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Modelling P-SV seismic wave in homogeneous media for the Brazilian territory

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Abstract. In the literature, there are few works of seismic modeling in Brazil. We know that Brazil is on the South American plate making the country have very few seismic events, but it is not a guarantee for some faults can collapse at any time, and earthquakes of magnitude greater than normal are possible. Earthquakes are violent vibratory phenomena of short duration and, at times, of great intensity generated around a point source called procedente, in which large displacements of masses are produced, generating longitudinal and transverse waves. The longitudinal waves vibrate in the direction of the wave propagation and they are the first to be perceived. The tranverse waves vibrate perpendicular to the propagation direction and they delay in relation to longitudinal waves. In this context, the mathematical modeling of seismic waves allows the elaboration of theorical seismograms that allow predicting the characteristics of earthquakes, depending on the geogical conditions. This work describes the propagation of P (longitudinal waves) and SV (transverse waves) seismic waves modeled by equations of motion in elastical media, since the Earth behaves as a deformable material. So, our partial differential equations (PDE) describe the propagation of seismic waves in a vertical two-dimensional system (x and z coordinates), in homogeneous media, given by a source, assuming Neumann boundary conditions. For simplicity, the vertical two-dimensional domain is considered rectangular. The point source is modeled using a Gaussian-type function, located at a point inside the domain. To solve this PDE system, the finite difference method (FDM) is used. The boundary conditions are also discretized by FDM. The simulations will be carried out for different media, described by different densities in the terrestrial layers, and for different vertical positions for the point source (hypocenter), generating different seismological maps.

Keywords: Seismic waves, Finite Difference Method, Theoretical seismograms.

1 Introduction

In the context of seismology, the earthquakes are vibratory movements caused by internal crumbling of the earth's crust that propagates in all directions in the form of seismic waves, which reach the surface. These can be caused by the movement of tectonic faults, volcanic explosions, landslides, and controlled explosions with a variable focal depth. Seismic waves can be P (primary) waves that are the first to reach the surface. In the case of S (secondary) waves, the particles move in the direction perpendicular to the propagation path, generating SV waves (movement of the particles is totally vertical) and SH (movement that occurs in the horizontal plane), inducing shear deformations [1],[2]. Earthquakes cause great material damage, numerous human and economic loss. Since the earthquakes are natural disasters against which there is no defense or protection, so the mitigation of this type of phenomenon becomes important. In this work we use a two-dimensional model to describe the vertical displacements generated by P and SV body seismic waves. These simulations are applied to the Brazilian territory for lithospheric forces close to the surface. Our goal is to describe the vertical propagation of displacement from the source to the surface, so that we analyze the movements of the P and SV waves. In this work, we do not describe movements of superficial waves of the Love and Rayleigh type [3]. In Brazil, the most intense earthquakes occur at a depth of less than 35km and greater than 35km on the Peru-Brazil-Acre border [CS-USP],[5]. Note that such a model can also describe tremors caused by soil accommodation due to the filling of hydroelectric plants and

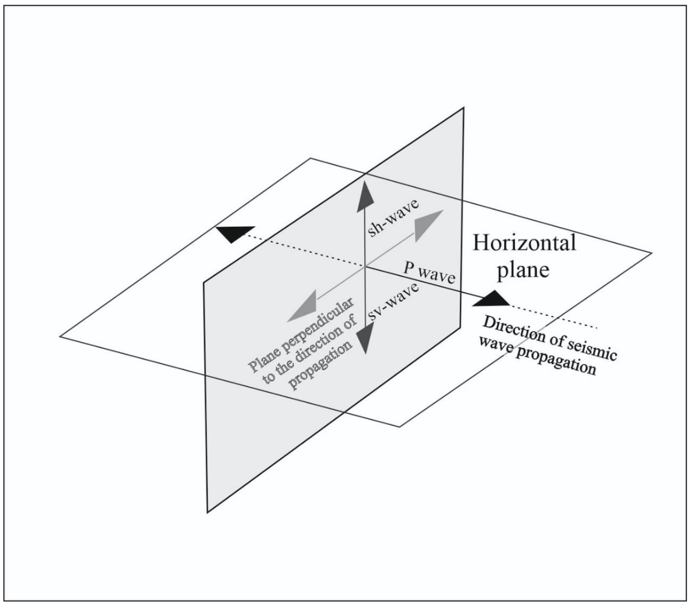

Figure 1. Seismic waves direction, SH: Displacements in the horizontal plane and SV: Dislocations perpendicular to those of SH.

Source:Adapted from [8]

reservoirs of mining debris [6].

2 Earthquake Modeling

In this section the modeling of seismic waves is developed and also the elements for solve the system equations of P-SV.

There are two types of body waves, P and S, the latter is polarized in SV and SH. The P-SV waves propagate on the plane $(X - Z)$ and cause displacements in the direction of X related to P and the direction Z related to SV. However, SH waves propagate in $(X - Z)$ and displacements occur only in the Y direction [7],as in the Figure 1.

2.1 P and SV seismic wave modeling with attenuation

First, the model will be carried out in a homogeneous and isotropic medium. The propagation of mechanical waves is polarized, being the P and SV waves described by a system of equations, and SH propagates along the axis that in our case is considered zero.

The deduction of this type of wave will be demonstrated below . Consider Newton's second law applied to a continuous medium. Consider the forces for an infinitesimal cube with coordinates (x, y, z). The set of forces on each side of a cube is represented as;

$$F_i = \partial_j \tau_{ij}\ dx_1\, dx_2\, dx_3 \tag{1}$$

There are also external forces acting on the cube, which are proportional to the volume of the material

$$(F)_i^{body} = f_i dx_1 dx_2 dx_3 \tag{2}$$

Using the Newton's second law, deriving the displacement twice, and substituting eq. (1) and eq. (2) in $ma = F$, where $m = \rho\, dx_1 dx_2 dx_3$ is the mass of the infinitesimal cube, with ρ the density , then

$$\rho\,\frac{\partial^2 u_i}{\partial t^2} = \partial_j \tau_{ij} + f_i + E_i \tag{3}$$

This is the fundamental equation in seismology[9], with more detailed and rigorous deduction of the equation eq. (3) is seen in [10]. In eq. (3), u_i are displacement, τ_{ij} are stress, f_i are a external attenuation force and E_i the components of the vector source $\vec{E}$

$$\begin{aligned} f_i &= -\alpha_i \frac{\partial u_i}{\partial t} \\ \vec{E} &= E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \end{aligned} \tag{4}$$

Then, using the relationship between stress and strain, which is given by in terms of the displacement u_i, given by Hooke's general law, for an isotropic medium, we have that:

$$\tau_{ij} = \lambda \delta_{ij} \partial_k u_k + \mu(\partial_i u_j + \partial_j u_i) \tag{5}$$

where α_i are attenuation constants, λ and μ are the Lame parameters and δ_{ij} is the Kronecker delta. Equations eq. (3), eq. (4) and eq. (5) are used to model the propagation of seismic waves, stress and displacement .For the P-SV seismic waves, only the X and Z plane will be to model the P and SV seismic waves, so on the Y axis, the displacement is equal to zero [11]. So considering $u = (u_x, 0, u_z)$ and $\vec{E} = (E_x, 0, E_z)$ we obtain, eq. (6). For our work, we use the components of the source vector like, $E_x = Ae^{-\kappa(x-x_0)^2}$ and $E_z = Ae^{-\kappa(z-z_0)^2}$, where $A = ae^{-c(t-t_0)^2}$ known as the Gaussian function [11], [7] and [12] , a is the source force, c is the speed of attenuation of the seismic source in time and κ is the speed of attenuation of the seismic source in space.

$$\begin{aligned} \rho\frac{\partial^2 u_x}{\partial t^2} &= (\lambda+2\mu)\frac{\partial^2 u_x}{\partial x^2} + \mu\frac{\partial^2 u_x}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u_z}{\partial z \partial x} - \alpha_1 \frac{\partial u_x}{\partial t} + E_x \\ \rho\frac{\partial^2 u_z}{\partial t^2} &= (\lambda+2\mu)\frac{\partial^2 u_z}{\partial z^2} + \mu\frac{\partial^2 u_z}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 u_x}{\partial z \partial x} - \alpha_2 \frac{\partial u_z}{\partial t} + E_z \end{aligned} \tag{6}$$

These system of equations eq. (6), describe the attenuated P-SV seismic waves with source. In the next section we will see the characteristics of the domain on its borders.

2.2 Geometric Domain of the Study

The choice of the domain is a very relevant topic in the study since the model will describe earthquakes near the surface. Earthquakes in Brazil occur at a depth less than 35km, small when compared to the Earth's radius of 6,371km. The figure3 show the rectangular domain that is very close to the free surface, where the source will be placed. In a retangular domain is adequate the use of a Cartesian system to model the propagation of the source.

2.3 Initial and boundary conditions

In Figure 2, it shows the four bondaries, which can be understood as a domain located within the surface layer of the earth. For the boundary conditions B1, B2, B3 and B4, in Figure3, we use free conditions. Neumann's conditions allow seismic waves to pass through the contour, avoiding artificial reflections on the problem, namely,

$$\begin{aligned} \frac{\partial u_x(B1)}{\partial n} = \frac{\partial u_x(B2)}{\partial n} = \frac{\partial u_x(B3)}{\partial n} = \frac{\partial u_x(B4)}{\partial n} = 0 \\ \frac{\partial u_z(B1)}{\partial n} = \frac{\partial u_z(B2)}{\partial n} = \frac{\partial u_z(B3)}{\partial n} = \frac{\partial u_z(B4)}{\partial n} = 0 \end{aligned} \tag{7}$$

For the initial condition, the medium is considered without changes at the start, as well

$$u_x(x,z,t) = 0 \qquad \text{and} \qquad u_z(x,z,t) = 0 \qquad \text{in} \qquad t_0 = 0 \tag{8}$$

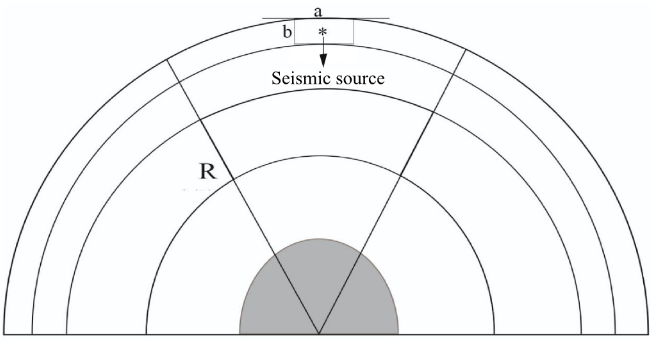

Figure 2. Location of the domain with lengths **a** and **b** with respect to the radius of the earth **R**
Source: Author

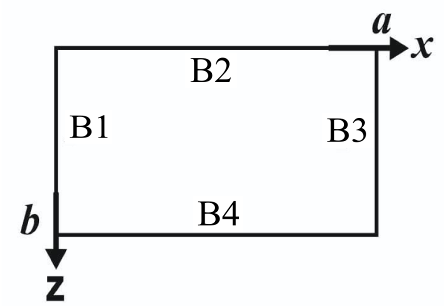

Figure 3. Domain representation, with borders B1, B2, B3 and B4 of sizes a and b.
Source: Author

2.4 Discretization of the Model

For the domain, where the finite difference method will be applied, a rectangular spatial discretization is used. The eq. (6) is discretized using second order finite differences with respect to space and time, like proposed in [13],

$$\begin{aligned} {u_x}_{i,k}^{l+1} = \frac{1}{A_{P1}}\left[A_{E1}{u_x}_{i+1,k}^{l+1} + A_{W1}{u_x}_{i-1,k}^{l+1} + A_{N1}{u_x}_{i,k+1}^{l+1} + A_{S1}{u_x}_{i,k-1}^{l+1} + b1 + ACu_z^{l+1}\right] + \\ + a\exp^{-c(l+1)^2}\exp^{-\kappa(i-i_0)^2} \end{aligned} \tag{9}$$

where;

$$\begin{aligned} A_{P1} &= \frac{\rho}{\triangle t^2} + \frac{3\alpha_1}{2\triangle t} + \frac{2(\lambda+2\mu)}{\triangle x^2} + \frac{2\mu}{\triangle z^2} \\ A_{E1} &= A_{W1} = \frac{\lambda+2\mu}{\triangle x^2} \\ A_{N1} &= A_{S1} = \frac{\mu}{\triangle z^2} \\ b1 &= \left(\frac{2\rho}{\triangle t^2} + \frac{4\lambda}{2\triangle t}\right){u_x}_{i,k}^{l} - \left(\frac{\rho}{\triangle t^2} + \frac{\lambda}{2\triangle t}\right){u_x}_{i,k}^{l-1} \\ ACu_z^{l+1} &= \frac{\lambda+\mu}{4\triangle x\triangle z}\left[{u_z}_{i+1,k+1}^{l+1} - {u_z}_{i+1,k-1}^{l+1} - {u_z}_{i-1,k-1}^{l+1} + {u_z}_{i-1,k+1}^{l+1}\right] \end{aligned} \tag{10}$$

The discretization of u_z is similar to discretization scheme as the equation u_x.

$$\begin{aligned} {u_z}_{i,k}^{l+1} = \frac{1}{A_{P2}}\left[A_{N2}{u_z}_{i,k+1}^{l+1} + A_{S2}{u_z}_{i,k-1}^{l+1} + A_{E2}{u_z}_{i+1,k}^{l+1} + A_{W2}{u_z}_{i-1,k}^{l+1} + b2 + ACu_x^{l+1}\right] + \\ + a\exp^{-c(l+1)^2}\exp^{-\kappa(k-k_0)^2} \end{aligned} \tag{11}$$

where;

$$
\begin{aligned}
A_{P2} &= \frac{\rho}{\triangle t^2} + \frac{3\alpha_2}{2\triangle t} + \frac{2(\lambda+2\mu)}{\triangle z^2} + \frac{2\mu}{\triangle x^2} \\
A_{E2} &= A_{W2} = \frac{\mu}{\triangle x^2} \\
A_{N2} &= A_{S2} = \frac{\lambda+2\mu}{\triangle z^2} \\
b2 &= \left(\frac{2\rho}{\triangle t^2} + \frac{4\lambda}{2\triangle t}\right) u_z{}^{l}_{i,k} - \left(\frac{\rho}{\triangle t^2} + \frac{\lambda}{2\triangle t}\right) u_z{}^{l-1}_{i,k} \\
ACu_x^{l+1} &= \frac{\lambda+\mu}{4\triangle x\triangle z}\left[u_x{}^{l+1}_{i+1,k+1} - u_x{}^{l+1}_{i+1,k-1} + u_x{}^{l+1}_{i-1,k-1} - u_x{}^{l+1}_{i-1,k+1}\right]
\end{aligned} \tag{12}
$$

The equations eq. (9) and eq. (11) were obtained by an implicit scheme. The discretization of the initial conditions eq. (8) are given by

$$
u_x{}^{1}_{i,k} = 0 \qquad \text{and} \qquad u_z{}^{1}_{i,k} = 0 \tag{13}
$$

For Neumann boundary conditions given in eq. (7), the derived terms are discretized by the central finite difference method as follows

$$
\begin{aligned}
\left.\frac{\partial u_x}{\partial n}\right|_{B1}^{l+1} = \left.\frac{\partial u_x}{\partial n}\right|_{B3}^{l+1} = \frac{u_x|_N^{l+1} - u_x|_S^{l+1}}{2\Delta z} = 0 \Rightarrow u_x|_N^{l+1} = u_x|_S^{l+1} \\
\left.\frac{\partial u_x}{\partial n}\right|_{B2}^{l+1} = \left.\frac{\partial u_x}{\partial n}\right|_{B4}^{l+1} = \frac{u_x|_E^{l+1} - u_x|_W^{l+1}}{2\Delta x} = 0 \Rightarrow u_x|_E^{l+1} = u_x|_W^{l+1}
\end{aligned} \tag{14}
$$

The same way for u_z.

3 Conclusions

The discretized equations eq. (9) and eq. (11), together with the initial conditions eq. (13) and boundary conditions eq. (14), describe the vertical displacements of P and SV waves in isotropic media. We will do simulations of these equations in different scenarios. We will study the propagation of these waves when the source of the tremor occurs at different depths, in 10km, 50km and 100km. We will also do simulations for the propagation of these waves in non-homogeneous media, considering different layers, with constant densities; or studies with media with varying densities.

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