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Structural optimization of trusses considering geometric stability

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Abstract. Structural optimization is a required area where the aim is to reduce project costs, reducing the amount of material used and meeting the requirements for the safety and use of the structure. The objective of the study is develop computerized routines for the structural optimization of trusses submitted to nodal loads considering geometric stability. The optimization problem seeks for an answer that minimizes the volume of trusses subjected to stress, displacement and geometric stability constraints. The design variables are those commonly used, given by the bars cross-section area and nodal coordinates. To solve this problem, computerized routines were developed using MASTAN2® program for the structural analysis and MATLAB® program, for numerical optimization. Different sample structures were created to analyze the resourcefulness of the routines. The program found difficulties in optimizes some structures that have bars that are not necessary in the final design, a classical difficulty in this area.

Keywords: Structural optimization, Truss structures, Geometric Stability

1 Introduction

Optimization is the act of obtaining the best result under given circumstances (Rao [1]). In engineering we ever aim to a solution that meets the pre-defined requirements and spends less materials and efforts as possible. There is a constant search for structures that are less costly and meet the established security and service criteria. In this scenario, structural optimization enters, with much more emphasis in the last decades due to the evolution and access to computers. In structural optimization we usually seek to minimize the cost of a structure, so we make a direct relationship between the cost and the volume of the structure. A structure is subject to a certain load and respecting restrictions of stress, displacements, etc. In structural optimization of trusses, the focus of this paper, the cross-sectional area of the bars and some nodal coordinates is usually taken as design variables, it is known that the nature of the problem is non-linear (Torii et al. [2]), having to apply more robust optimization methods to find the optimal solution.

We seek to analyze and find the optimal configuration of plane trusses, minimizing the volume of the structure considering maximum stress constraint, maximum displacement constraint and a global stability constraint, in addition to minimum and maximum limits for the cross-sectional area and movement of the structure's nodes. Two sample structures are proposed to assess the resourcefulness of computational routines.

2 Optimization problem formulation

The formulation of the problem is the one commonly seen, described below

$$\begin{gathered}\text{Find } \mathbf{X} = (x_1, ..., x_n) \\ \text{that minimize } f(\mathbf{x}) \\ \text{subject to } \mathbf{g}(\mathbf{x}) = \{g_1(\mathbf{x}), ..., g_m(\mathbf{x})\} \leq 0 \\ \text{and } l_i \leq x_i \leq u_i, i = 1, 2, ..., k\end{gathered} \tag{1}$$

being $\mathbf{X}$ the vector of the design variables, $f(\mathbf{x})$ the objective function, $\mathbf{g}$ the vector of inequality constraints, l_i the lower bounds of the design variables and u_i the upper bounds of the design variables. The design variables are the cross-sectional radius of the structure bars and some nodal coordinates, the objective function is the total volume of the structure, the restrictions will be described later.

All computational routines were developed using MATLAB software, and the structural analysis using MASTAN2 software. The advantage found in the use of these softwares is that MASTAN2 is implemented within MATLAB, facilitating the development of routines and reducing the computational cost. The method used for optimization are those available in the fmincon function.

2.1 Problem constraints

Although the structure is a truss, we use frame elements to model and analyze it, so that some obstacles that the algorithm may encounter, as described by Torii et al. [3], can be crossed. Unlike truss elements, frame elements are bars subject to axial forces, shear forces, bending moments, torsional moments, etc., while truss elements are only capable of capturing axial forces and displacements. More details on the modeling of structures using elements of frames and trusses is presented in McGuire et al. [4]. We know that when using frame elements, the tension in the bars is not only the result of axial force, but also of the bending and torsion moments, however, in the present work we are more interested in discussing other aspects of the problem, such as the stability of the structure and some problems that the modeling can cause, therefore, to make the problem simpler but without losing the rigor, we define the stress constraint written using the relaxation ε, presented by Cheng and Guo [5]. So we have :

$$g_i = A_i(\sigma_i - \sigma_{\max}) - \varepsilon, \tag{2}$$

being A_i the area, σ_i the stress, $\sigma_{\max}$ the allowable stress and ε a relaxation factor. The multiplication by the bar area and the subtraction of the ε factor are used to allow the constraint to be respected when the area is reduced indefinitely. If relaxation is not performed and the constraint is written as $g_i = \sigma_i - \sigma_{\max}$, Cheng and Guo [5] demonstrated that the stress increases indefinitely when the area of the bar is reduced, making it impossible for the bar be removed from the structure.

For the displacement constraint, the expression adopted is as follows:

$$g_i = \frac{|U_i|}{U_{\max}} - 1, \tag{3}$$

with U_i being the displacement considered and $U_{\max}$ the allowable displacement.

Finally, the stability constraint is written as

$$g = 1 - P_{cr}, \tag{4}$$

with P_{cr} being the elastic critical load, evaluated as described in the next section.

2.2 Global stability model

To quickly explain where the critical load constraint (P_{Cr}) comes from, we turn to the global analisys of the structure. In the global analysis it is assumed that the tangent stiffness matrix of the structure $\mathbf{K}_t$ is written as the sum of the elastic stiffness matrix $\mathbf{K}_e$ with the linearized geometric stiffness matrix $\mathbf{K}_g$ multiplied by the load scale factor.

$$\mathbf{K}_t = \mathbf{K}_e + P_{Cr} \cdot \mathbf{K}_g. \tag{5}$$

The structure is considered unstable when the tangent stiffness matrix becomes singular [4], so to find the load scale factor, the following eigenvalue problem must be solved:

$$(\mathbf{K}_e + P_{Cr} \cdot \mathbf{K}_g)\phi = \mathbf{0}. \tag{6}$$

Solving the eigenvalue problem, we take the smallest positive eigenvalue found to be P_{Cr}, as it is the smallest proportion of the possible load capable to make the structure unstable.

3 Numerical examples

In all sample structures, the following assumptions have been made:

I - The structure consists of circular cross-section bars and rigid connections;

II - The design variables are some nodal coordinates of the structure and the cross-sectional radius of the structure bars. The cross-section properties of the elements are constant in each member;

III - The objective function assumed is the volume of the structure;

IV - Stability must be guaranteed, making $P_{Cr} \geq 1$;

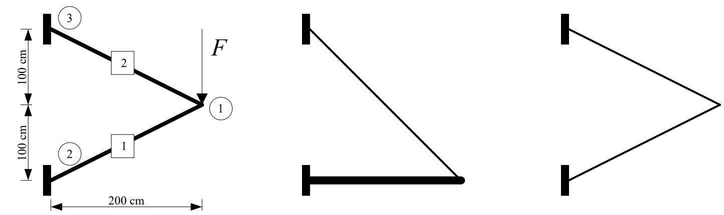
V - The material from which bars are modeled is steel, with an elasticity modulus $E = 210$ GPa, yielding stress $\sigma_y = 200$MPa, maximum allowed displacement $U = 0.05$m and Poisson's coefficient $v = 0.3$;

VI - The mathematical modeling of the structure and its stiffness matrices are proposed by McGuire et al. [4];

Although they are plane, the structures in the examples were calculated as spatial frames, so it is possible to capture the instability of the structure outside its plane.

3.1 Example 1: 2 bars planar truss sizing-geometry optimization

The structure shown in the first example is a plane truss made up of two bars, with restricted supports, not allowing their displacement or rotation. The total length and height of the structure is 2m. In this example, a force $F = 5$kN is applied. The design variables are the cross-sectional radius of the bars and the vertical coordinate of the node where the force is being applied. The radius of the cross section of the bars has a lower bound $l = 10^{-3}$cm and an upper bound $u = 20$cm. The vertical coordinate of the node where the force is applied can move between the heights of the supports.



(a) Initial design. (b) Final design. (c) Final design without stability constraint.

Figure 1. Example 1 truss design before optimization (a), after the optimization (b), and after optimization without stability constraint (c)

After the algorithm was executed, the new geometric configuration of the structure was found, the node where the force is applied moved up to the height of the first support, with a configuration very similar to a beam with a steel cable connected to the end free.

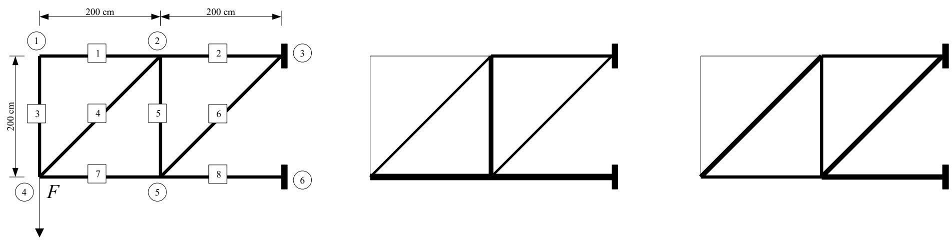
3.2 Example 3: 8 bars planar truss sizing optimization

In the second and last example, we have a truss composed of 8 bars and 6 nodes. A cantilevered structure, with both rigid supports as well. The structure has a total length of 4m and a total height of 2m, with a force $F = 5$kN applied on node 4. Here we also have the lower bounds $l = 10^{-3}$cm and upper bounds $u = 20$cm for the cross-sectional radius. In this example, the algorithm will not work with nodal coordinates as design variables.

Table 1. Example 1: optimum bar's radius, critical load scale factor (P_{Cr}), maximum absolute displacement (U_{max}), maximum absolute stress (σ_{max}) and total volume

Case	With stability constraint	Without Stability Constraint
Bar 1 (cm)	1.663	0.595
Bar 2 (cm)	0.670	0.595
P_{Cr}	1.000	0.015
U_{max} (cm)	0.404	0.478
σ_{max} (MPa)	200.57	200.718
Total volume (cm^3)	534.146	124.552

First, we will execute the algorithm with all restrictions, after that we will only execute with the stress and displacement restrictions and with the lower bound $l = 2.5 \cdot 10^{-2}$cm.



(a) Initial design. (b) Final design. (c) Final design without stability constraint.

Figure 2. Example 2 truss design before optimization (a), after the optimization (b), and after optimization without stability constraint (c).

Table 2. Example 3: optimum bar's radius, critical load scale factor (P_{Cr}), maximum absolute displacement (U), maximum absolute stress (σ_{max}) and total volume

Case	With stability constraint	Without stability constraint
Bar 1 (cm)	0.100	0.025
Bar 2 (cm)	1.026	1.045
Bar 3 (cm)	0.100	0.025
Bar 4 (cm)	0.955	1.242
Bar 5 (cm)	1.332	1.045
Bar 6 (cm)	0.947	1.242
Bar 7 (cm)	3.212	1.045
Bar 8 (cm)	3.236	1.477
P_{Cr}	1.002	0.036
U_{max} (cm)	0.500	0.500
σ_{max} (MPa)	100.241	58.336
Total volume (cm^3)	4114.600	1543.000

Doing the truss analysis we found the result that the bars 1 and 3 are not under tension, therefore they could be removed from the structure without affecting the structure, knowing that the algorithm should reduce the radius of the cross section to the minimum proposed for the problem. However, looking at Table 2 we see that the bars have a radius of cross-section far from being close to the proposed minimum. Running the same example only with the maximum stress and maximum displacement restrictions, we can see in Table 2 that bars 1 and 3 have been reduced to the minimum allowable radius, which is considered the removal of these bars.

4 Conclusions

In the present work, an approach for the optimization of truss structures was presented considering the geometric stability of the structure. It was shown that it is possible to optimize structures using global stability constraints and modeling truss structures with frame elements, thus simplifying the process, refining the structural model, in return increasing the computational cost.

We believe that this work shows that the global stability of the structure is an important factor for structural optimization, since there is still no knowledge of efficient relaxation techniques to deal with this kind of constraint.

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