

# Numerical analysis of a beam with and without viscoelastic treatment subject to cyclic loading in the time domain

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**Abstract.** It is well-known that the use of viscoelastic materials has been one efficient strategy to minimize fatigue in structures. They have the characteristic of absorbing part of the vibratory energy of mechanical systems and dissipating it in the heat form. Thus, the lower the vibrations produced, the lower the cyclic stresses and consequently, the longer the beam supports the loading, increasing its fatigue life. Thus, in the context of fatigue analysis, this work aims to analyze two scenarios: (1) the estimation of the fatigue life of an untreated beam made of 1020 steel subjected to cyclic loadings; (2) the analysis of a beam treatment with passive constraining viscoelastic layers. For this, the analysis of the results is performed in the time domain using the finite element method. For the viscoelastic structure, a new fractional derivative model (FDM) is used to describe the time-domain behavior of the viscoelastic part. For the fatigue analysis, the Rainflow method is used herein, based on the stress-strain behavior of the material. Using this method, it was possible to determine the accumulated damage by the Palmgren-Miner technique and, consequently, the fatigue life of the beam with and without viscoelastic damping.

**Keywords:** fatigue analysis, viscoelastic materials, finite element, fractional derivative model.

## 1 Introduction

Fatigue can be defined as a progressive failure in a component under repetitive, cyclic or oscillating random loadings (Bosco Júnior, 2007 [1]). According to recent studies, fatigue failures of mechanical components and equipments represent almost all of the problems of engineering systems in U.S. However, many researches state that, these failures can be reduced by 30% with the use of specialized control technology to increase the life of the component subjected to fatigue (Bosco Junior, 2007 [1]). For instance, this technology consists of eliminating undesirable vibrations or at least reducing their effects on the structure. Therefore, it is essential to understand the control techniques and include vibration analysis as an integral part of the design of a structure.

Several vibration control techniques have been proposed in the last decades, and they can be divided into active, passive or semi-active control methods (Rao, 2003 [2]). Active control strategies involve the use of elements such as actuators and microprocessors to produce an out-of-phase signal to electronically cancel the undesirable disturbance. Traditional passive control methods use normally absorbers, barriers, mufflers and silencers. In semi-active methods, active control is used to improve the damping properties of passive elements, and can be applied in the form of electrorheological fluids (ER), magneto-rheological (MR) or active constraining layer damping (ACLD) using piezoelectric materials (Rao, 2003 [2]).

Among the passive control techniques available in the open literature, those that use viscoelastic materials have a great interest from academics and industrials, due to their low cost of application and maintenance, in addition to their efficiency and robustness for a large frequency band (Ramos, 2014 [3]). Thus, it has been motivated its use in a variety of applications. This is due to the fact that these materials have the characteristic of absorbing part of the vibratory energy of mechanical systems and dissipating it in a heat form (de Lima, 2007 [4]).

Thus, in quest for fatigue, it is not difficult to expect that, the lower the vibrations produced, the lower the cyclic stresses and consequently, the longer the fatigue life of a structure subjected to this loading condition. Hence, the

viscoelastic materials can be used in such situations with advantages, but few works have been addressed this study, especially for dealing with the fatigue analysis of viscoelastic systems in the time domain, which motivates the present study.

The first models to describe the behavior of viscoelastic materials began to be developed at the end of the 19th century and are simpler one-dimensional models, represented by associations of springs and shock absorbers. These are called Maxwell, Kelvin-Voigt, and Zener models (also called the standard linear models). Maxwell and Kelvin-Voigt's formulations, however, are not able to represent with accuracy the dissipation mechanism of the viscoelastic materials, and therefore, Zener's model can be considered the first approximation to be used to overcome this drawback, but at the cost of great number of variables to be identified (de Lima, 2007 [4]). Another approach, similar to what is observed by Hooke's law for elasticity, is to relate stress and strain through a complex modulus. For viscoelasticity, however, this modulus contains an imaginary component. The complex modulus, despite being a widely used method, has some limitations: it is a non-causal model and unique to the frequency domain (Bagley and Torvik, 1983 [5]). Thus, the improved Fractional Derivative Model (FDM) developed by the authors in [Cunha-Filho, 2019 [6]] seems to be an interesting strategy to describe the behavior of viscoelastic materials in the context of the present study, since it requires few parameters and the stress field of the viscoelastic part is not self-dependent as in the classical FDM [Nashif et al., 1985 [7]]. This new approach has been shown to be an important tool, still relatively new and little explored, that is capable of generating good results when compared to experiments (Cunha-Filho, 2019 [6]).

Therefore, the objective of this work is to numerically analyze a sandwich beam with viscoelastic material to estimate its fatigue life in time-domain. It is used herein a simple beam made of 1020 steel and a sandwich beam. To perform the numerical analyses, it was necessary to create a finite element (FE) model in the time domain to incorporate the FDM model and the mechanisms necessary to estimate the fatigue, such as the Rainflow method and the Palmgren-Miner technique. Thus, it is investigate the possibility of increasing the fatigue life of the beam by adding the passive constraining layer.

## **2 Modes of failure in structures**

Fatigue can be defined as a progressive failure in a component under repetitive, cyclic or oscillating random loads (Bosco Júnior, 2007 [1]; Gonçalves L.K.S et al.; 2019 [8]). In functional terms, the failure fails to meet and perform the function for which it was designed. It can be placed as the loss of the ability to perform the required function. In physical terms, functional failure occurs because the system's ability to withstand a given demand has been exceeded, which in structural problems can be placed in the form that the resistance of the system,  $R$ , is not sufficient to support the request,  $S$ . Thus, the failure is defined by the event  $S > R$  (Da Rosa, 2002 [9]).

Also, failure are time-dependent or time-independent. Failure modes considered to be time-independent are those that do not consider the lifetime or use of the component. These are treated as deterministic and occur when the structure is subjected to a request overload, and this exceeds the material limit. Failure modes considered time-dependent are those where the time of use and life of the component influences the response and resistance of the structure. It is characterized by aging or wear of the material where there is a degradation of the original properties of the same (Da Rosa, 2002 [9]).

In the present study, time-dependent failures were treated for which the fatigue life is altered when time changes, generating, as it was said, a change in the resistance of the material in use. In addition, there is a need to include the time-dependent viscoelastic behavior on the system.

## **3 Review of theoretical foundations**

### **3.1 Sandwich viscoelastic beams**

Sandwich beams are structures composed of two thin faces of elastic material, separated by a viscoelastic core, having a lower stiffness and lower strength (de Lima, 2007 [4]). The use of this type of structure has been increased in the last decades due to its ability to combine high stiffness to bending and low weight. The main function of the faces in this type of structure is to provide the required stiffness and shear and therefore withstand the bending and shear loads in the plane. Typical materials used in the making of faces include aluminum alloys,

titanium, steel, fiber-reinforced plastics and plywood. In the case of the nucleus, foam polymers, synthetic rubbers, low density wood and inorganic cements can be used (from Lima, 2007 [4]). In the figure below, the illustration of the beam with the three layers is shown.

At this time, it is important to emphasized that, the FE model of the sandwich beam depicted in Fig. 1 and used in the present study has been fully developed in the work by Guedri et al. (2009). However, it is important to note that, the element used herein is formed by two nodes and ten degrees of freedom per element, as shown in the same figure.

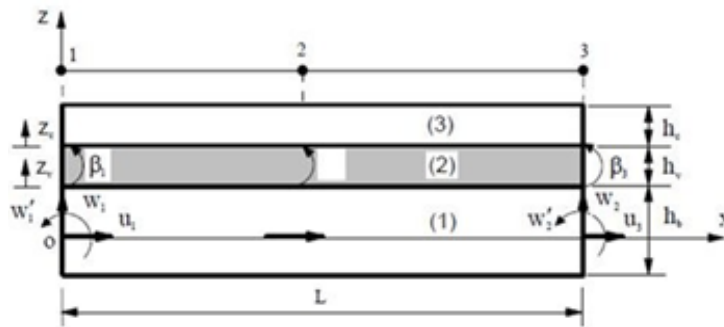


Figure 1. Illustration of a three-layer sandwich beam.

Source: Adapted from Lima, 2007 [4]

### 3.2 Fundamentals of Linear Viscosity

Sandwich beams are structures composed of two thin faces of elastic material, separated by a viscoelastic core, having a lower stiffness and lower strength (de Lima, 2007 [4]). The use of this type of structure has been increased in the last decades due to its ability to combine high stiffness to bending

Viscoelastic materials are some glasses and polymers that, when subjected to mechanical excitations, deform exhibiting elastic and viscous behaviors. The challenge when modeling the behavior of these materials is the fact that their properties are dependent on environmental and operational factors, especially temperature and excitation frequency. Figure 2 illustrates the influence of temperature, it is possible to observe that viscoelastic materials presents four different states as their temperatures varies: the vitreous state, in which the storage module has a high value, varying little with temperature, and the loss factor increases substantially; the transition state, where there is a considerable decrease in the storage module, and the loss factor reaches its maximum value; the rubber state, in which there is little variation of the properties of the material with temperature; and fluid state, which is typical of few materials and usually has little applicability in projects (Nashif et al., 1985 [7]).

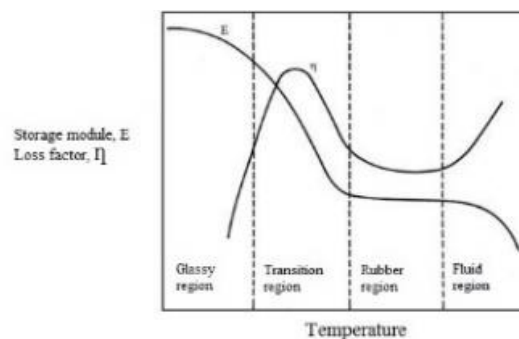


Figure 2. Variation of storage module and loss factor with temperature

Source: Adapted from Nashif et al. (1985) [7]

As for frequency, its main effect on the properties of the viscoelastic material is the increase in the storage module. The frequency has an opposite effect to that of the temperature. Nashif et al. (1985) [7] state that this phenomenon is one of the most important aspects of linear viscoelasticity, since it is possible to establish the Frequency-Temperature Superposition Principle (PSFT), which enables the transformation of the viscoelastic properties of the temperature to the frequency and the opposite is also true.

Regarding the time-domain behavior of a viscoelastic material, when mechanically requested, it behaves in an intermediate way between an elastic solid and a viscous fluid. The elastic materials obey Hooke's law and, through the application of a tension, they deform practically instantly and, when the load application ceases, the material recovers its former unformed shape. Viscous materials, on the other hand, deform continuously while the tension is applied and there is no recovery after withdrawal of the request. In viscoelastic materials, in turn, the application of the load results in an instantaneous elastic deformation and an anelastic deformation, which is dependent on time. Once the load is removed, the deformation is recovered and the material returns to its initial configuration, however, this recovery does not occur instantly either. In this way, viscoelasticity can be defined as the characteristic that certain materials have to deform through elastic and viscous components

## 4 Methodology

The methodology adopted in this work is represented by the flowchart shown in Fig. 3. The objective of numerical simulations is to demonstrate the efficiency of the viscoelastic treatment in relation to the untreated beam for the fatigue. The untreated beam is made of 1020 steel and the treated beam has three layers, as mentioned in Section 3.1. The face layers are made of 1020 steel, i.e. the beam-base layer and the constraining layer. The middle layer, is made of 3M ISD-112 material.

The numerical simulations consisted in the use of the FE model depicted in Fig. 1 to discretize the system in the time-domain. This model was used both for the beam with and without viscoelastic treatment. For the viscoelastic beam, another model was used, called FDM (Fractional Derivative Model) based on the fractional calculation between three and five parameters. This model has a great advantage, since the stresses and deformations in a viscoelastic material depend not only on the loading conditions at a given moment of time, but also on the entire history of the loads to which the material was subjected. In addition, the constituent law that describes the field of stresses of these materials has a self-dependence of tension, but recurrence eliminates this self-independence, making the numerical integration process more efficient, facilitating the implementation of computational code.

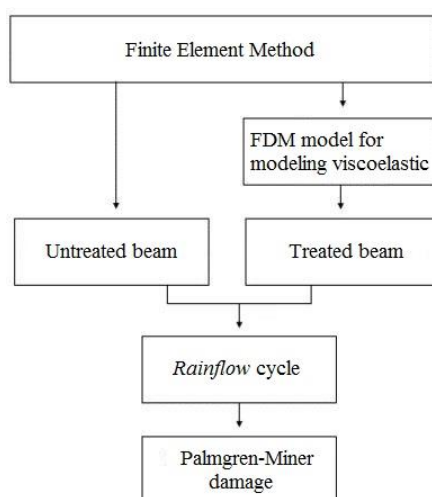


Figure 3. Methodology adopted in this work

To estimate the fatigue of the system with and without viscoelastic treatment, the Rainflow method was implemented herein. This method is most popular and probably the best method which has been used in the open

literature to compute the cycles (Passos, 2016 [10]).

By using the Palmgren-Miner technique, it is possible to determine the damage for a given cyclic loading during a period of time and, consequently, the fatigue life of the structure. This counting method is based on the stress-strain behavior of the material, since in a fatigue analysis it is intended to determine the relationship between stresses and strains, since fatigue damage is influenced by the alternation of plastic deformations, being, therefore, a hysteresis process (Passos, 2016 [10]). The counting process of this method allows the construction of a histogram of stress cycles, which is a representation of the distribution of the 48 amplitudes of tension in ranges (Ariduru, 2004 [11]; Bishop, 1989 [12]). The data from this histogram can then be used together with the Palmgren-Miner method to estimate the fatigue life of the structure. This process can be seen in Fig. 4.

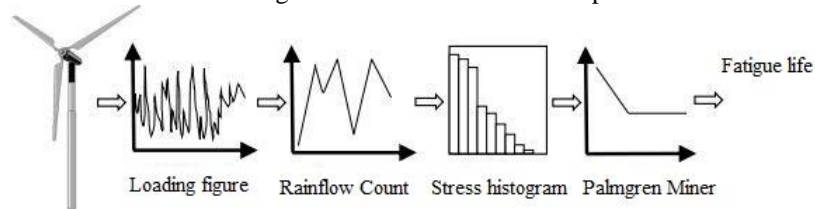


Figure 4. Steps for estimating fatigue life in the domain of time

Source: Adapted from Ariduru, 2004

Since its development in 1924 by Palmgren, and modified later in 1945 by Miner, the linear accumulation of Palmgren-Miner damage has been widely used in methods and criteria for fatigue analyses by random loadings. Even with its linear limitation and not considering the combined effects of loading, the Palmgren-Miner linear has extensive application due to its facilitation in numerical and analytical implementations (Bosco Júnior, 2007 [1]).

We analyzed the estimation of fatigue life of the beam with and without viscoelastic material and the influence of the temperature and the thicknesses on the estimation of fatigue life, using the proposed methodology.

## 5 Results and discussions

### 5.1 Estimation of the fatigue life

Using the proposed method, the following results were obtained in terms of the fatigue life of the beam with and without passive constraining damping layer:

Table 1. Estimation of Fatigue Life for the beam with and without viscoelastic treatment

Estimation of Fatigue Life in Cycles	
Beam without treatment	$6,07 \cdot 10^7$
Beam with treatment	$1,02 \cdot 10^8$

It can be clearly perceived that, the beam with treatment presents a higher fatigue life when compared to the beam without treatment, demonstrating the efficiency of the viscoelastic material as control strategy to increase the fatigue of structures. This is due to the fact that these materials have the characteristic of absorbing part of the vibration energy of the beam and dissipating it in the a heat form, as discussed before. Thus, the lower the vibrations produced, the lower the cyclic stresses and consequently, increasing its fatigue life.

### 5.2 Influence of the operation temperature on the fatigue life

Figure 5 shows the influence of the temperature of the viscoelastic part on the fatigue life of the sandwich

beam. It is evident that, as the temperature increases, the fatigue of the structure becomes smaller. It can be explained by a significant reduction on the performance of the viscoelastic material for higher values of operating temperature. Also, the time-domain responses of the sandwich beam was obtained for different temperatures, as shown in Fig. 6. It founds a good performed of the viscoelastic system for lower values of operating temperature, as expected.

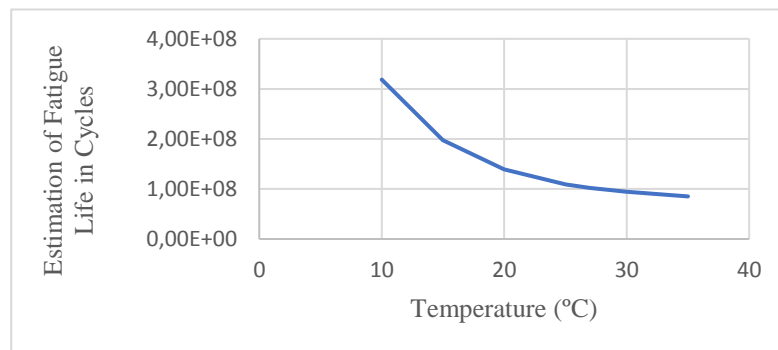


Figure 5. Influence of Temperature on Fatigue Life Estimation

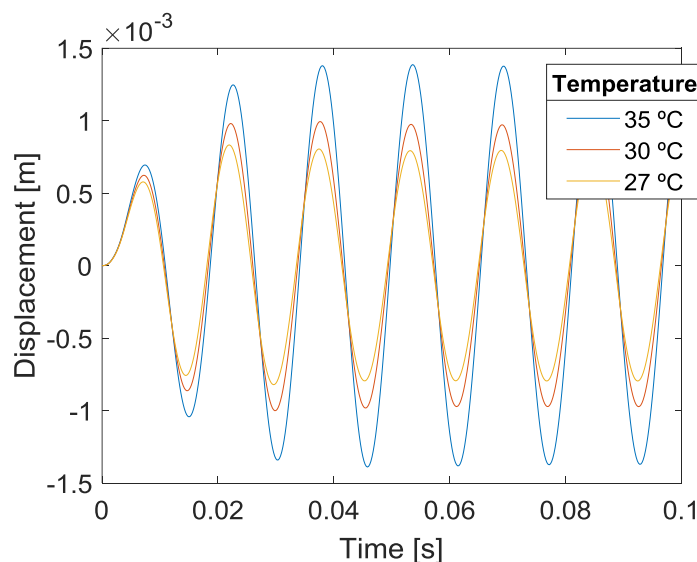


Figure 6. Time-domain responses of the sandwich beam for various values of temperature

### 5.3 Influence of the thicknesses on the fatigue life

Here, it is interesting to investigate the influence of the layer thicknesses on the fatigue life, as shown in Table 2. It can be concluded that, the thicknesses of the layers affect significantly the fatigue life of the beam.

Table 2. Parametric study.

Thickness	Life Estimation (Cycles)	
	Before	Increase 10%
Base Layer	3, $19 \cdot 10^8$	5, $50 \cdot 10^8$
Viscoelastic Layer	3, $19 \cdot 10^8$	3, $75 \cdot 10^8$
Restricting Layer	3, $19 \cdot 10^8$	3, $34 \cdot 10^8$

## 6 Conclusions

The results obtained in the present study has demonstrated that, in terms of fatigue, the use of viscoelastic materials applied as passive constraining damping layers, is efficient to increase the fatigue life of engineering structures. In this study, it has been used a sandwich viscoelastic beam, which has a much higher efficiency when compared to the beam without treatment due to an increasing in its fatigue life. This is due to the fact that viscoelastic materials have the characteristic of absorbing part of the vibratory energy of the beam. Thus, the lower the vibrations produced, the lower the cyclic stresses and consequently, the longer the beam supports the loading, increasing the fatigue life of it. Additionally, it can be perceived a significant influence of the operation temperature of the viscoelastic part on the fatigue: as the temperature of the viscoelastic increases, the fatigue life of the beam becomes smaller due to a loss of efficiency of the viscoelastic. Moreover, it was shown that, an increasing in the thicknesses of the layers, mainly of the base and constraining layers, leads to reasonable increasing in the fatigue life of the beam.

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## References

- [1] Bosco Junior, R., *Análise Numérico-Experimental de Componentes Sujeitos à Fadiga Por Solicitações Aleatórias - Avaliação de Modelos*. Master thesis, Federal University of Santa Catarina, 2007.
- [2] Rao, M. D., “Recent applications of viscoelastic damping for noise control in automobiles and commercial airplanes”. *Journal of Sound and Vibration*, vol. 262, pp. 457-474, 2003.
- [3] Ramos, A.P.R., *Contribuição à análise de fadiga de sistemas estruturais incorporando materiais viscoelásticos*. PhD thesis, Federal University of Uberlândia, 2014.
- [4] de Lima, A. M. G., *Modélisation et Optimisation Robuste de Systèmes Mécaniques en Présence d’Amortissement Viscoélastique*. PhD thesis, Federal University of Uberlândia, 2007.
- [5] Bagley, R. L. and Torvik, P. J., “Fractional calculus—A different approach to the analysis of viscoelastically damped structures”. *AIAA Journal*, vol. 21, n. 5, pp. 741–748, 1983
- [6] Cunha-Filho A. G., *Abordagem Transiente sobre os Efeitos do Amortecimento Viscoelástico na Estabilidade Aeroelástica de Estruturas Aeronáuticas*. PhD thesis, Federal University of Uberlândia, 2019.
- [7] Nashif, A. D. et. al., *Vibration Damping*. John Wiley & Sons, Inc., 1985
- [8] Gonçalves L. K. S., Rosa U. L. and de Lima A. M. G., “Fatigue damage investigation and optimization of a viscoelastically damped system with uncertainties”. *Journal of the Brazilian Society of Mechanical*, vol. 41, n. 9, p. 382, 2019
- [9] da Rosa, E., “Análise de Resistência Mecânica”. *Apostila da disciplina de Mecânica da Fratura e Fadiga*, 2002.
- [10] Passos, J. P., *Estimativa da vida em fadiga através da análise dinâmica global-local no domínio da frequência*. Master thesis, Federal University of Rio Grande do Sul, 2016.
- [11] Ariduru, S., *Fatigue Life Calculation by Rainflow Cycle Counting Method*. Master thesis, The Graduate School of Natural and Applied Sciences of Middle East Technical University, 2004.
- [12] Bishop, N. W. M. and Sherratt, F., “Fatigue life prediction from power spectral density data”. *Environmental Engineering*, vol. 2, n. 1-2, 1989.