentration (ϵ_{s0})	0.14000% v/v
Volumetric concentration (ϵ_{sm})	0.50000% v/v
Reference concentration (ϵ_{sref})	0.14700% v/v
Suspension density (ρ_{susp})	$1.14590 \cdot 10^3$ kg.m^{-3}
Shear rate (λ)	1.60612 s^{-1}
Consistence index (M)	1.25000 Pa.s^n
Fluid behavior index (n)	0.38000 kg.m^{-2}.s^{-2}
Initial suspension height (L_0)	0.21000 m
Sphericity (ϕ_{esf})	0.80000
Final time (t_f)	$3.15400 \cdot 10^7 s$

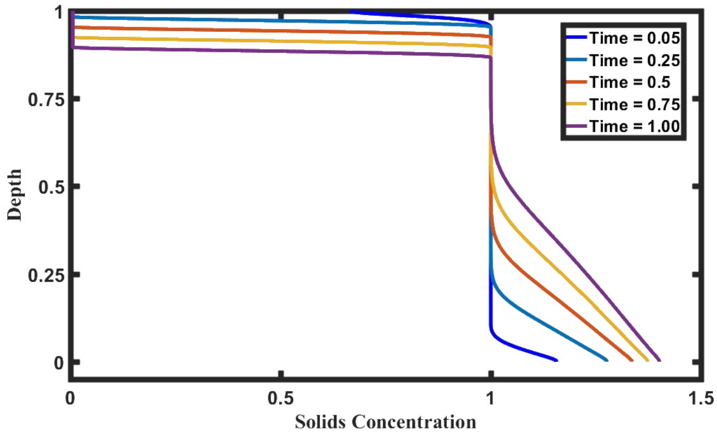
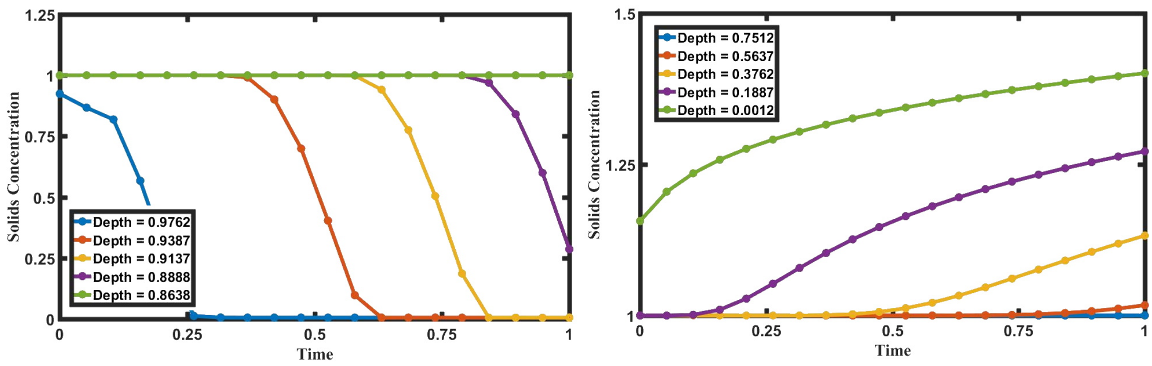


Figure 2. Axial concentration profile for different dimensionless time.

behavior in settling processes presents three different regions, where a free settling zone, a non compression zone and a compression zone are identified. Therefore, the solution seems to predict the phenomenon. Moreover, the sedimentation process increases both regions.

The Figures 3a and 3b shows the evaluation of dimensionless solid concentration in different depth positions where the Figure 3a present the behavior in clarified zone while Figure 3b in concentrated zone. As shown by the results, the concentration in the clarified zone tends to be zero along the time. Otherwise, the concentration zone increases significantly during the sedimentation process. The prediction is similar to batch tests presented by McCabe et al. [14]. They identified a free settling zone, where particles falls with no contact with others; a non-compression zone, where there are increases in the concentration and the sedimentation rate of the particles decreases; and a compression regime, where particles accumulates as they have contact with each other. We can identify similarities in the data with the experimental description of how the sedimentation phenomenon occurs. Therefore, we can assume that the phenomenological model proposed may predict with reliability the settling of particles.



(a) (b)

Figure 3. Concentration time evaluation for different depth positions for clarified zone (a) and concentrated zone (b).

4 Conclusions

This research presented a numerical simulation of the batch settling in non-Newtonian fluid. The mathematical formulation developed by Rocha [3] was solved with a non-oscillatory Nessyahu-Tadmor scheme. As shown by the results, the simulation solution converged as the grid size rises. Then, we can expect that the program proposed is capable of simulate the phenomenon without discontinuities that causes error in the prediction.

It is also notable that the data illustrates a solids deposition in the bottom of the well, suggesting that there is a solids accumulation. Moreover, in the clarified zone the solids concentration tends to be zero in the solution. These results seems to satisfy theories of the sedimentation phenomenon behavior. However, the predictions of this study are restricted to theoretical analysis. A experimental work is required to provide reliability in the solution proposed.

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6 Authorship statement

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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